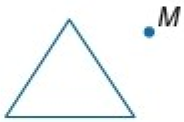
9-6 Dilations

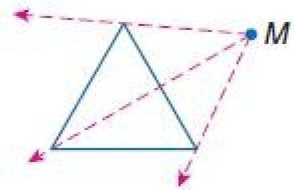
Copy the figure and point *M*. Then use a ruler to draw the image of the figure under a dilation with center *M* and the scale factor *k* indicated.

1. $k = \frac{1}{4}$

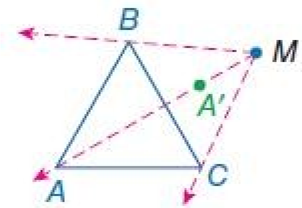


SOLUTION:

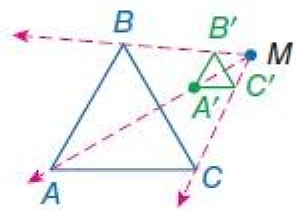
Step 1: Draw rays from *M* though each vertex.



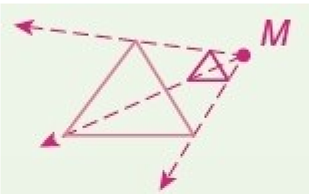
Step 2: Locate *A′* on $\overrightarrow{MA}$ such that $MA' = \frac{1}{2}MA$.



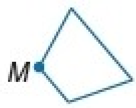
Step 3: Locate *B′* on $\overrightarrow{MB}$ and *C′* on $\overrightarrow{MC}$ in the same way. Then draw Δ*A′B′C′*.



ANSWER:

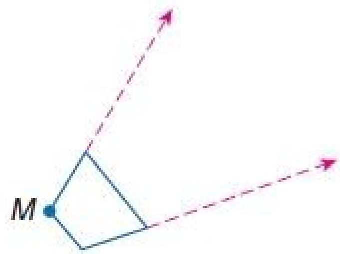


2. $k = 2$

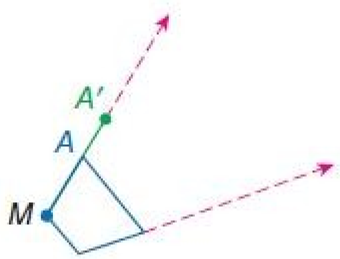


SOLUTION:

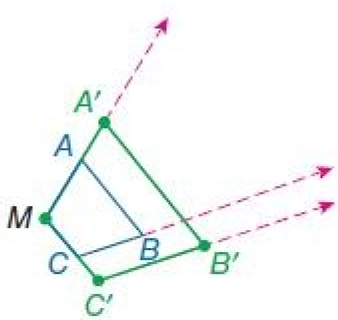
Step 1: Draw rays from *M*.



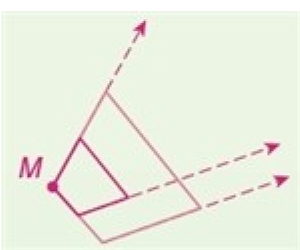
Step 2: Locate *A′* on $\overrightarrow{MA}$ such that $MA' = 2MA$.



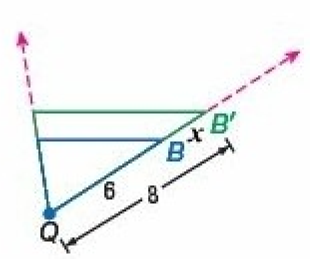
Step 3: Locate *B′* on $\overrightarrow{MB}$, and *C′* on $\overrightarrow{MC}$, an the same way. Then draw *A'B'C'M*.



ANSWER:



3. Determine whether the dilation from Figure B to B' is an e*nlargement* or a reduction. Then find the scale factor of the dilation and x.



SOLUTION:

The figure B' is larger than the figure B, so the dilation an enlargement.

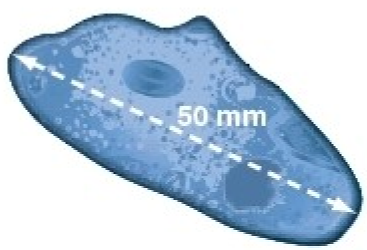
The scale factor is $\frac{8}{6}$ or $\frac{4}{3}$.

The value of x is 8 – 6 or 2.

