Mid-Chapter Quiz: Lessons 8-1 through 8-4

Find the geometric mean between each pair of numbers.

1. 12 and 3

SOLUTION:

The geometric mean between *a* and *b* is given by, $x = \sqrt{ab}$.
Substitute 12 for *a* and 3 for *b*.

$$\begin{aligned} x &= \sqrt{12 \cdot 3} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

ANSWER:

6

2. 63 and 7

SOLUTION:

The geometric mean between *a* and *b* is given by, $x = \sqrt{ab}$.
Substitute 63 for *a* and 7 for *b*.

$$\begin{aligned} x &= \sqrt{63 \cdot 7} \\ &= \sqrt{441} \\ &= 21 \end{aligned}$$

ANSWER:

21

3. 45 and 20

SOLUTION:

The geometric mean between *a* and *b* is given by, $x = \sqrt{ab}$.
Substitute 45 for *a* and 20 for *b*.

$$\begin{aligned} x &= \sqrt{45 \cdot 20} \\ &= \sqrt{900} \\ &= 30 \end{aligned}$$

ANSWER:

30

4. 50 and 10

SOLUTION:

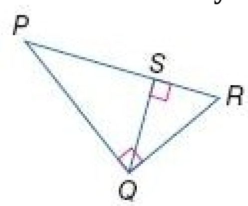
The geometric mean between *a* and *b* is given by, $x = \sqrt{ab}$.
Substitute 50 for *a* and 10 for *b*.

$$\begin{aligned} x &= \sqrt{50 \cdot 10} \\ &= \sqrt{500} \\ &= \sqrt{100 \cdot 5} \\ &= 10\sqrt{5} \end{aligned}$$

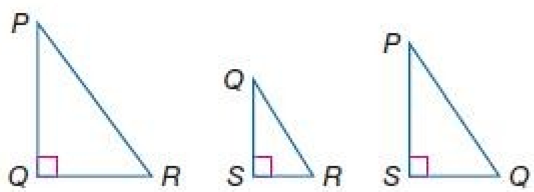
ANSWER:

$10\sqrt{5}$

Write a similarity statement identifying the three similar triangles in each figure.

5. 

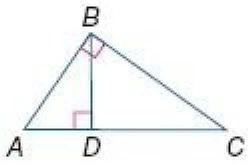
SOLUTION:



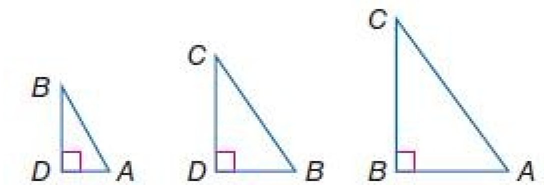
$\Delta PRQ \sim \Delta QRS \sim \Delta PQS$

ANSWER:

$\Delta PRQ \sim \Delta QRS \sim \Delta PQS$

6. 

SOLUTION:

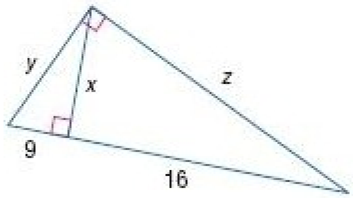


$\Delta ABD \sim \Delta BCD \sim \Delta ACB$

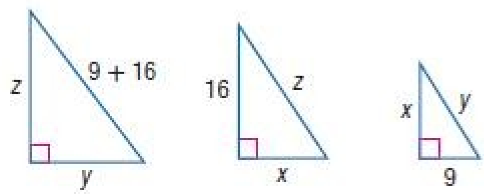
ANSWER:

$\Delta ABD \sim \Delta BCD \sim \Delta ACB$

Find *x*, *y*, and *z*.

7. 

SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

$$\frac{hypotenuse}{longer\ leg} = \frac{hypotenuse}{longer\ leg} \qquad \frac{hypotenuse}{shorter\ leg} = \frac{hypotenuse}{shorter\ leg}$$

Set up a proportion and solve for z and y:

$$\frac{25}{z} = \frac{z}{16} \quad \text{and} \quad \frac{25}{y} = \frac{y}{9}.$$

$$z^2 = 400 \qquad y^2 = 225$$

$$z = \sqrt{400} \quad \text{and} \quad y = \sqrt{225}$$

$$= 20 \qquad = 15$$

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

Solve for *x*:

$$\frac{16}{x} = \frac{x}{9}$$

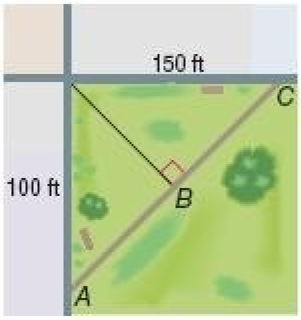
$$x^2 = 144$$

$$x = 12$$

ANSWER:

$x = 12, y = 15, z = 20$

8. **PARKS** There is a small park in a corner made by two perpendicular streets. The park is 100 ft by 150 ft, with a diagonal path, as shown below. What is the length of path $\overline{AC}$?



SOLUTION:

Use Pythagorean theorem to find the length of the path $\overline{AC}$.

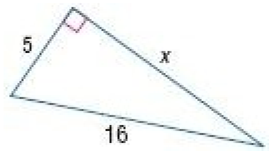
$$100^2 + 150^2 = AC^2$$
$$10000 + 22500 = AC^2$$
$$32500 = AC^2$$
$$\sqrt{32500} = AC$$
$$180.3 \approx AC$$

The length of path $\overline{AC}$ is 180.3 ft.

ANSWER:

180.3 ft

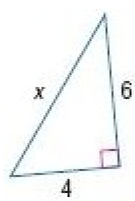
Find *x*. Round to the nearest hundredth.

9. 

SOLUTION:
Use Pythagorean theorem to find the length *x*.

$$x^2 + 5^2 = 16^2$$
$$x^2 + 25 = 256$$
$$x^2 = 231$$
$$x = \sqrt{231}$$
$$x \approx 15.20$$

ANSWER:
$\sqrt{231} \approx 15.20$

10. 

SOLUTION:
Use Pythagorean theorem to find the length of the hypotenuse, *x*.

$$4^2 + 6^2 = x^2$$
$$16 + 36 = x^2$$
$$52 = x^2$$
$$\sqrt{52} = x$$
$$x = 2\sqrt{13} \approx 7.21$$

ANSWER:
$2\sqrt{13} \approx 7.21$

11. **MULTIPLE CHOICE** Which of the following sets of numbers is not a Pythagorean triple?
A 9, 12, 15
B 21, 72, 75
C 15, 36, 39
D 8, 13, 15

SOLUTION:
By Pythagorean theorem,
$a^2 + b^2 = c^2$, where c is the hypotenuse of the right triangle.
In choice **A**,

$$9^2 + 12^2 \stackrel{?}{=} 15^2$$
$$81 + 144 \stackrel{?}{=} 225$$
$$225 \stackrel{?}{=} 225 \checkmark$$

In choice **B**,

$$21^2 + 72^2 \stackrel{?}{=} 75^2$$
$$441 + 5184 \stackrel{?}{=} 5625$$
$$5625 \stackrel{?}{=} 5625 \checkmark$$

In choice **C**,

$$15^2 + 36^2 \stackrel{?}{=} 39^2$$
$$225 + 1296 \stackrel{?}{=} 1521$$
$$1521 \stackrel{?}{=} 1521 \checkmark$$

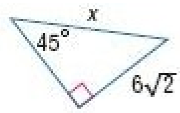
In choice **D**,

$$8^2 + 13^2 \stackrel{?}{=} 15^2$$
$$64 + 169 \stackrel{?}{=} 225$$
$$233 \stackrel{?}{=} 225 \times$$

Therefore, the correct choice is **D**.

ANSWER:
D

Find *x*.

12. 

SOLUTION:

Use special right triangles to find the value of *x*. In 45-45-90 triangles, the hypotenuse is $\sqrt{2}$ times the leg lengths *(l)*, which are equal.

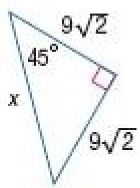
Therefore, since the legs of this triangle are $6\sqrt{2}$, the hypotenuse would be $h = \left(6\sqrt{2}\right)\sqrt{2}$.

Simplify:

$$\begin{aligned} h &= \left(6\sqrt{2}\right)\sqrt{2} \\ &= 6\left(\sqrt{2}\cdot\sqrt{2}\right) \\ &= 6\cdot 2 \\ &= 12 \end{aligned}$$

ANSWER:

12

13. 

