Choose the term that best completes each sentence.

When a transformation is applied to a figure, and then another transformation is applied to its image, this is a(n) (composition of transformations, order of symmetries).

SOLUTION:

Composition of transformations; the order of symmetries is the number of times a figure maps to itself when rotated.

ANSWER:

composition of transformations

2. If a figure is folded across a straight line and the halves match exactly, the fold line is called the (line of reflection, line of symmetry).

SOLUTION:

Line of symmetry; the line of reflection is the line in which a figure is reflected.

ANSWER:

line of symmetry

3. A (dilation, glide reflection) enlarges or reduces a figure proportionally.

SOLUTION:

Dilation; a glide reflection is a composition of a translation followed by a reflection.

ANSWER:

dilation

4. The number of times a figure maps onto itself as it rotates from 0° to 360° is called the (magnitude of symmetry, order of symmetry).

SOLUTION:

Order of symmetry; the magnitude of symmetry is the smallest angle through which a figure can be rotated.

ANSWER:

order of symmetry

5. A (line of reflection, translation vector) is the same distance from each point of a figure and its image.

SOLUTION:

Line of reflection; a translation vector is a vector in which a translation maps each point to its image.

ANSWER:

line of reflection

6. A figure has (a center of rotation, symmetry) if it can be mapped onto itself by a rigid motion.

SOLUTION:

Symmetry; a center of rotation is the point through which an angle maps a point to its image.

ANSWER:

symmetry

7. A glide reflection includes both a reflection and a (rotation, translation).

SOLUTION:
Translation, rotations are not involved in a glide reflection.

ANSWER:
translation

8. To rotate a point (90°, 180°) counterclockwise about the origin, multiply the y-coordinate by –1 and then interchange the x- and y-coordinates.

SOLUTION:
90°; a 180 degree rotation can be done by multiplying the x- and y-coordinates by -1.

ANSWER:
90°

9. A (vector, reflection) is a congruence transformation.

SOLUTION:
Reflection; a vector is a quantity that has both magnitude and direction.

ANSWER:
reflection

10. A figure has (plane symmetry, rotational symmetry) if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

SOLUTION:
Rotational symmetry; a figure has plane symmetry if it can be mapped to itself by a reflection.

ANSWER:
rotational symmetry

Graph each figure and its image under the given reflection.

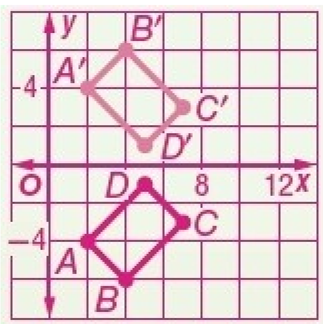
11. rectangle *ABCD* with *A*(2, –4), *B*(4, –6), *C*(7, –3), and *D*(5, –1) in the *x*-axis.

SOLUTION:

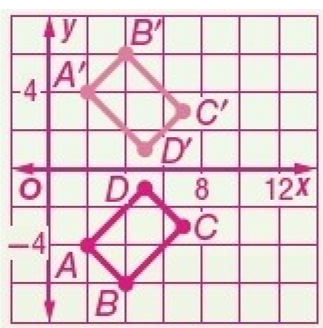
To reflect over the x-axis, multiply the y-coordinate of each vertex by –1.

$(x, y) \rightarrow (-x, y)$
$A(2, -4) \rightarrow A'(2, 4)$
$B(4, -6) \rightarrow B'(4, 6)$
$C(7, -3) \rightarrow C'(7, 3)$
$D(5, -1) \rightarrow D'(5, 1)$

Plot the points. Then connect the vertices, A', B', C', and D' to form the reflected image.



ANSWER:



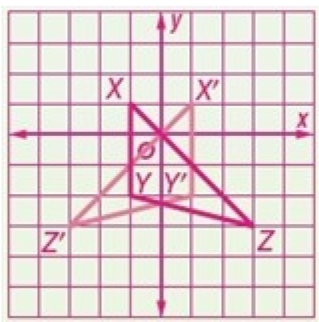
12. triangle XYZ with $X(–1, 1)$, $Y(–1, –2)$, and $Z(3, –3)$ in the y-axis.

