State whether each sentence is *true* or *false*. If *false*, replace the underlined word or phrase to make a true sentence.

1. Any segment with both endpoints on the circle is a radius of the circle.

SOLUTION:
false; chord

ANSWER:
false; chord

2. A chord passing through the center of a circle is a diameter.

SOLUTION:
true

ANSWER:
true

3. A central angle has the center as its vertex and its sides contain two radii of the circle.

SOLUTION:
true

ANSWER:
true

4. An arc with a measure of less than 180° is a major arc.

SOLUTION:
false; minor arc

ANSWER:
false; minor arc

5. An intercepted arc is an arc that has its endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle.

SOLUTION:
true

ANSWER:
true

6. A common tangent is the point at which a line in the same plane as a circle intersects the circle.

SOLUTION:
false; point of tangency

ANSWER:
false; point of tangency

7. A secant is a line that intersects a circle in exactly <u>one</u> point.

SOLUTION:
false; two

ANSWER:
false; two

8. A secant segment is a segment of a <u>diameter</u> that has exactly one endpoint on the circle.

SOLUTION:
false; secant line

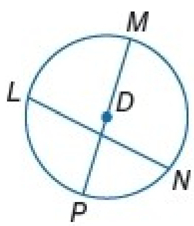
ANSWER:
false; secant line

9. Two circles are <u>concentric</u> circles if and only if they have congruent radii.

SOLUTION:
false; congruent

ANSWER:
false; congruent

For Exercises 10–12, refer to $\odot D$.



10. Name the circle.

SOLUTION:
The center of the circle is *D*. So, the name of the circle is $\odot D$.

ANSWER:
$\odot D$

11. Name a radius.

SOLUTION:
A radius is a segment with endpoints at the center and on the circle. Here, $\overline{DM}$ or $\overline{DP}$ are radii.

ANSWER:
$\overline{DM}$ or $\overline{DP}$

12. Name a chord that is not a diameter

SOLUTION:

A chord is a segment with endpoints on the circle. A diameter of a circle is a chord that passes through the center. Here, $\overline{LN}$ is a chord which is not a diameter.

ANSWER:

$\overline{LN}$

Find the diameter and radius of a circle with the given circumference. Round to the nearest hundredth.

13. C = 43 cm

SOLUTION:

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ 43 &= \pi d && \text{Substitution} \\ \frac{43}{\pi} &= d && \text{Divide each side by } \pi. \\ 13.69 &\approx d && \text{Use a calculator.} \end{aligned}$$

So, the diameter of the circle is about 13.69 centimeters.
The radius is half the diameter. So, the radius of the circle is about 6.84 cm.

ANSWER:

13.69 cm; 6.84 cm

14. C = 26.7 yd

SOLUTION:

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ 26.7 &= \pi d && \text{Substitution} \\ \frac{26.7}{\pi} &= d && \text{Divide each side by } \pi. \\ 8.50 &\approx d && \text{Use a calculator.} \end{aligned}$$

Therefore, the diameter is about 8.50 yards.
The radius is half the diameter. So, the radius of the circle is $\frac{1}{2}(8.50)$ or about 4.25 yards.

ANSWER:

8.50 yd; 4.25 yd

15. C = 108.5 ft

SOLUTION:

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ 108.5 &= \pi d && \text{Substitution} \\ \frac{108.5}{\pi} &= d && \text{Divide each side by } \pi. \\ 34.54 &\approx d && \text{Use a calculator.} \end{aligned}$$

So, the diameter of the circle is about 34.54feet.
The radius is half the diameter. So, the radius of the circle is $\frac{1}{2}$(34.54) or about 17.27 feet.

ANSWER:
34.54 ft; 17.27 ft

16. C = 225.9 mm

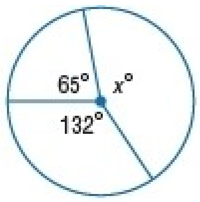
SOLUTION:

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ 225.9 &= \pi d && \text{Substitution} \\ \frac{225.9}{\pi} &= d && \text{Divide each side by } \pi. \\ 71.91 &\approx d && \text{Use a calculator.} \end{aligned}$$

So, the diameter of the circle is about 71.91 millimeters.
The radius is half the diameter. So, the radius of the circle is $\frac{1}{2}$(71.91) or about 35.95 millimeters.

ANSWER:
71.91 mm; 35.95 mm

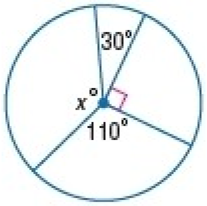
Find the value of *x*.

