Practice Test - Chapter 8

Find the geometric mean between each pair of numbers.

1. 7 and 11

SOLUTION:

By the definition, the geometric mean *x* of any two numbers *a* and *b* is given by $x = \sqrt{ab}$. Therefore, the geometric mean of 7 and 11 is
$\sqrt{(7)(11)} = \sqrt{77} \approx 8.8.$

ANSWER:

$\sqrt{77} \approx 8.8$

2. 12 and 9

SOLUTION:

By the definition, the geometric mean *x* of any two numbers *a* and *b* is given by $x = \sqrt{ab}$. Therefore, the geometric mean of 12 and 9 is
$\sqrt{(12)(9)} = 6\sqrt{3} \approx 10.4.$

ANSWER:

$6\sqrt{3} \approx 10.4$

3. 14 and 21

SOLUTION:

By the definition, the geometric mean *x* of any two numbers *a* and *b* is given by $x = \sqrt{ab}$. Therefore, the geometric mean of 14 and 21 is
$\sqrt{(14)(21)} = 7\sqrt{6} \approx 17.1.$

ANSWER:

$7\sqrt{6} \approx 17.1$

4. $4\sqrt{3}$ and $10\sqrt{3}$

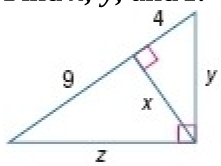
SOLUTION:

By the definition, the geometric mean *x* of any two numbers *a* and *b* is given by $x = \sqrt{ab}$. Therefore, the geometric mean of $4\sqrt{3}$ and $10\sqrt{3}$ is
$\sqrt{\left(4\sqrt{3}\right)\left(10\sqrt{3}\right)} = 2\sqrt{30} \approx 11.$

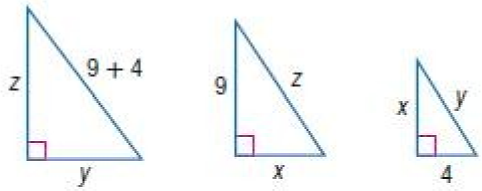
ANSWER:

$2\sqrt{30} \approx 11.0$

Find *x*, *y*, and *z*.

5. 

SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

$$\frac{hypotenuse}{longer\ leg} = \frac{hypotenuse}{longer\ leg} \qquad \frac{hypotenuse}{shorter\ leg} = \frac{hypotenuse}{shorter\ leg}$$

Set up a proportion and solve for z and y:

$$\frac{13}{z} = \frac{z}{9} \quad \text{and} \quad \frac{13}{y} = \frac{y}{4}.$$

$$z^2 = 117 \qquad y^2 = 52$$

$$z = \sqrt{117} \quad \text{and} \quad y = \sqrt{52}$$

$$= 3\sqrt{13} \qquad = 2\sqrt{13}$$

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

Solve for x

$$\frac{9}{x} = \frac{x}{4}$$

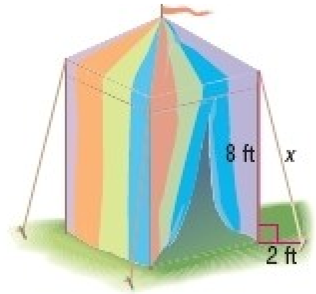
$$x^2 = 36$$

$$x = 6$$

ANSWER:

$x = 6,\ y = 2\sqrt{13}, z = 3\sqrt{13}$

6. **FAIRS** Blake is setting up his tent at a renaissance fair. If the tent is 8 feet tall, and the tether can be staked no more than two feet from the tent, how long should the tether be?



SOLUTION:

Use the Pythagorean Theorem to find *x*.

$$2^2 + 8^2 = x^2$$
$$4 + 64 = x^2$$
$$68 = x^2$$
$$\sqrt{68} = x$$
$$x = 2\sqrt{17} \approx 8.2\ ft$$

ANSWER:

8.2 ft

Use a calculator to find the measure of $\angle R$ to the nearest tenth.

7. 

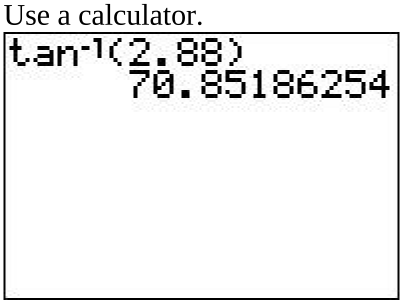
SOLUTION:

The measures given are those of the legs opposite and adjacent to $\angle R$, so write an equation using the tangent ratio.

$$\tan R = \frac{PQ}{QR} = \frac{72}{25} = 2.88.$$

If $\tan R = 2.88$, then $R = \tan^{-1}(2.88)$.

