State whether each sentence is true or false. If false, replace the underlined word or phrase to make a true sentence.

1. The arithmetic mean of two numbers is the positive square root of the product of the numbers.

SOLUTION:
false, geometric

ANSWER:
false, geometric

2. Extended ratios can be used to compare three or more quantities.

SOLUTION:
true

ANSWER:
true

3. To find the length of the hypotenuse of a right triangle, take the square root of the difference of the squares of the legs.

SOLUTION:
false, sum

ANSWER:
false, sum

4. An angle of elevation is the angle formed by a horizontal line and an observer's line of sight to an object below the horizon.

SOLUTION:
false, depression

ANSWER:
false, depression

5. The sum of two vectors is the resultant.

SOLUTION:
true

ANSWER:
true

6. Magnitude is the angle a vector makes with the *x*-axis.

SOLUTION:
false, length of the vector

ANSWER:
false, length of the vector

7. A vector is in standard position when the initial point is at the origin.

SOLUTION:
true

ANSWER:
true

8. The component form of a vector describes the vector in terms of change in x and change in y.

SOLUTION:
true

ANSWER:
true

9. The Law of Sines can be used to fi nd an angle measure when given three side lengths.

SOLUTION:
false, Law of Cosines

ANSWER:
false, Law of Cosines

10. A trigonometric ratio is a ratio of the lengths of two sides of a right triangle.

SOLUTION:
true

ANSWER:
true

Find the geometric mean between each pair of numbers.

11. 9 and 4

SOLUTION:
By definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.
Therefore, the geometric mean of 9 and 4 is
$\sqrt{(9)(4)} = \sqrt{36} = 6.$

ANSWER:
6

12. $\sqrt{20}$ and $\sqrt{80}$

SOLUTION:

By definition, the geometric mean *x* of any two numbers *a* and *b* is given by $x = \sqrt{ab}$.
Therefore, the geometric mean of $\sqrt{20}$ and $\sqrt{80}$ is
$\sqrt{\left(\sqrt{20}\right)\left(\sqrt{80}\right)} = \sqrt{\sqrt{1600}} = \sqrt{40}$.

ANSWER:

$\sqrt{40}$

13. $\frac{8\sqrt{2}}{3}$ and $\frac{4\sqrt{2}}{3}$

SOLUTION:

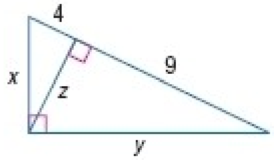
By definition, the geometric mean *x* of any two numbers *a* and *b* is given by $x = \sqrt{ab}$.
Therefore, the geometric mean of $\frac{8\sqrt{2}}{3}$ and $\frac{4\sqrt{2}}{3}$ is

$$\sqrt{\left(\frac{8\sqrt{2}}{3}\right)\left(\frac{4\sqrt{2}}{3}\right)} = \sqrt{\frac{32 \cdot \sqrt{4}}{9}} = \sqrt{\frac{32 \cdot 2}{9}} = \sqrt{\frac{64}{9}} = \frac{8}{3}$$

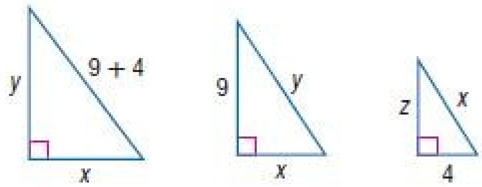
ANSWER:

$\frac{8}{3}$

14. **Find *x*, *y*, and *z*.**



SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

$$\frac{hypotenuse}{longer\ leg} = \frac{hypotenuse}{longer\ leg} \qquad \frac{hypotenuse}{shorter\ leg} = \frac{hypotenuse}{shorter\ leg}$$

Set up a proportion and solve for y and x:

$$\begin{aligned} \frac{13}{y} &= \frac{y}{9} \\ y^2 &= 117 \\ y &= \sqrt{117} \\ &= 3\sqrt{13} \end{aligned} \quad \text{and} \quad \begin{aligned} \frac{13}{x} &= \frac{x}{4} \\ x^2 &= 52 \\ x &= \sqrt{52} \\ &= 2\sqrt{13} \end{aligned}$$