SOLUTION:

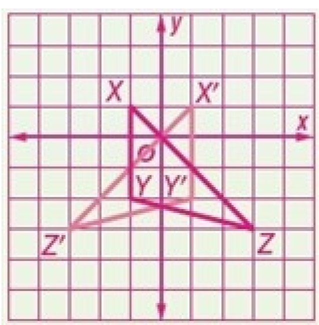
To reflect over the y-axis, multiply the x-coordinate of each vertex by –1.

$(x, y) \rightarrow (-x, y)$
$X(–1, 1) \rightarrow X'(1, 1)$
$Y(–1, –2) \rightarrow Y'(1, –2)$
$Z(3, –3) \rightarrow Z'(–3, –3)$

Plot the points. Then connect the vertices, X', Y', and Z' to form the reflected image.



ANSWER:



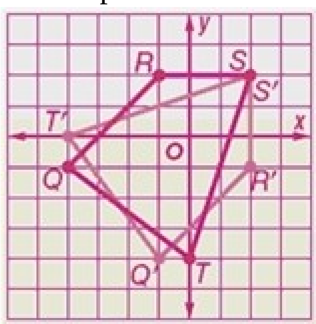
13. quadrilateral $QRST$ with $Q(-4, -1)$, $R(-1, 2)$, $S(2, 2)$, and $T(0, -4)$ in the line $y = x$.

SOLUTION:

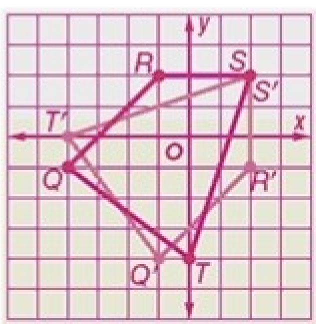
To reflect over the line $y = x$, interchange the x- and y-coordinates.

$(x, y) \rightarrow (-x, y)$
$Q(-4, -1) \rightarrow Q'(-1, -4)$
$R(-1, 2) \rightarrow R'(2, -1)$
$S(2, 2) \rightarrow S'(2, 2)$
$T(0, -4) \rightarrow T'(-4, 0)$

Plot the points. Then connect the vertices, Q', R', S', and T' to form the reflected image.



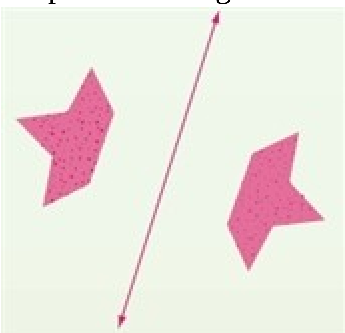
ANSWER:



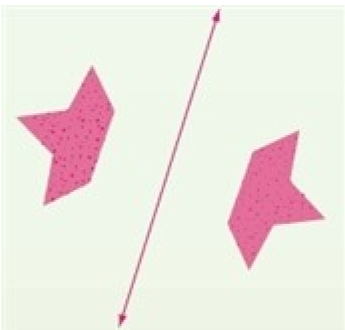
14. **ART** Anita is making the two-piece sculpture shown for a memorial garden. In her design, one piece of the sculpture is a reflection of the other, to be placed beside a sidewalk that would be located along the line of reflection. Copy the figures and draw the line of reflection.

SOLUTION:

The sidewalk will serve as a line of reflection in the art piece. In a line of reflection, each point of the preimage and its corresponding point on the image are the same distance from this line. So, to find the line of reflection, find the midpoint of the segments connecting corresponding points.