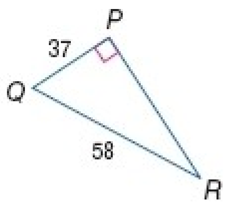
Use a calculator.



$m\angle R \approx 70.9°$

ANSWER:

70.9

8. 

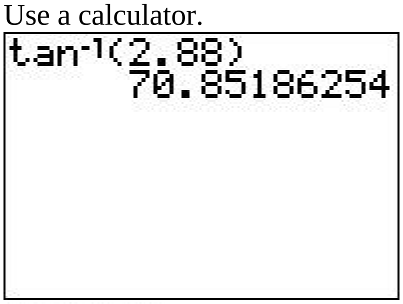
SOLUTION:

The measures given are those of the leg opposite to $\angle R$ and the hypotenuse, so write an equation using the sine ratio.

$$\sin R = \frac{PQ}{QR} = \frac{37}{58}$$

If $\sin R = \frac{37}{58}$, then $\sqrt{T} = \sin^{-1}\left(\frac{37}{58}\right)$.

Use a calculator.

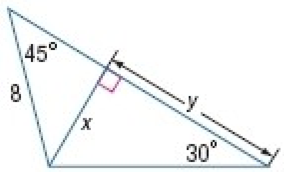


$m\angle R \approx 39.6°$

ANSWER:

39.6

Find *x* and *y*.

9. 

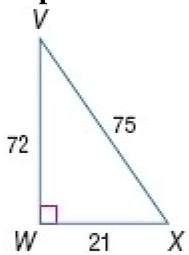
SOLUTION:

$$\sin 45° = \frac{x}{8}$$
$$x = 8\sin 45°$$
$$x = 8\left(\frac{1}{\sqrt{2}}\right)$$
$$x = \frac{8}{\sqrt{2}}$$
$$x = \frac{8\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$$
$$x = \frac{8\sqrt{2}}{2}$$
$$x = 4\sqrt{2}$$
$$\tan 30° = \frac{x}{y}$$
$$\frac{1}{\sqrt{3}} = \frac{4\sqrt{2}}{y}$$
$$y = 4\sqrt{2}\cdot\sqrt{3}$$
$$y = 4\sqrt{6}$$

ANSWER:

$x = 4\sqrt{2}, y = 4\sqrt{6}$

Express each ratio as a fraction and as a decimal to the nearest hundredth.



10. cos *X*

SOLUTION:

The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse. So,

$\cos X = \frac{WX}{VX} = \frac{21}{75} = 0.28.$

ANSWER:

$\frac{21}{75} = 0.28$

11. tan *X*

SOLUTION:

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$\tan X = \frac{VW}{WX} = \frac{72}{21} \approx 3.43.$

ANSWER:

$\frac{72}{21} \approx 3.43$

12. tan *V*

SOLUTION:

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$\tan V = \frac{WX}{VW} = \frac{21}{72} \approx 0.29.$

ANSWER:

$\frac{21}{72} \approx 0.29$

13. sin *V*

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. So,

$\sin V = \frac{WX}{VX} = \frac{21}{75} = 0.28.$

ANSWER:

$\frac{21}{75} = 0.28$

Find the magnitude and direction of each vector.

14. $\overrightarrow{JK}$: *J*(–6, –4) and *K*(–10, –4)

SOLUTION:

Use the distance formula to find the vector's magnitude.

$$\begin{aligned}\text{magnitude of } \overrightarrow{JK} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-10+6)^2 + (-4+4)^2} \\ &= \sqrt{(-4)^2} \\ &= 4\end{aligned}$$

Use trigonometry to find the vector's direction.

$$\begin{aligned}\tan J &= \left|\frac{y_2 - y_1}{x_2 - x_1}\right| \\ &= \left|\frac{-4+4}{-10+6}\right| \\ &= \left|\frac{0}{-4}\right| \\ &= 0 \\ J &= \tan^{-1}(0) \\ &= 0°\end{aligned}$$

When repositioned so that its initial point is at the origin, $\overrightarrow{JK}$ lies on the negative *x*-axis and forms an angle of $0°$. So, the direction of $\overrightarrow{JK}$ is the angle it makes with the positive *x*-axis which is $0° + 180°$ or $180°$.