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

Solve for z

$$\begin{aligned} \frac{9}{z} &= \frac{z}{4} \\ z^2 &= 36 \\ z &= 6 \end{aligned}$$

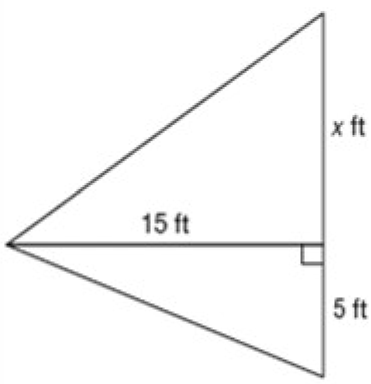
ANSWER:

$x = 2\sqrt{13}, y = 3\sqrt{13}, z = 6$

15. **DANCES** Mike is hanging a string of lights on his barn for a square dance. Using a book to sight the top and bottom of the barn, he can see he is 15 feet from the barn. If his eye level is 5 feet from the ground, how tall is the barn?

SOLUTION:

Draw a rough figure for the given information.



By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

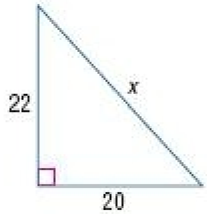
$$15 = \sqrt{x \cdot 5}$$
$$225 = x \cdot 5$$
$$45 = x$$

The total height of the barn is 45 + 5 = 50 ft.

ANSWER:

50 ft

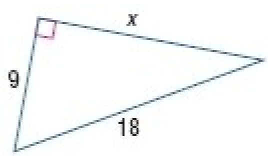
Find *x*.

16. 

SOLUTION:
Use the Pythagorean theorem to find the hypotenuse of the right triangle.

$$22^2 + 20^2 = x^2$$
$$484 + 400 = x^2$$
$$884 = x^2$$
$$\sqrt{884} = x$$
$$x = 2\sqrt{221} \approx 29.7$$

ANSWER:
$2\sqrt{221} \approx 29.7$

17. 

SOLUTION:
Use the Pythagorean theorem to find the missing side of the right triangle.

$$x^2 + 9^2 = 18^2$$
$$x^2 + 81 = 324$$
$$x^2 = 243$$
$$x = \sqrt{243}$$
$$x = 9\sqrt{3} \approx 15.6$$

ANSWER:
$9\sqrt{3} \approx 15.6$

Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute, obtuse,* or *right*. Justify your answer.

18. 7, 24, 25

SOLUTION:

By the Triangle Inequality Theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$$7 + 24 > 25 \checkmark$$
$$24 + 25 > 7 \checkmark$$
$$7 + 25 > 24 \checkmark$$

Therefore, the set of numbers can be measures of a triangle.

Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$25^2 \stackrel{?}{=} 24^2 + 7^2$$
$$625 \stackrel{?}{=} 576 + 49$$
$$625 = 625 \checkmark$$

Therefore, by the converse of Pythagorean Theorem, a triangle with the given measures will be a right triangle.

ANSWER:

Yes; right

$$25 \stackrel{?}{=} 7^2 + 24^2$$
$$625 = 49 + 576$$

19. 13, 15, 16

SOLUTION:

By the Triangle Inequality Theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$$13 + 15 > 16 \checkmark$$
$$13 + 16 > 15 \checkmark$$
$$15 + 16 > 13 \checkmark$$

Therefore, the set of numbers can be measures of a triangle.

Now, classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$16^2 \stackrel{?}{=} 13^2 + 15^2$$
$$256 \stackrel{?}{=} 169 + 225$$
$$256 < 394$$

Therefore, by Pythagorean Inequality Theorem, a triangle with the given measures will be an acute triangle.