SOLUTION:

Use special right triangles to find the value of *x*. In 45-45-90 triangles, the hypotenuse is $\sqrt{2}$ times the leg lengths *(l)*, which are equal.

Therefore, since the legs of this triangle are $9\sqrt{2}$, the hypotenuse would be $h = (9\sqrt{2})\sqrt{2}$.

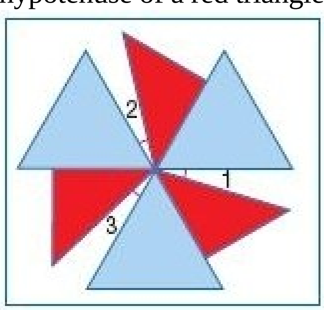
Simplify:

$$\begin{aligned} h &= (9\sqrt{2})\sqrt{2} \\ &= 9(\sqrt{2} \cdot \sqrt{2}) \\ &= 9 \cdot 2 \\ &= 18 \end{aligned}$$

ANSWER:

18

14. **DESIGN** Jamie designed a pinwheel to put in her garden. In the pinwheel, the blue triangles are congruent equilateral triangles, each with an altitude of 4 inches. The red triangles are congruent isosceles right triangles. The hypotenuse of a red triangle is congruent to a side of the blue triangle.



a. If angles 1, 2, and 3 are congruent, find the measure of each angle.
b. Find the perimeter of the pinwheel.

SOLUTION:

a) Since the base angles of an isosceles triangle are congruent, the measure of each acute angle is $45°$. The three angles of an equilateral triangle are congruent and each angle has a measures of $60°$.

Let *x* represent the $m\angle 1, m\angle 2, \text{ or } m\angle 3$. Set all the measures of all the angles in the pinwheel equal to 360 and solve for *x*:

$$\begin{aligned}3(x)+3(45)+3(60)&=360\\3x+135+180&=360\\3x+315&=360\\3x&=45\\x&=15\end{aligned}$$

Therefore, the angles 1, 2, and 3 each have a measure of $15°$.

b) The altitude of an equilateral triangle divides the triangle in to two 30-60-90 right triangles with a side of the equilateral triangle as the hypotenuse.The altitude (the longer leg) is 4 inches.
If *s* is the shortest side of the 30-60-90 triangle, then the length of the hypotenuse is 2*s* and the length of the longer leg is $s\sqrt{3}$.

Solve for *s*:

$$\begin{aligned}s\sqrt{3}&=4\\s&=\frac{4}{\sqrt{3}}\\&=\frac{4\sqrt{3}}{3}\end{aligned}$$

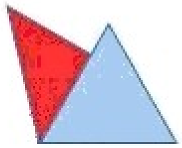
The length of the hypotenuse (and the side of the equilateral triangle) is $2\left(\frac{8\sqrt{3}}{3}\right)=\frac{8\sqrt{3}}{3}$.

The red triangles are isosceles triangles with hypotenuse measuring $\frac{8\sqrt{3}}{3}$.

So, the length of each leg of the red triangle is:

$$\begin{aligned}\frac{\frac{8\sqrt{3}}{3}}{\sqrt{2}}&=\frac{8\sqrt{3}}{3}\cdot\frac{1}{\sqrt{2}}\\&=\frac{8\sqrt{3}}{3\sqrt{2}}\\&=\frac{8\sqrt{3}}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}\\&=\frac{8\sqrt{6}}{6}\text{ or }\frac{4\sqrt{6}}{3}.\end{aligned}$$

Find the perimeter of the one of the wings.



$$\left.\begin{array}{l}\text{Perimeter of}\\ \text{this figure}\end{array}\right\} = \frac{8\sqrt{3}}{3} + \frac{8\sqrt{3}}{3} + \left(\frac{8\sqrt{3}}{3} - \frac{4\sqrt{6}}{3}\right) + \frac{4\sqrt{6}}{3} + \frac{8\sqrt{3}}{3}$$

Simplify.

$$= \frac{8\sqrt{3}}{3} + \frac{8\sqrt{3}}{3} + \frac{8\sqrt{3}}{3} + \frac{8\sqrt{3}}{3}$$
$$= 4 \cdot \frac{8\sqrt{3}}{3}$$
$$= \frac{32\sqrt{3}}{3}$$

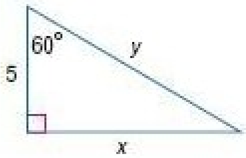
There are 3 such wings, therefore the perimeter of the pinwheel is $3 \cdot \frac{32\sqrt{3}}{3} = 32\sqrt{3} \approx 55$.