ANSWER:

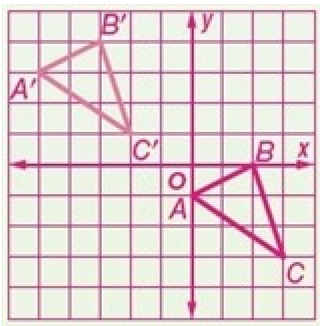


15. Graph ΔABC with vertices $A(0, –1)$, $B(2, 0)$, $C(3, –3)$ and its image along $\langle -5, 4 \rangle$.

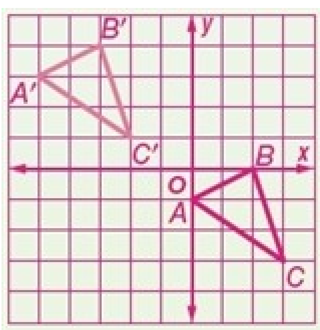
SOLUTION:

vertices $A(0, –1)$, $B(2, 0)$, $C(3, –3)$ along $\langle -5, 4 \rangle$

$A' = (–5, 3)$, $B' = (–3, 4)$, and $C' = (–2, 1)$



ANSWER:



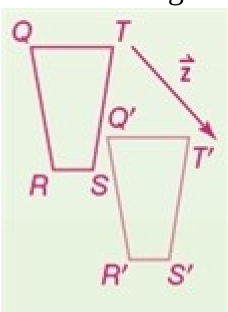
16. Copy the figure and the given translation vector Then draw the translation of the figure along the translation vector.

SOLUTION:

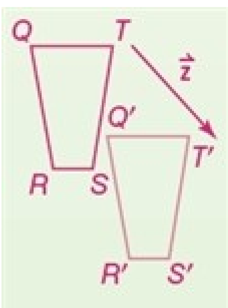
Step 1: Draw a line through each vertex parallel to vector $\vec{z}$.

Step 2 : Measure the length of vector $\vec{z}$. Locate point *Q*' by marking off this distance along the line through vertex *Q*, starting at *Q* and in the same direction as the vector.

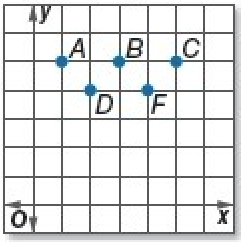
Step 3: Repeat Step 2 to locate points *R*', *S*', and *T*'. Then connect vertices *Q*', *R*', *S*', and *T*' to form the translated image.



ANSWER:



17. **DANCE** Five dancers are positioned onstage as shown. Dancers *B*, *F*, and *C* move along $\langle 0, -2 \rangle$ while dancer *A* moves along $\langle 5, -1 \rangle$. Draw the dancers' final positions.



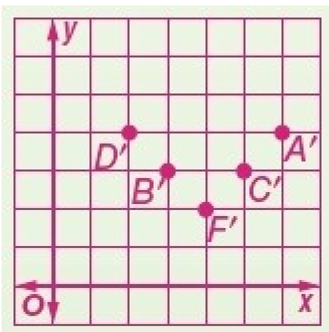
SOLUTION:

The first transformation is a translation along $\langle 0, -2 \rangle$, so $(x, y) \rightarrow (x, y - 2)$.

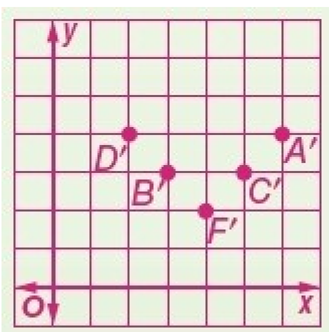
$B\ (3, 5) \rightarrow (3, 3)$
$F\ (4, 4) \rightarrow (4, 2)$
$C\ (5, 5) \rightarrow (5, 3)$

The next transformation is a translation along $\langle 5, -1 \rangle$, so $(x, y) \rightarrow (x + 5, y - 1)$.

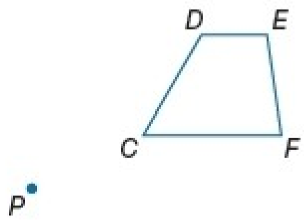
$A\ (1, 5) \rightarrow (6, 4)$



ANSWER:

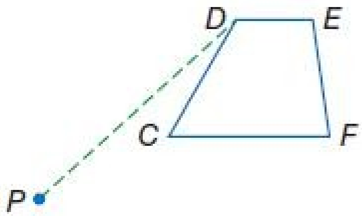


18. Copy trapezoid *CDEF* and point *P*. Then use a protractor and ruler to draw a 50° rotation of *CDEF* about point *P*.