ANSWER:

Yes; acute

$$16^2 \stackrel{?}{=} 13^2 + 15^2$$
$$256 < 169 + 225$$

20. 65, 72, 88

SOLUTION:

By the Triangle Inequality Theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$65 + 72 > 88$ ✓
$72 + 88 > 65$ ✓
$65 + 88 > 72$ ✓

Therefore, the set of numbers can be measures of a triangle.

Now, classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$88^2 \stackrel{?}{=} 65^2 + 72^2$$
$$7744 \stackrel{?}{=} 4225 + 5184$$
$$7744 < 9409$$

Therefore, by Pythagorean Inequality Theorem, a triangle with the given measures will be an acute triangle.

ANSWER:

Yes; acute

$$88^2 \stackrel{?}{=} 65^2 + 72^2$$
$$7744 \stackrel{?}{=} 4225 + 5184$$
$$7744 < 9409$$

21. **SWIMMING** Alexi walks 27 meters south and 38 meters east to get around a lake. Her sister swims directly across the lake. How many meters to the nearest tenth did Alexi's sister save by swimming?

SOLUTION:

Let *x* be the distance across the river.

$$\begin{aligned} 27^2 + 38^2 &= x^2 \\ 729 + 1444 &= x^2 \\ 2173 &= x^2 \\ x &= \sqrt{2173} \\ x &\approx 46.6 \end{aligned}$$

The distance saved would be the difference between the sum of the two legs of the triangle and the hypotenuse (distance across the river).

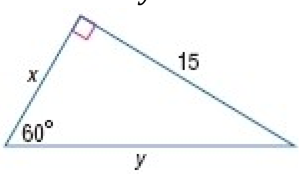
distance saved = (27+38) – 46.6 = 18.4

Alexi's sister saved about 18.4 meters by swimming across the river.

ANSWER:

18.4 m

Find *x* and *y*.

22. 

SOLUTION:

In a 30°-60°-90° triangle, the length of the hypotenuse *h* is 2 times the length of the shorter leg *s*, and the length of the longer leg *l* is $\sqrt{3}$ times the length of the shorter leg.

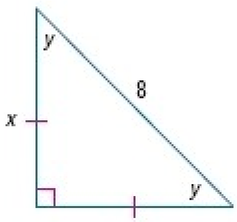
Here, the length of the hypotenuse is y, the shorter leg is *x* and the longer leg is 15. So, $\sqrt{3}x = 15.$

Then, $x = \frac{15}{\sqrt{3}}$ or $5\sqrt{3}$.

Also we have, $y = 2\left(5\sqrt{3}\right) = 10\sqrt{3}.$

ANSWER:

$x = 5\sqrt{3}, y = 10\sqrt{3}$

23. 

SOLUTION:

The given triangle is an isosceles right triangle, that is, 45°-45°-90° triangle. So, $y = 45°$.

In a 45°-45°-90° triangle, the legs *l* are congruent and the length of the hypotenuse *h* is $\sqrt{2}$ times the length of a leg. Here, $x\sqrt{2} = 8.$

Solve for *x*.

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{8}{\sqrt{2}}$$

$$x = \frac{8}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

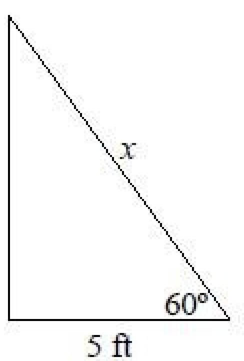
$$= 4\sqrt{2}$$

ANSWER:

$x = 4\sqrt{2}, y = 45°$

24. **CLIMBING** Jason is adding a climbing wall to his little brother's swing-set. If he starts building 5 feet out from the existing structure, and wants it to have a $60°$ angle, how long should the wall be?

SOLUTION:



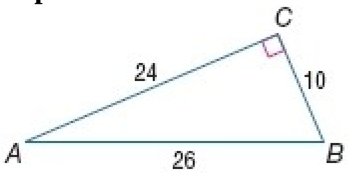
10 ft.

Since the ground and the side of the play structure make a $90°$ angle, this is a $30° - 60° - 90°$ triangle. The short side is 5 feet. The climbing wall will be the hypotenuse, so it is 10 feet.

