NAME Answer Key DATE ____________ PERIOD ________

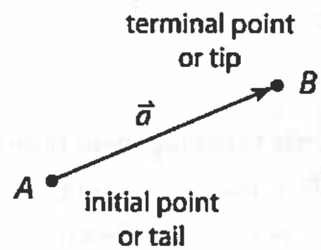
8-7 Geometric Vector Operations

Some quantities are described by real number quantities known as a *scalar*, which describes the *magnitude* or size of the quantity.

Vectors describe both the magnitude and direction of the quantity.

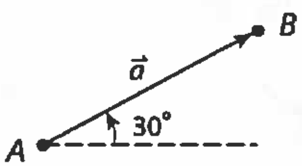
Examples:

Scalar: a speed of 5 miles per hour
Vector: 5 miles per hour due north

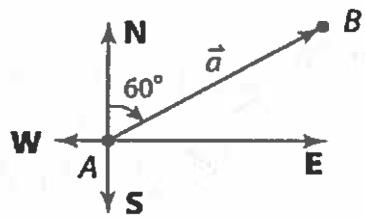


The magnitude of $\overline{AB}$, denoted $|\overline{AB}|$, is the length of the vector from the initial point to its terminal point.

The direction of a vector can be expressed as the angle it forms with the horizontal or as a measurement between 0° and 90° east or west of the north-south line.



The direction of $\vec{a}$ is 30° relative to the horizontal.

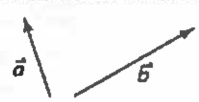


The direction of $\vec{a}$ is 60° east of north.

The sum of two or more vectors is a single vector called the resultant.

KeyConcept Vector Addition

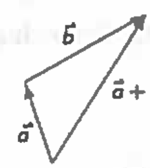
To find the resultant of $\vec{a}$ and $\vec{b}$, use one of the following methods.



Triangle Method

Step 1 Translate $\vec{b}$ so that the tail of $\vec{b}$ touches the tip of $\vec{a}$.

Step 2 Draw the resultant vector from the tail of $\vec{a}$ to the tip of $\vec{b}$.



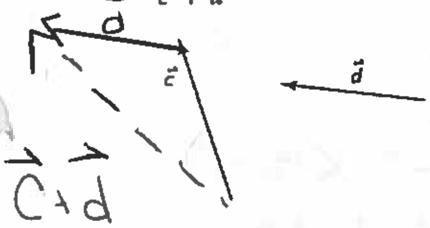
KeyConcept Vector Operations

If $\langle a, b\rangle$ and $\langle c, d\rangle$ are vectors and k is a scalar, then the following are true.

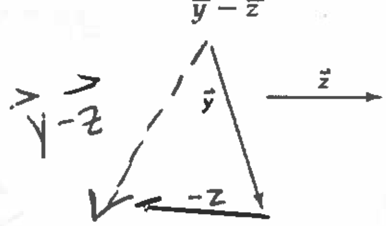
Vector Addition	$\langle a, b\rangle + \langle c, d\rangle = \langle a+c, b+d\rangle$
Vector Subtraction	$\langle a, b\rangle - \langle c, d\rangle = \langle a-c, b-d\rangle$
Scalar Multiplication	$k\langle a, b\rangle = \langle ka, kb\rangle$

Copy the vectors. Then find each sum or difference.

1. $\vec{c}+\vec{d}$



2. $\vec{y}-\vec{z}$



8-7 Vectors on the Coordinate Plane

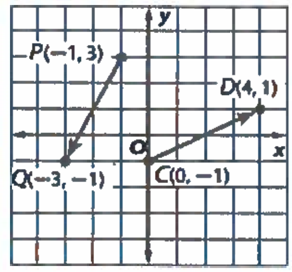
A vector in standard position is if its initial point is at the origin. In this position, a vector can be uniquely described by its terminal point $P(x,y)$.

To describe a vector with any initial point, you can use the component form $\langle x, y \rangle$, which describes the vector in terms of its horizontal component x and vertical component y.

To write the component form of a vector with initial point (x_1,y_1) and terminal point (x_2,y_2), find $\langle x_2 - x_1, y_2 - y_1 \rangle$.

Write the component form of $\overrightarrow{CD}$.

$\overrightarrow{CD} = \langle x_2 - x_1, y_2 - y_1 \rangle$	Component form of a vector
$= \langle 4 - 0, 1 - (-1) \rangle$	$(x_1, y_1) = (0, -1)$ and $(x_2, y_2) = (4, 1)$
$= \langle 4, 2 \rangle$	Simplify.



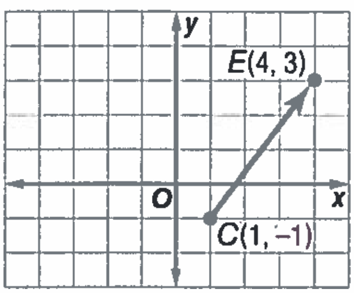
3. Write the component form of $\overrightarrow{PQ}$.

$\langle -1+3, 3+1 \rangle$

$\langle 2, 4 \rangle$

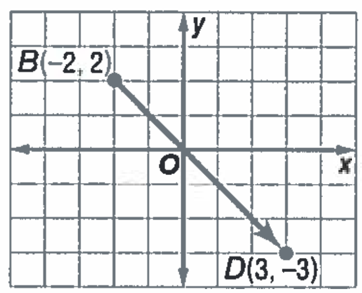
Write the component form of each vector.

4.