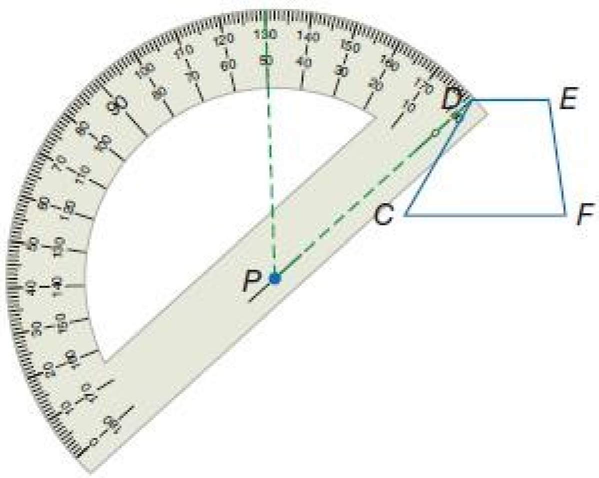


SOLUTION:

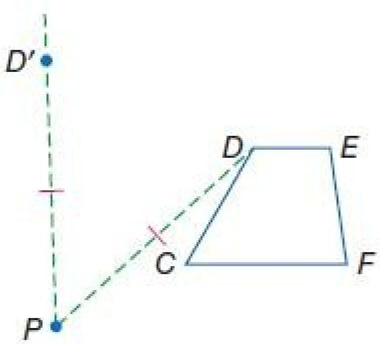
Step 1: Draw line *DP*.



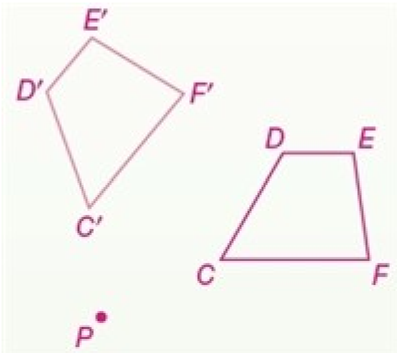
Step 2: Use a protractor to create a 50° angle with *DP*.



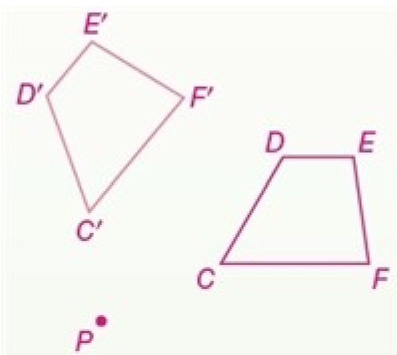
Step 3: Use a ruler to draw *D*' such that *DP* = *D'P*.



Step 4: Repeat Steps 1-3 for vertices *C*, *E*, and *F*, to finish the trapezoid.



ANSWER:



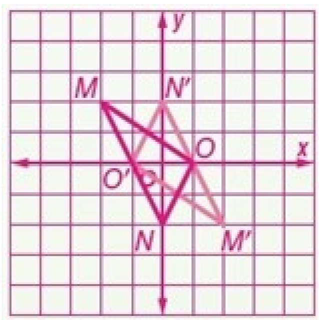
Graph each figure and its image after the specified rotation about the origin.

19. ΔMNO with vertices $M(–2, 2)$, $N(0, –2)$, $O(1, 0)$; 180°

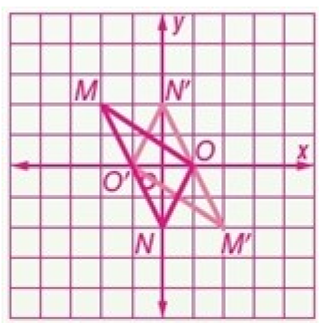
SOLUTION:

This transformation is a 180º rotation, so $(x, y) \rightarrow (–x, –y)$.