17. 

SOLUTION:
The sum of the measures of the central angles of a circle with no interior points in common is 360.

$$\begin{aligned} 65+132+x &= 360 && \text{Sum of Central Angles} \\ 197+x &= 360 && \text{Simplify.} \\ x &= 163 && \text{Subtract 197 from each side.} \end{aligned}$$

ANSWER:
163

18. 

SOLUTION:

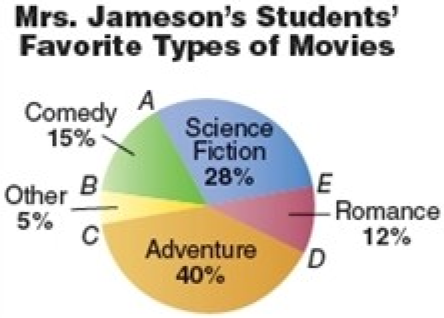
The sum of the measures of the central angles of a circle with no interior points in common is 360.

$$\begin{aligned} 90+30+110+x &= 360 && \text{Sum of Central Angles} \\ 230+x &= 360 && \text{Simplify.} \\ x &= 130 && \text{Subtract 230 from each side.} \end{aligned}$$

ANSWER:

130

19. **MOVIES** The pie chart below represents the results of a survey taken by Mrs. Jameson regarding her students' favorite types of movies. Find each measure.



a. $m\widehat{AE}$

b. $m\widehat{BC}$

c. Describe the type of arc that the category Adventure represents.

SOLUTION:

a.

$$\begin{aligned} m(\text{arc}\,AE) &= 28\% \text{ of } 360 \\ &= 0.28(360) \\ &= 100.8 \end{aligned}$$

b.

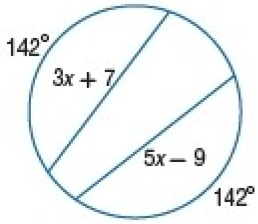
$$\begin{aligned} m(\text{arc}\,BC) &= 5\% \text{ of } 360 \\ &= 0.05(360) \\ &= 18 \end{aligned}$$

c. Adventure would have a measure of 40% of 360 or 144, so it is a minor arc.

ANSWER:

a. 100.8
b. 18
c. minor arc

Find the value of *x*.

20. 

SOLUTION:

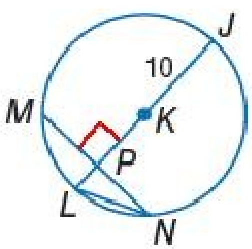
In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

$$\begin{aligned} 3x + 7 &= 5x - 9 && \text{Equal arcs cut equal chords} \\ 16 &= 2x && \text{Add } -3x \text{ and 9 to each side.} \\ 8 &= x && \text{Divide each side by 2.} \end{aligned}$$

ANSWER:

8

In $\odot K$, *MN* = 16 and $m\widehat{MN} = 98$. Find each measure. Round to the nearest hundredth.



21. $m\widehat{NJ}$

SOLUTION:

Since $\overline{LJ}$ is a diameter and $\overline{LJ} \perp \overline{MN}$, then arc *LNJ* is a semicircle and $\overline{LJ}$ bisects arc *MN*. So, *m*(arc *LN*) = $\frac{1}{2}m$(arc *MN*) or 49.

$$\begin{aligned} m(\text{arc } LN) + m(\text{arc } NJ) &= m(\text{arc } LNJ) && \text{Arc Addition Postulate} \\ 49 + m(\text{arc } NJ) &= 180 && m(\text{arc } LN) = 49,\ m(\text{arc } LNJ) = 180 \\ m(\text{arc } NJ) &= 131 && \text{Subtract 49 from each side.} \end{aligned}$$

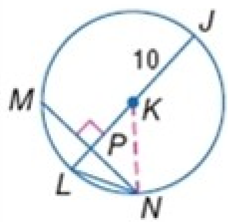
ANSWER:

131

22. *LN*

SOLUTION:

Draw $\overline{KN}$ to complete right triangle *KPN*.