$\langle 3, 4 \rangle$

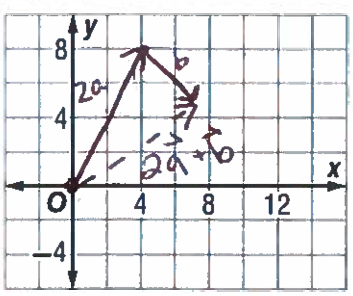
5.



$\langle 5, -5 \rangle$

Find each of the following for $\vec{a} = \langle 2, 4 \rangle$, $\vec{b} = \langle 3, -3 \rangle$, and $\vec{c} = \langle 4, -1 \rangle$. Check your answers graphically.

6. $2\vec{a} + \vec{b}$

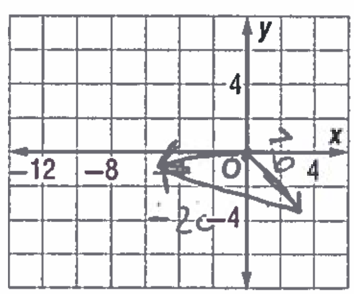


$2\langle 2,4 \rangle + \langle 3,-3 \rangle$

$\langle 4,8 \rangle + \langle 3,-3 \rangle$

$\boxed{\langle 7,5 \rangle}$

7. $\vec{b} - 2\vec{c}$



$-2\langle c \rangle = \langle -8, 2 \rangle$

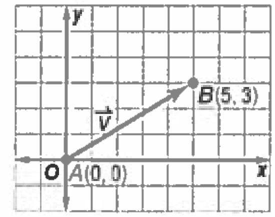
$\langle 3,-3 \rangle - 2\langle 4,-1 \rangle$

$\langle 3,-3 \rangle + \langle -8, 2 \rangle$

$\boxed{\langle -5, -1 \rangle}$

8-7 Magnitude and Direction of a Vector

The vector at the right can be expressed as $\vec{v} = \langle 5, 3\rangle$.

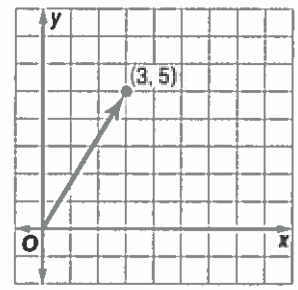


You can use the Distance Formula to find the magnitude $|\overline{AB}|$ of a vector.

You can describe the direction of a vector by measuring the angle that the vector forms with the positive x-axis or with any other horizontal line using trigonometric ratios.

Example: Find the magnitude and direction of $\vec{a} = \langle 3, 5\rangle$.

Find the magnitude.



$\vec{a} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(3-0)^2 + (5-0)^2}$	$(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (3, 5)$
$= \sqrt{34}$ or about 5.8	Simplify.

To find the direction, use the tangent ratio.

$\tan\theta = \frac{5}{3}$	The tangent ratio is opposite over adjacent.
$m\angle\theta \approx 59.0$	Use a calculator.

The magnitude of the vector is about 5.8 units and its direction is 59°.

Find the magnitude and direction of each vector.

8. $\vec{b} = \langle -5, 2\rangle$

$\sqrt{(-5-0)^2 + (2-0)^2}$

$\sqrt{29} \approx \boxed{5.4}$

$\tan\theta = \frac{2}{5}$

$\theta = \tan^{-1}\left(\frac{2}{5}\right)$

$\boxed{\theta = 21.8^\circ}$

$\theta = 180 - 21.8 = 158.2^\circ$

9. $\vec{c} = \langle -2, 1\rangle$ $\boxed{mag = 2.2}$ $\boxed{\theta = 153.4^\circ}$

$\tan^{-1}\left(\frac{1}{2}\right) = \alpha$

$26.6 = \alpha$

$180 - 26.6 = \boxed{153.4^\circ}$

10. $\vec{d} = \langle 3, 4\rangle$

$\boxed{mag = 5}$

$\tan\theta = \frac{4}{3}$

$\boxed{\theta = 53.1^\circ}$

11. $\vec{m} = \langle 5, -1\rangle$ $\boxed{mag = 5.1}$ $\boxed{\theta = 348.7^\circ}$

$\alpha = \tan^{-1}\left(\frac{1}{5}\right)$

$\alpha = 11.3$

$360 - 11.3 = 347.7^\circ$

12. $\vec{r} = \langle -3, -4\rangle$

$\boxed{mag = 5}$ $\boxed{\theta = 233.1^\circ}$

$\tan(\alpha) = \frac{4}{3}$

$\alpha = 53.1^\circ$

$\theta = 180 + 53.1 = 233.1$

13. $\vec{v} = \langle -4, 1\rangle$ $\boxed{mag = 4.1}$ $\boxed{\theta = 166.0^\circ}$

$\alpha = \tan^{-1}\left(\frac{1}{4}\right)$

$\alpha = 14.0^\circ$

$180 - 14.0^\circ = \boxed{166^\circ}$

14. $\vec{t} = \langle 2, 12\rangle$

$\boxed{mag \approx 12.2}$

$\tan\theta = \frac{12}{2}$

$\boxed{\theta = 80.5^\circ}$

15. $\vec{k} = \langle -8, -3\rangle$ $\boxed{mag = 8.5}$ $\boxed{\theta = 200.6^\circ}$

$\alpha = \tan^{-1}\left(\frac{3}{8}\right)$

$\alpha = 20.6$

$180 + 20.6 = 200.6^\circ$