M (–2, 2) → (2, –2)
N (0, –2) → (0, 2)
O (1, 0) → (–1, 0)



ANSWER:

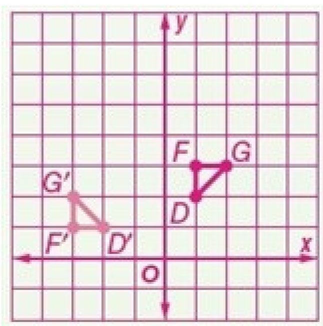


20. ΔDGF with vertices $D(1, 2)$, $G(2, 3)$, $F(1, 3)$; 90°

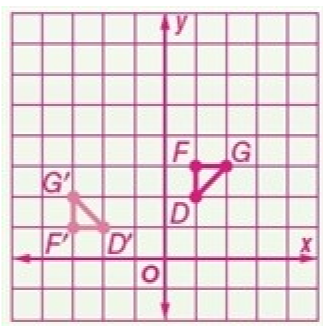
SOLUTION:

This transformation is a 90º rotation, so $(x, y) \rightarrow (-y, x)$.

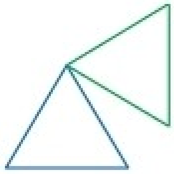
D (1, 2) → (–2, 1)
G (2, 3) → (–3, 2)
F (1, 3) → (–3, 1)



ANSWER:

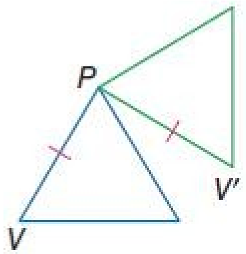


Each figure shows a preimage and its image after a rotation about a point *P*. Copy each figure, locate point *P*, and find the angle of rotation.

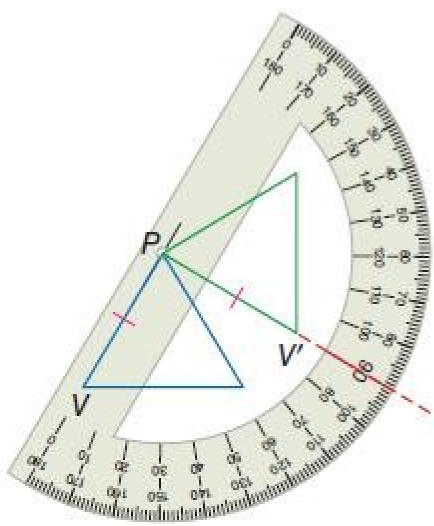
21. 

SOLUTION:

Use a protractor and ruler. Notice that $VP = V'P$. Then P is the center of rotation.

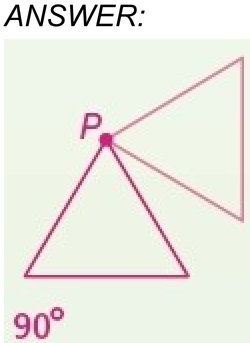


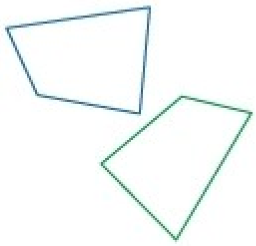
Use a protractor to determine the angle of rotation. Place the protractor on *VP* with the center point on *P*. Then determine the angle.



The angle is 90 degrees.

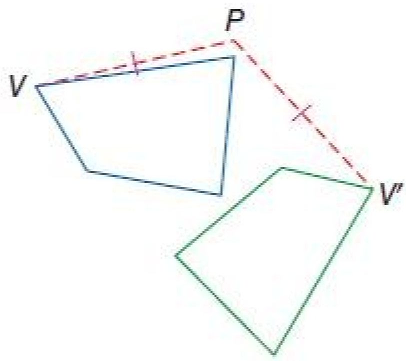
ANSWER:



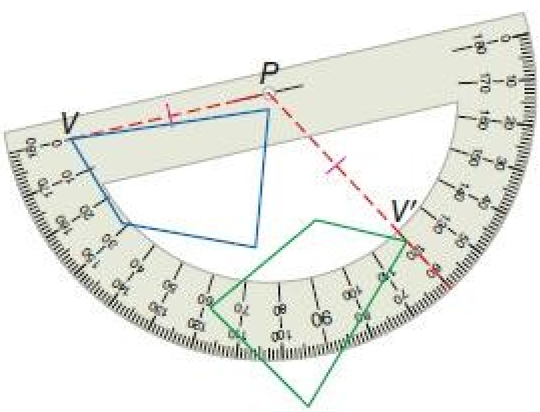
22. 