ANSWER:

10 ft. sample answer: since the ground and the side of the play structure make a $90°$ angle, this is a $30° - 60° - 90°$ triangle. The short side is 5 feet. The climbing wall will be the hypotenuse, so it is 10 feet.

Express each ratio as a fraction and as a decimal to the nearest hundredth.



25. sin *A*

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. So,

$$\sin A = \frac{BC}{AB} = \frac{10}{26} \approx 0.38.$$

ANSWER:

$\frac{5}{13}, 0.38$

26. tan *B*

SOLUTION:

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$$\tan B = \frac{AC}{BC} = \frac{24}{10} = 2.4.$$

ANSWER:

$\frac{12}{5}, 2.40$

27. sin *B*

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. So,

$$\sin B = \frac{AC}{AB} = \frac{24}{26} \approx 0.92.$$

ANSWER:

$\frac{12}{13}, 0.92$

28. cos *A*

SOLUTION:

The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse. So,

$$\cos A = \frac{AC}{AB} = \frac{24}{26} \approx 0.92.$$

ANSWER:

$\frac{12}{13}, 0.92$

29. tan *A*

SOLUTION:

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$\tan A = \frac{BC}{AC} = \frac{10}{24} \approx 0.42.$

ANSWER:

$\frac{5}{12}, 0.42$

30. cos *B*

SOLUTION:

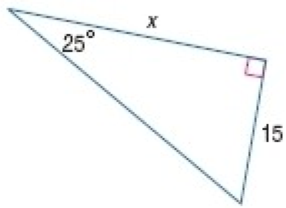
The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse. So,

$\cos B = \frac{AB}{AB} = \frac{10}{26} \approx 0.38.$

ANSWER:

$\frac{5}{13}, 0.38$

Find *x*.

31. 

SOLUTION:

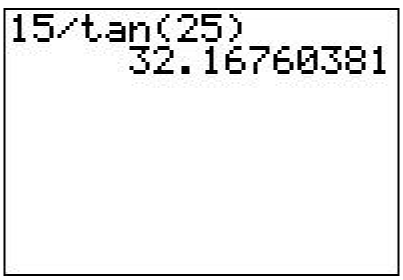
The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$$\tan 25° = \frac{15}{x}.$$

Solve for *x*:

$$x = \frac{15}{\tan 25°}$$

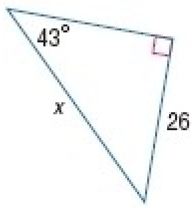
Use a calculator to find the value of *x*. Make sure your calculator is in degree mode.



$x \approx 32.2$

ANSWER:

32.2

32. 

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. So,

$$\sin 43° = \frac{26}{x}.$$

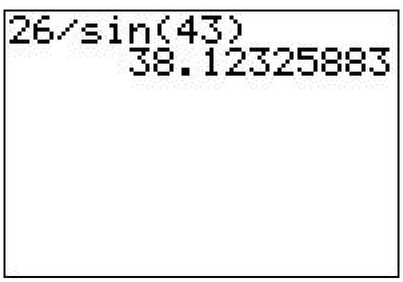
First, multiply both sides by *x*.

$$x \cdot \sin 43° = 26$$

Divide both sides by sin 43°.

$$x = \frac{26}{\sin 43°}$$

Use a calculator to find the value of *x*. Make sure your calculator is in degree mode.

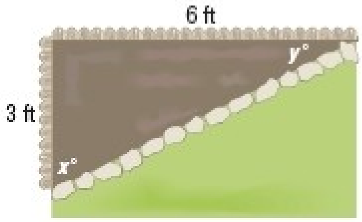


$$x \approx 38.1$$

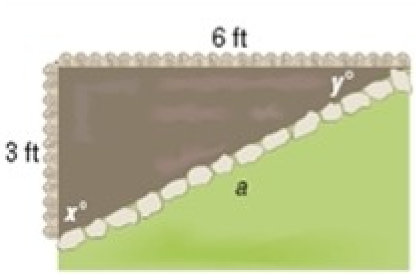
ANSWER:

38.1

33. **GARDENING** Sofia wants to put a flower bed in the corner of her yard by laying a stone border that starts 3 feet from the corner of one fence and ends 6 feet from the corner of the other fence. Find the angles, x and y, the fence make with the border.