ANSWER:

4 units; $180°$

15. $\overrightarrow{RS}$: $R(1, 0)$ and $S(-2, 3)$

SOLUTION:

Use the distance formula to find the vector's magnitude.

$$\begin{aligned}\text{magnitude of } \overrightarrow{RS} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2-1)^2 + (3-0)^2} \\ &= \sqrt{(-3)^2 + (3)^2} \\ &= \sqrt{9+9} \\ &= \sqrt{18} \\ &= 3\sqrt{2}\end{aligned}$$

Use trigonometry to find the vector's direction.

$$\begin{aligned}\tan R &= \left|\frac{y_2 - y_1}{x_2 - x_1}\right| \\ &= \left|\frac{-2-1}{3-0}\right| \\ &= \left|\frac{-3}{3}\right| \\ &= 1 \\ R &= \tan^{-1}(1) \\ &= 45°\end{aligned}$$

When repositioned so that its initial point is at the origin, $\overrightarrow{RS}$ lies in the same quadrant and forms an angle with the positive y-axis $45°$ counter clockwise.

So, the direction of $\overrightarrow{RS}$ is the angle it makes with the positive x-axis counter clockwise, which is $90° + 45°$ or $135°$.

ANSWER:

$3\sqrt{2}, 135°$

16. **SPACE** Anna is watching a space shuttle launch 6 miles from Cape Canaveral in Florida. When the angle of elevation from her viewing point to the shuttle is $80°$, how high is the shuttle, if it is going straight up?

SOLUTION:

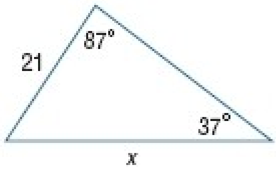
Let x be the unknown.

$$\begin{aligned}\tan 80° &= \frac{x}{6} \\ x &= 6 \tan 80° \\ x &\approx 34 \text{ mi}\end{aligned}$$

ANSWER:

34 mi

Find *x*. Round angle measures to the nearest degree and side measures to the nearest tenth.

17. 

SOLUTION:

By the Law of Sines, $\frac{\sin 37°}{21} = \frac{\sin 87°}{x}$.

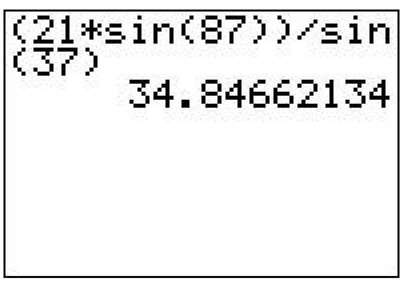
Take the cross products.

$x \cdot \sin 37° = 21 \cdot \sin 87°$

Divide each side by sin 37°.

$x = \frac{21 \cdot \sin 87°}{\sin 37°}$

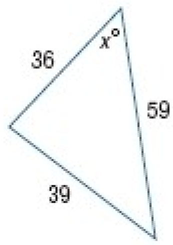
Use a calculator, in degree mode.



$x \approx 34.8$

ANSWER:

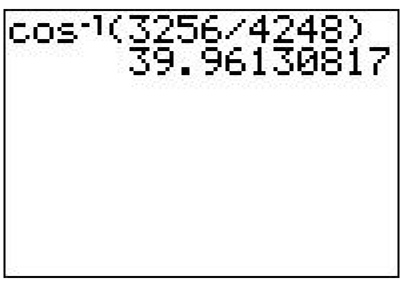
34.8

18. 

SOLUTION:

We are given the measures of two sides and their included angle, so use the Law of Cosines.

$$39^2 = 36^2 + 59^2 - 2(36)(59)\cos x$$
$$1521 = 1296 + 3481 - 4248\cos x$$
$$1521 = 4777 - 4248\cos x$$
$$4248\cos x = 4777 - 1521$$
$$4248\cos x = 3256$$
$$\cos x = \frac{3256}{4248}$$
$$x = \cos^{-1}\left(\frac{3256}{4248}\right)$$



$x \approx 40.0$

ANSWER:

40

19. **MULTIPLE CHOICE** Which of the following is the length of the leg of a $45° - 45° - 90°$ triangle with a hypotenuse of 20?

A 10

B $10\sqrt{2}$

C 20

D $20\sqrt{2}$

SOLUTION:

Let *x* be the unknown.
Use the Pythagorean Theorem.

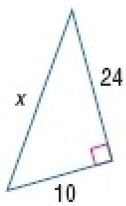
$$x^2 + x^2 = 20^2$$
$$2x^2 = 400$$
$$x^2 = 200$$
$$x = 10\sqrt{2}$$

So, the correct choice is B.

ANSWER:

B

Find *x*.

20. 