SOLUTION:

Use a protractor and ruler. Connect V and V'. Notice that $VP = V'P$. Then P is the center of rotation.

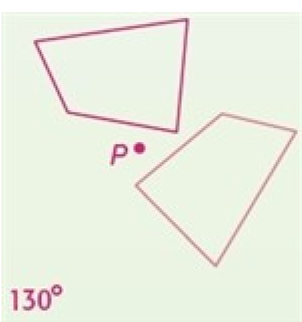


Use a protractor to determine the angle of rotation. Place the protractor on line VP with the center on P. Then determine the angle through point V'.



The angle of rotation is 130 degrees.

ANSWER:



Graph each figure with the given vertices and its image after the indicated transformation.

23. $\overline{CD}$: $C(3, 2)$ and $D(1, 4)$
Reflection: in $y = x$ Rotation: 270° about the origin.

SOLUTION:

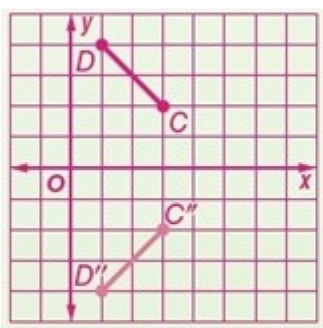
Step 1 reflection in $y = x$

$(x, y) \rightarrow (x, -y)$
$C(3, 2) \rightarrow (3, -2)$
$D(1, 4) \rightarrow (1, -4)$

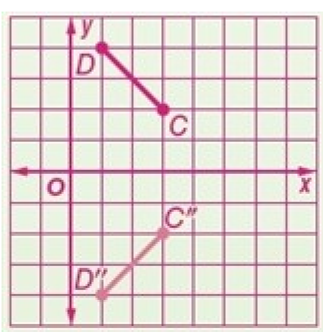
Step 2 rotation 270º about the origin

$(x, y) \rightarrow (y, -x)$
$C(3, 2) \rightarrow (2, -3)$
$D(1, 4) \rightarrow (4, -1)$

Step 3 Graph $\overline{CD}$ and its image $\overline{C''D''}$.



ANSWER:



24. $\overline{GH}$: $G(–2, –3)$ and $H(1, 1)$
Translation: along $\langle 4, 2 \rangle$ Reflection: in the x-axis

SOLUTION:

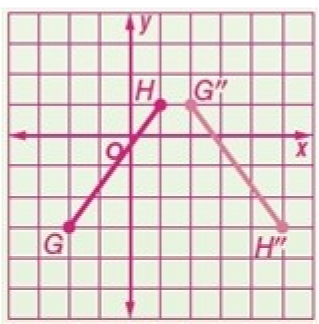
Step 1 translation along $\langle 4, 2 \rangle$

$(x, y) \rightarrow (x + 4, y + 2)$
$G(–2, –3) \rightarrow (2, –1)$
$H(1, 1) \rightarrow (5, 3)$

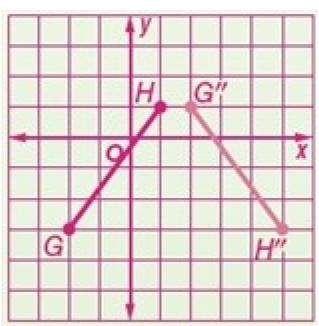
Step 2 Reflection in the x-axis

$(x, y) \rightarrow (x, –y)$
$G(–2, –3) \rightarrow (–2, 3)$
$H(1, 1) \rightarrow (1, –1)$

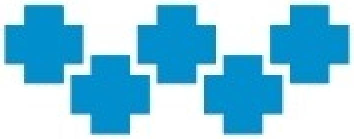
Step 3 Graph $\overline{GH}$ and its image $\overline{G''H''}$.



ANSWER:



25. **PATTERNS** Jeremy is creating a pattern for the border of a poster using a stencil. Describe the transformation combination that he used to create the pattern below.