SOLUTION:



Use tangent.

$$\tan x = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan x = \frac{6}{3}$$

$$\tan x = 2$$

$$x = \tan^{-1} 2$$

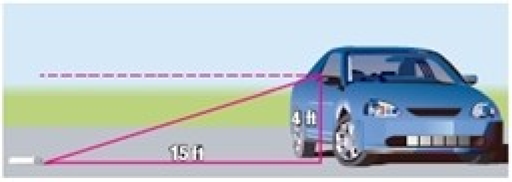
$$x \approx 63.4$$

$y = 90 - 63.4$ or 26.6

ANSWER:

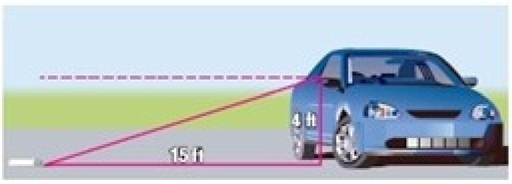
$63.4°$ and $26.6°$

34. **JOBS** Tom delivers papers on a rural route from his car. If he throws a paper from a height of 4 feet, and it lands 15 feet from the car, at what angle of depression did he throw the paper to the nearest degree?



SOLUTION:

Let *x* be the unknown.



$\tan x = \frac{4}{15}$

$x = \tan^{-1}\left(\frac{4}{15}\right)$

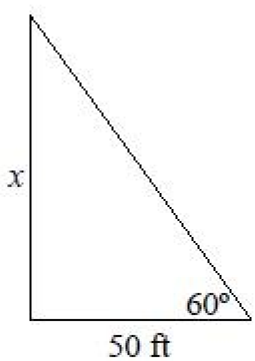
$x \approx 15°$

ANSWER:

15°

35. **TOWER** There is a cell phone tower in the field across from Jen's house. If Jen walks 50 feet from the tower, and finds the angle of elevation from her position to the top of the tower to be $60°$, how tall is the tower?

SOLUTION:



Let *x* be the height of the tower.

$\tan 60° = \frac{x}{50}$

$x = 50\tan 60°$

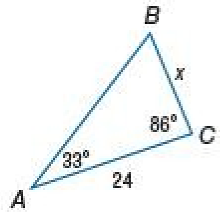
$x \approx 86.6$

So, the height of the tower is 86.6 feet.

ANSWER:

86.6 ft

Find *x*. Round angle measures to the nearest degree and side measures to the nearest tenth.

36. 

SOLUTION:

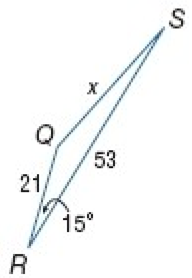
Since we are given one side and two angles, we can set up a proportion using the Law of Sines.

The angle opposite the side of length 73 has a degree measure of 180° – 33° – 86° = 61°.

$$\begin{aligned} \frac{\sin 33}{x} &= \frac{\sin 61}{24} && \text{Substitution} \\ x \cdot \sin 61 &= 24 \cdot \sin 33 && \text{Cross Products Property} \\ x &= \frac{24 \cdot \sin 33}{\sin 61} && \text{Solve for } \sin x \\ x &= 15.0 && \text{Use a calculator} \end{aligned}$$

ANSWER:

15.0

37. 