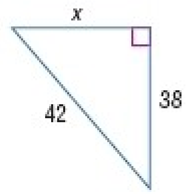
SOLUTION:

Use the Pythagorean Theorem to solve for *x*.

$$10^2 + 24^2 = x^2$$
$$100 + 576 = x^2$$
$$676 = x^2$$
$$\sqrt{676} = x$$
$$x = 26$$

ANSWER:

26

21. 

SOLUTION:

Use the Pythagorean Theorem to solve for *x*.

$$x^2 + 38^2 = 42^2$$
$$x^2 + 1444 = 1764$$
$$x^2 = 1764 - 1444$$
$$x^2 = 320$$
$$x = \sqrt{320} \approx 17.9$$

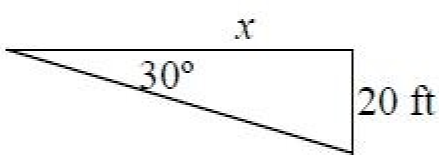
ANSWER:

$\sqrt{320} \approx 17.9$

22. **WHALE WATCHING** Isaac is looking through binoculars on a whale watching trip when he notices a sea otter in the distance. If he is 20 feet above sea level in the boat, and the angle of depression is $30°$, how far away from the boat is the otter to the nearest foot?

SOLUTION:

Let *x* be the unknown.

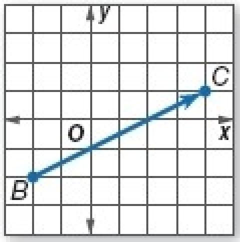


$$\tan 30° = \frac{20}{x}$$
$$x = \frac{20}{\tan 30°}$$
$$x \approx 35 \text{ ft}$$

ANSWER:

35 ft

Write the component form of each vector.

23. 

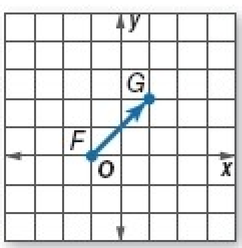
SOLUTION:

Use component notation and find the change in *x*-values and the change in *y*-values. Here, the coordinates of *B* and *C* are (–2, –2) and (4, 1) respectively.

$$\begin{aligned}\overline{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 4-(-2), 1-(-2) \rangle \\ &= \langle 6, 3 \rangle\end{aligned}$$

ANSWER:

$\langle 6, 3 \rangle$

24. 

SOLUTION:

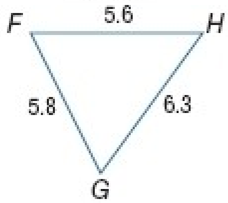
Use component notation and find the change in *x*-values and the change in *y*-values. Here, the coordinates of *F* and G are (–1, 0) and (1, 2) respectively.

$$\begin{aligned}\overline{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 1-(-1), 2-0 \rangle \\ &= \langle 2, 2 \rangle\end{aligned}$$

ANSWER:

$\langle 2, 2 \rangle$

25. Solve ΔFGH . Round to the nearest degree.



SOLUTION:

Since we are given three sides and no angle measures, we can solve this triangle using the Law of Cosines.

$$\begin{aligned} 6.3^2 &= 5.6^2 + 5.8^2 - 2(5.6)(5.8)\cos F \\ 6.3^2 - (5.6^2 + 5.8^2) &= -64.96\cos F \\ \frac{6.3^2 - (5.6^2 + 5.8^2)}{-64.96} &= \cos F \\ \cos x^{-1}\left[\frac{6.3^2 - (5.6^2 + 5.8^2)}{-64.96}\right] &= F \\ 67.1 &\approx F \end{aligned}$$

Similarly, we can solve for the measure of angle G, using the Law of Cosines.

$$\begin{aligned} 5.6^2 &= 6.3^2 + 5.8^2 - 2(6.3)(5.8)\cos G \\ 5.6^2 - (6.3^2 + 5.8^2) &= -73.08\cos G \\ \frac{5.6^2 - (6.3^2 + 5.8^2)}{-73.08} &= \cos G \\ \cos x^{-1}\left[\frac{5.6^2 - (6.3^2 + 5.8^2)}{-73.08}\right] &= G \\ 54.9 &\approx G \end{aligned}$$

We know that the sum of the measures all interior angles of a triangle is 180.

$$\begin{aligned} m\angle F + m\angle G + m\angle H &= 180 \\ 67 + 55 + m\angle H &= 180 \\ m\angle H &= 58 \end{aligned}$$

ANSWER:

$m\angle F = 67°, m\angle G = 55°, m\angle H = 58°$