SOLUTION:

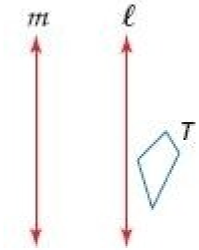
Look for a pattern from the first image to the second, then from the second to the third, and so on.

Sample answer: translation right and down, translation of result right and up.

ANSWER:

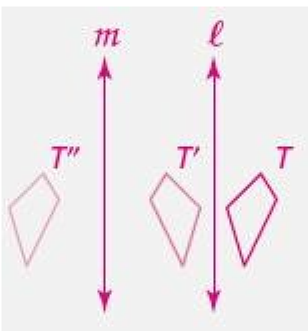
Sample answer: translation right and down, translation of result right and up

26. Copy and reflect figure *T* in line *l* and then line *m*. Then describe a single transformation that maps *T* onto T''.



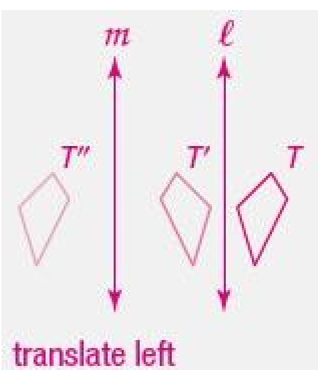
SOLUTION:

The first reflection flips the object, the second reflection flips it again, reversing the first flip and returning the image to its original orientation. After the reflections, the only change in the image is that it moved to the left. This can be replicated was a translation.

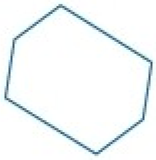


The transformation is equivalent to translating the figure

ANSWER:



State whether each figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.

27. 

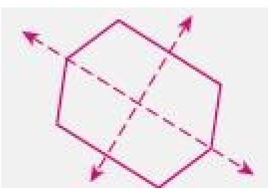
SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

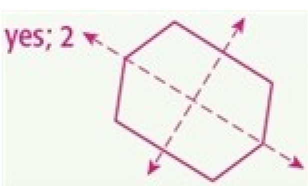
The figure has line symmetry.

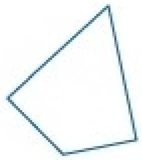
In order for the figure to map onto itself, the line of reflection must go through the center point.

One line of reflection goes through the top and bottom of the tilted hexagon. Another line of reflection goes through the vertices in the middle.



ANSWER:



28. 

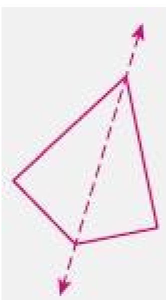
SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

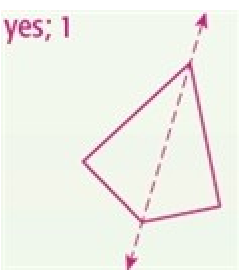
The figure has line symmetry.

In order for the figure to map onto itself, the line of reflection must go through the center point.

One line of reflection goes through the top and bottom vertices of the quadrilateral.



ANSWER:



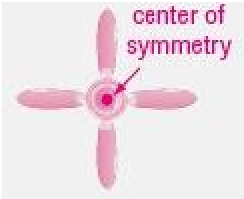
State whether each figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

29. 

SOLUTION:

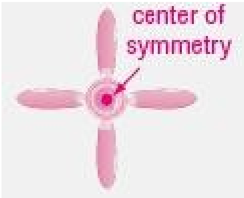
A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure. This figure has rotational symmetry.
The number of times a figure maps onto itself as it rotates form 0° and 360° is called the order of symmetry. The order of symmetry is 4 because the figure maps to itself at 90º, 180º, 270º, and 360º rotations.
The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself. The smallest angle through which this figure maps onto itself is 90º.

yes; 4; 90º



ANSWER:

yes; 4; 90º



30. 

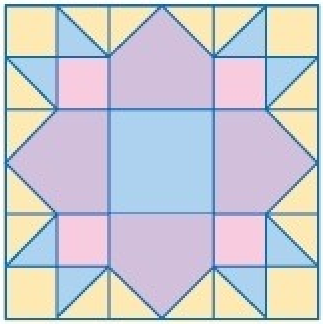
SOLUTION:

No, this figure does not have rotational symmetry. There is no center of symmetry about which a rotation would map the figure onto itself.