The perimeter of the pinwheel is about 55 inches.

ANSWER:

a. 15
b. 55 in.

Find x and y.

15. 

SOLUTION:

In 30-60-90 right triangles, the hypotenuse is twice the shorter leg ($h = 2s$), and the longer leg is $\sqrt{3}$ times the shorter leg ($l = s\sqrt{3}$).

Solve for x:

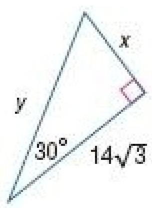
$x = 5\sqrt{3}$

Solve for y:

$y = 2(5) = 10$

ANSWER:

$x = 5\sqrt{3}$; $y = 10$

16. 

SOLUTION:

In 30-60-90 right triangles, the hypotenuse is twice the shorter leg ($h = 2s$), and the longer leg is $\sqrt{3}$ times the shorter leg ($l = s\sqrt{3}$).

Solve for x:

$x\sqrt{3} = 14\sqrt{3}$
$x = 14$

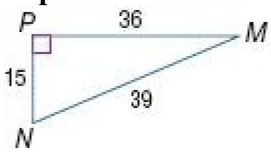
Solve for y:

$y = 2(14) = 28$

ANSWER:

$x = 14$; $y = 28$

Express each ratio as a fraction and as a decimal to the nearest hundredth.



17. tan M

SOLUTION:

$$\begin{aligned}\tan M &= \frac{\text{opposite}}{\text{adjacent}}\\ &= \frac{15}{36}\\ &\approx 0.42\end{aligned}$$

ANSWER:

$\frac{15}{36} = 0.42$

18. cos M

SOLUTION:

$$\begin{aligned}\cos M &= \frac{\text{adjacent}}{\text{hypotenuse}}\\ &= \frac{36}{39}\\ &\approx 0.92\end{aligned}$$

ANSWER:

$\frac{36}{39} = 0.92$

19. cos N

SOLUTION:

$$\begin{aligned}\cos N &= \frac{\text{adjacent}}{\text{hypotenuse}}\\ &= \frac{15}{39}\\ &\approx 0.38\end{aligned}$$

ANSWER:

$\frac{15}{39} \approx 0.38$

20. $\sin N$

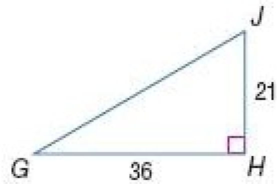
SOLUTION:

$$\begin{aligned}\sin N &= \frac{\text{opposite}}{\text{hypotenuse}}\\ &= \frac{36}{39}\\ &\approx 0.92\end{aligned}$$

ANSWER:

$\mathbf{\frac{36}{39} \approx 0.92}$

21. Solve the right triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.



SOLUTION:

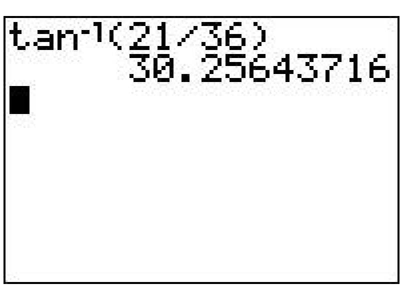
Use Pythagorean theorem to find GJ.

$$\begin{aligned}21^2 + 36^2 &= GJ^2\\ 441 + 1296 &= GJ^2\\ GJ &= \sqrt{1737}\\ GJ &\approx 41.7\end{aligned}$$

Use the tangent ratio to find $m\angle G$ and $m\angle J$.

$$\tan G = \frac{\text{opposite}}{\text{adjacent}} = \frac{21}{36}$$

$$\begin{aligned}G &= \tan^{-1}\left(\frac{21}{36}\right)\\ &\approx 30°\end{aligned}$$



$$\tan J = \frac{\text{opposite}}{\text{adjacent}} = \frac{36}{21}$$

$$J = \tan^{-1}\left(\frac{36}{21}\right)$$
$$\approx 60°$$

ANSWER:

JG = 41.7; $m\angle G = 30°$; $m\angle J = 60°$