SOLUTION:

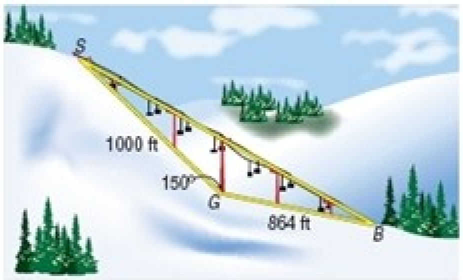
We are given the measures of two sides and their included angle, so use the Law of Cosines.

$$
\begin{aligned}
33.2^2 &= (21)^2 + (53)^2 - 2(21)(53)\cos(x) && \text{Substitute.}\\
1102.24 &= 3250 - 2226\cos(x) && \text{Simplify}\\
-2147.76 &= -2226\cos(x) && \text{Take the square root of each side}\\
\frac{-2147.76}{-2226} &= \cos(x) && \text{Take the square root of each side}\\
x &= \cos^{-1}\left(\frac{2147.76}{2226}\right) && \text{Simplify}\\
x &\approx 15.2° && \text{Simplify}
\end{aligned}
$$

ANSWER:

15.2

38. **SKIING** At Crazy Ed's Ski resort, Ed wants to put in another ski lift for the skiers to ride from the base to the summit of the mountain. The run over which the ski lift will go is represented by the figure below. The length of the lift is represented by *SB*. If Ed needs twice as much cable as the length of $\overline{SB}$, how much cable does he need?



SOLUTION:

We are given the measures of two sides and their included angle, so use the Law of Cosines.

$$\begin{aligned} SB^2 &= 1000^2 + 864^2 - 2(1000)(864)\cos 150° \\ &= 1,000,000 + 746,496 - 1,728,000\cos 150° \\ &\approx 1,000,000 + 746,496 + 1,496,491.9 \end{aligned}$$

$$SB \approx 3,242,987.9$$

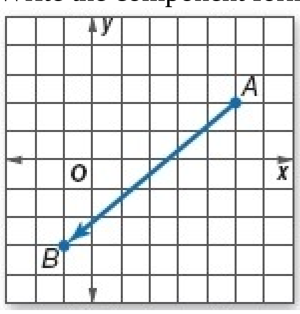
$$SB \approx \pm 1800.83$$

Ed needs 2(1800.83) or about 3601.7 feet cable.

ANSWER:

3601.7 ft

39. Write the component form of the vector shown.



SOLUTION:

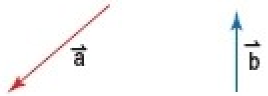
Use component notation and find the change in *x*-values and the change in *y*-values. The coordinates of *A* and *B* are (5, 2) and (–1, –3) respectively.

$$\begin{aligned} \overline{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle -1 - 5, -3 - 2 \rangle \\ &= \langle -6, -5 \rangle \end{aligned}$$

ANSWER:

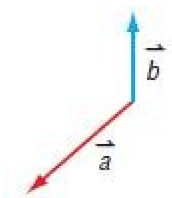
$\langle -6, -5 \rangle$

40. Copy the vectors to find $\overrightarrow{a} + \overrightarrow{b}$.

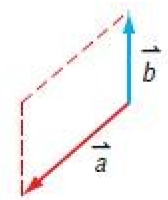


SOLUTION:

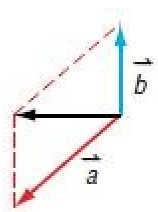
Using the Parallelogram method, start by translating $\vec{b}$ until its initial point touches the initial point of $\vec{a}$.



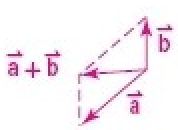
Next, complete the parallelogram, as shown:



Draw in the diagonal, which represents the resulting vector when adding $\vec{a}$ and $\vec{b}$.



ANSWER:



41. Given that $\overrightarrow{s}$ is $\langle 2, -6\rangle$ and $\overrightarrow{t}$ is $\langle -10, 7\rangle$, find the component form of $\overrightarrow{s} + \overrightarrow{t}$.

SOLUTION:

$$\begin{aligned}\overrightarrow{s} + \overrightarrow{t} &= \langle 2, -6\rangle + \langle -10, 7\rangle \\ &= \langle 2 - 10, -6 + 7\rangle \\ &= \langle -8, 1\rangle\end{aligned}$$

ANSWER:

$\langle -8, 1\rangle$