ANSWER:

no

31. **KNITTING** Amy is creating a pattern for a scarf she is knitting for her friend. How many lines of symmetry are there in the pattern?



SOLUTION:

The two diagonals of the square are lines of symmetry. Also, the lines joining the midpoints of the opposite sides also form lines of symmetry. Therefore, the total number of lines of symmetry is 4.

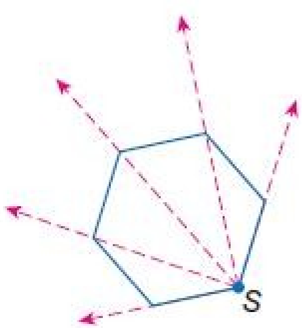
ANSWER:

4

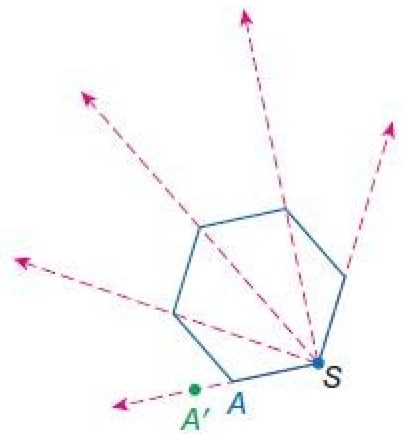
32. Copy the figure and point *S*. Then use a ruler to draw the image of the figure under a dilation with center *S* and scale factor *r* = 1.25.

SOLUTION:

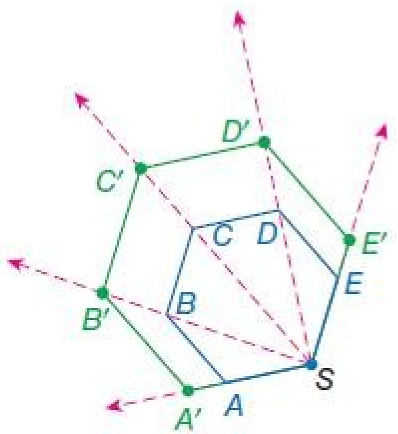
Step 1: Draw rays from *S* though each vertex.



Step 2: Locate A' on $\overrightarrow{SA}$ such that $SA' = 1.25SA$.

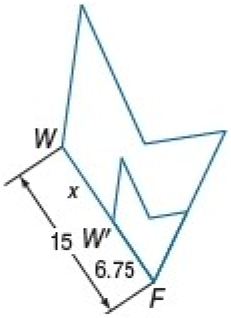


Step 3: Locate *B′* on $\overrightarrow{SB}$, *C′* on $\overrightarrow{SC}$, *D′* on $\overrightarrow{SD}$ and *E′* on $\overrightarrow{SE}$ in the same way. Then draw *A'B'C'D'E'*.



ANSWER:

33. Determine whether the dilation from figure *W* to W' is an enlargement or a reduction. Then find the scale factor of the dilation and *x*.



SOLUTION:

The figure *W'* is smaller than the figure *W*. So, the dilation is a reduction.
The scale factor *k* of the enlargement or reduction is the ratio of a length on the image to a corresponding length on the preimage. Here, *W'F* = 6.75 and *WF* = 15. So, the scale factor is $k = \frac{6.75}{15} = 0.45.$

$x = 15 - 6.75 = 8.25$

ANSWER:

reduction; 8.25, 0.45

34. **CLUBS** The members of the Math Club use an overhead projector to make a poster. If the original image was 6 inches wide, and the image on the poster is 4 feet wide, what is the scale factor of the enlargement?

SOLUTION:

The scale factor *k* of the enlargement or reduction is the ratio of a length on the image to a corresponding length on the preimage. Here, the original image was 6 inches wide and the image on the poster is 4 feet = 48 inches wide. So, the scale factor is $k = \frac{48}{6} = 8.$

ANSWER:

8