$KJ = KL = KN = 10$ (all radii are equal)

$PN = \frac{1}{2}(16)$ or 8 (a diameter drawn perpendicular to a chord bisects the chord)

Use the Pythagorean Theorem to find *KP*.

$$\begin{aligned} KP^2 + PN^2 &= KN^2 && \text{Pythagorean Theorem} \\ KP^2 + 8^2 &= 10^2 && \text{Substitution} \\ KP^2 &= 36 && \text{Simplify.} \\ KP &= 6 && \text{Take the positive square root of each side.} \end{aligned}$$

Use the Segment Addition Postulate to find *PL* .

$$\begin{aligned} KP + PL &= KL && \text{Segment Addition Postulate} \\ 6 + PL &= 10 && KP = 6, KL = 10 \\ PL &= 4 && \text{Subtract 6 from each side.} \end{aligned}$$

Use right triangle *LPN* and the Pythagorean Theorem to find *LN*.

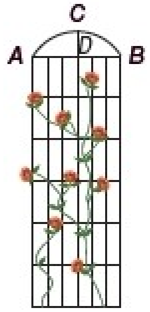
$$\begin{aligned} LN^2 &= PL^2 + PN^2 && \text{Pythagorean Theorem} \\ LN^2 &= 4^2 + 8^2 && PL = 4, PN = 8 \\ LN^2 &= 80 && \text{Simplify.} \\ LN &\approx 8.94 && \text{Take the positive square root of each side.} \end{aligned}$$

So, *LN* is about 8.94.

ANSWER:

8.94

23. **GARDENING** The top of the trellis shown is an arc of a circle in which $\overline{CD}$ is part of the diameter and $\overline{CD} \perp \overline{AB}$. If $\widehat{ACB}$ is about 28% of a complete circle, what is $m\widehat{CB}$?



SOLUTION:

Use a percent of a number problem to find m(arc ACB).

$$\begin{aligned} m(\text{arc}\,ACB) &= 28\%\text{ of }360 && \text{Original problem in equation form} \\ &= 0.28(360) && \text{Change percent to a decimal.} \\ &= 100.8 && \text{Multiply.} \end{aligned}$$

$\overline{CD}$ bisects arc ACB since $\overline{CD}$ is part of the diameter and $\overline{CD}$ is perpendicular to $\overline{AB}$.

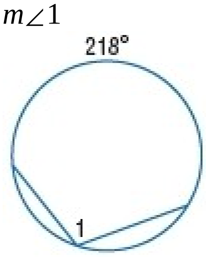
$$\begin{aligned} m(\text{arc}\,CD) &= \frac{1}{2}m(\text{arc}\,ACB) \\ &= \frac{1}{2}(100.8) \\ &= 50.4 \end{aligned}$$

ANSWER:

50.4

Find each measure.

24. $m\angle 1$



SOLUTION:

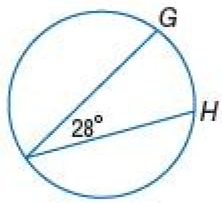
An inscribed angle equals one half the measure of its intercepted arc.

$$\begin{aligned} m\angle 1 &= \frac{1}{2}[218] && \text{Theorem 10.6} \\ &= 109 && \text{Multiply.} \end{aligned}$$

ANSWER:

109

25. $m\widehat{GH}$



SOLUTION:

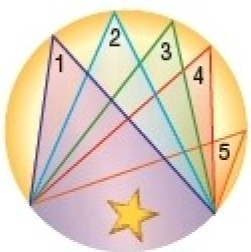
An inscribed angle equals one half the measure of its intercepted arc.

$$28 = \frac{1}{2}m(\text{arc}\,GH) \quad \text{Theorem 10.6}$$
$$56 = m(\text{arc}\,GH) \quad \text{Multiply each side by 2.}$$

ANSWER:

56

26. **MARKETING** In the logo, $m\angle 1 = 42$. Find $m\angle 5$.



SOLUTION:

By Theorem 10.7, the measures of two inscribed angles of a circle that intercept the same arc or congruent arcs are congruent. So, since $\angle 1$ and $\angle 5$ intercept the same arc on this circle, $m\angle 5 = m\angle 1$. Therefore, $m\angle 5 = 42$.

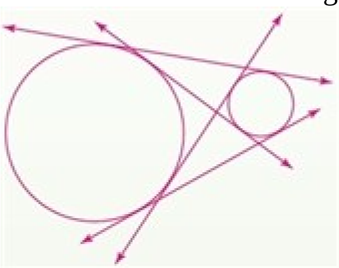
ANSWER:

42

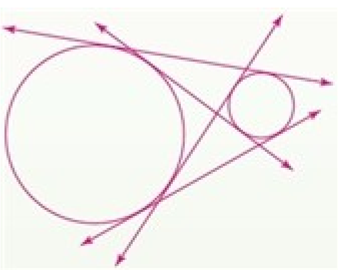
27. **SCIENCE FICTION** In a story Todd is writing, instantaneous travel between a two-dimensional planet and its moon is possible when the time-traveler follows a tangent. Copy the figures below and draw all possible travel paths.

SOLUTION:

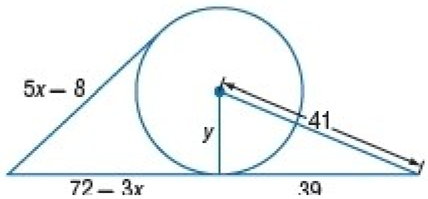
There are four common tangent lines that can be drawn for the two circles representing the planet and its moon.



ANSWER:



28. Find x and y. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.



SOLUTION:

Tangent segments drawn from the same exterior point are congruent.

$$\begin{aligned} 5x - 8 &= 72 - 3x && \text{Theorem 10.11} \\ 8x &= 80 && \text{Add 8 and 3x to each side.} \\ x &= 10 && \text{Divide each side by 8.} \end{aligned}$$

Since y is a radius, it is perpendicular to the tangent. Use the right triangle formed and the Pythagorean Theorem to find the value of y.

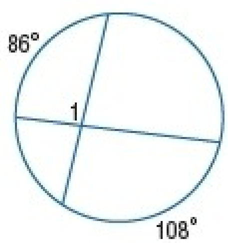
$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagorean Theorem} \\ y^2 + 39^2 &= 41^2 && \text{Substitution} \\ y^2 &= 160 && \text{Simplify.} \\ y &\approx 12.6 && \text{Take the positive square root of each side.} \end{aligned}$$

ANSWER:

$x = 10, y = 12.6$

Find each measure. Assume that segments that appear to be tangent are tangent.

29. $m\angle 1$



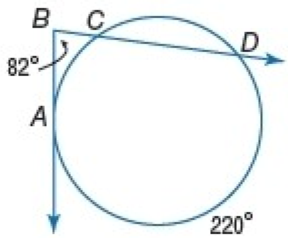
SOLUTION:

$$\begin{aligned} m\angle 1 &= \frac{1}{2}[86 + 108] && \text{Theorem 10.12} \\ &= \frac{1}{2}[194] && \text{Simplify.} \\ &= 97 && \text{Multiply.} \end{aligned}$$

ANSWER:

97

30. $m\widehat{AC}$

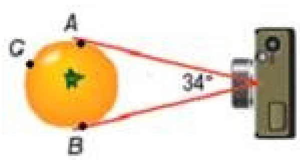


SOLUTION:

$$82 = \frac{1}{2}[220 - m(\text{arc}\,AC)] \quad \text{Theorem 10.14}$$
$$164 = 220 - m(\text{arc}\,AC) \quad \text{Multiply each side by 2.}$$
$$m(\text{arc}\,AC) = 56 \quad \text{Add } m(\text{arc}\,AC) \text{ and } -164 \text{ to each side.}$$

ANSWER:
56

31. **PHOTOGRAPHY** Ahmed needs to take a close-up shot of an orange for his art class. He frames a shot of an orange as shown below, so that the lines of sight form tangents to the orange. If the measure of the camera's viewing angle is 34°, what is $m\widehat{ACB}$?



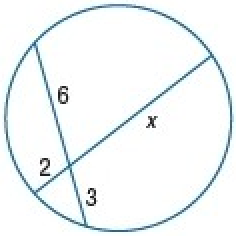
SOLUTION:
Major arc *ACB* shares the same endpoints as minor arc *AB*, so *m*(arc *ACB*) = 360 – *m*(arc *AB*).

$$34 = \frac{1}{2}[m(\text{arc}\,ACB) - m(\text{arc}\,AB)] \quad \text{Theorem 10.14}$$
$$34 = \frac{1}{2}[360 - m(\text{arc}\,AB) - m(\text{arc}\,AB)] \quad \text{Substitution}$$
$$68 = 360 - 2m(\text{arc}\,AB) \quad \text{Simplify and multiply each side by 2.}$$
$$-292 = -2m(\text{arc}\,AB) \quad \text{Subtract 360 from each side.}$$
$$146 = m(\text{arc}\,AB) \quad \text{Divide each side by } -2.$$

Therefore, *m*(arc *ACB*) = 360 – 146 or 214.

ANSWER:
214

Find *x*. Assume that segments that appear to be tangent are tangent.

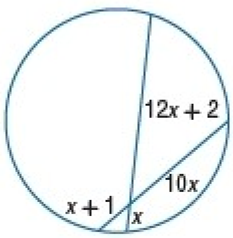
32. 

SOLUTION:

$$\begin{aligned} 2 \cdot x &= 6 \cdot 3 && \text{Theorem 10.15} \\ 2x &= 18 && \text{Multiply.} \\ x &= 9 && \text{Divide each side by 2.} \end{aligned}$$

ANSWER:

9

33. 

SOLUTION:

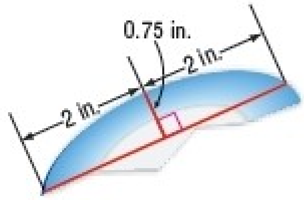
$$\begin{aligned} x(12x+2) &= 10x(x+1) && \text{Theorem 10.15} \\ 12x^2+2x &= 10x^2+10x && \text{Multiply.} \\ 2x^2-8x &= 0 && \text{Subtract } 10x^2 \text{ and } 10x \text{ from each side.} \\ 2x(x-4) &= 0 && \text{Factor.} \\ x &= 0, 4 && \text{Zero Product Property} \end{aligned}$$

The length of a segment must be a positive number, so $x = 4$.

ANSWER:

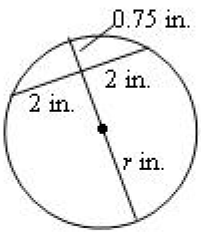
4

34. **ARCHAEOLOGY** While digging a hole to plant a tree, Henry found a piece of a broken saucer. What was the circumference of the original saucer? Round to the nearest hundredth.



SOLUTION:

Complete the circle represented by the plate and since the 0.75 inch segment is perpendicular to and bisects the chord, it will be a diameter.



Use the intersecting chords to find the radius of the circle. Let *r* be the radius of the circle. The length of the diameter is twice the radius, or 2*r*. The length of the longer segment for the diameter is given by 2*r* – 0.75.

$$\begin{aligned} 0.75(2r - 0.75) &= 2(2) && \text{Theorem 10.15} \\ 1.5r - 0.5625 &= 4 && \text{Multiply.} \\ 1.5r &= 4.5625 && \text{Add 0.5625 to each side.} \\ r &\approx 3.04 && \text{Divide each side by 1.5.} \end{aligned}$$

Use the radius to find the circumference.

$$\begin{aligned} C &= 2\pi r && \text{Circumference Formula} \\ &= 2\pi(3.04) && \text{Substitution} \\ &\approx 19.1 && \text{Use a calculator.} \end{aligned}$$

Therefore, the circumference of the original saucer was about 19.1 inches.

ANSWER:

19.1 in.

Write the equation of each circle.

35. center at (–2, 4), radius 5

SOLUTION:

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 && \text{Equation of a circle} \\ (x-(-2))^2 + (y-4)^2 &= 5^2 && (h, k) = (-2, 4), r = 5 \\ (x+2)^2 + (y-4)^2 &= 25 && \text{Simplify.} \end{aligned}$$

ANSWER:

$(x+2)^2 + (y-4)^2 = 25$

36. center at (1, 2), diameter 14

SOLUTION:

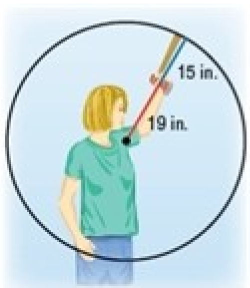
If the diameter is 14, then the radius is $\frac{1}{2}(14)$ or 7.

$$(x-h)^2+(y-k)^2=r^2 \quad \text{Equation of a circle}$$
$$(x-1)^2+(y-2)^2=7^2 \quad (h,k)=(1,2),\ r=7$$
$$(x-1)^2+(y-2)^2=49 \quad \text{Simplify.}$$

ANSWER:

$(x-1)^2+(y-2)^2=49$

37. **FIREWOOD** In an outdoor training course, Kat learns a wood-chopping safety check that involves making a circle with her arm extended, to ensure she will not hit anything overhead as she chops. If her reach is 19 inches, the hatchet handle is 15 inches, and her shoulder is located at the origin, what is the equation of Kat's safety circle?



SOLUTION:

The radius of Kat's safety circle is 19 + 15 or 34 inches. Write an equation using h = 0, k = 0, and r = 34.

$$(x-h)^2+(y-k)^2=r^2 \quad \text{Equation of a circle}$$
$$(x-0)^2+(y-0)^2=34^2 \quad h=0,\ k=0,\ r=34$$
$$x^2+y^2=1156 \quad \text{Simplify.}$$

ANSWER:

$x^2+y^2=1156$