ANSWER:

enlargement; $\frac{4}{3}$; 2

4. **BIOLOGY** Under a microscope, a single-celled organism 200 microns in length appears to be 50 millimeters long. If 1 millimeter = 1000 microns, what magnification setting (scale factor) was used? Explain your reasoning.



SOLUTION:

250×; The organism's length in millimeters is 200 ÷ 1000 or 0.2 mm.

The scale factor of the dilation is $\frac{50}{0.2}$ or 250.

ANSWER:

250×; The organism's length in millimeters is 200 ÷ 1000 or 0.2 mm. The scale factor of the dilation is $\frac{50}{0.2}$ or 250.

Graph the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.

5. $W(0, 0)$, $X(6, 6)$, $Y(6, 0)$; $k = 1.5$

SOLUTION:

Multiply the x- and y-coordinates of each vertex by the scale factor k.

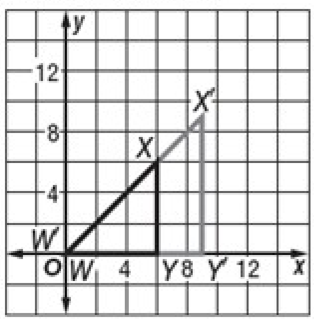
$(x, y) \rightarrow (kx, ky)$

Multiply the x- and y- coordinates by 1.5.

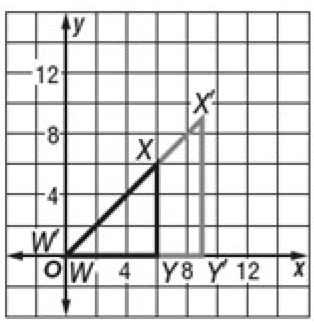
$W(0,0) \rightarrow W'(0,0)$

$X(6,6) \rightarrow X'(9,9)$

$Y(6,0) \rightarrow Y'(9,0)$



ANSWER:



6. $Q(-4, 4)$, $R(-4, -4)$, $S(4, -4)$, $T(4, 4)$; $k = \frac{1}{2}$

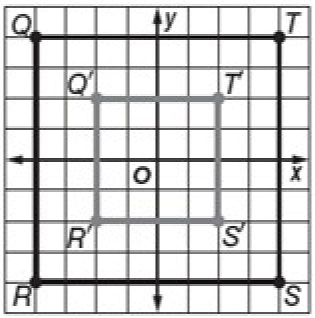
SOLUTION:

Multiply the x- and y-coordinates of each vertex by the scale factor k.

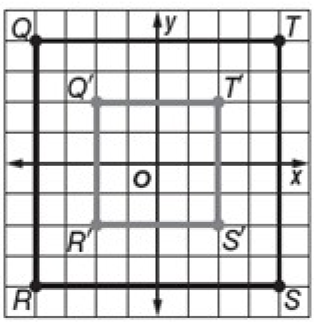
$(x, y) \to (kx, ky)$.

Multiply the x- and y-coordinates by 1/2.

$$Q(-4,4) \to Q'(-2,2)$$
$$R(-4,-4) \to R'(-2,-2)$$
$$S(4,-4) \to S'(2,-2)$$
$$T(4,4) \to T'(2,2)$$



ANSWER:



7. $A(–1, 4)$, $B(2, 4)$, $C(3, 2)$, $D(–2, 2)$; $k = 2$

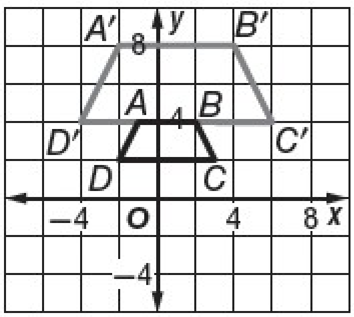
SOLUTION:

Multiply the x- and y-coordinates of each vertex by the scale factor k.

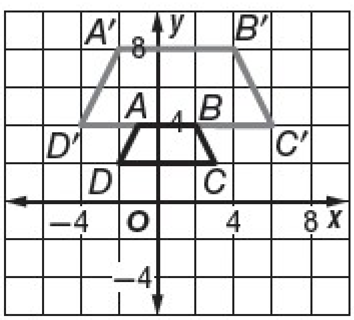
$(x, y) \rightarrow (kx, ky)$.

Multiply the x- and y- coordinates by 2.

$$A(-1,4) \rightarrow A'(-2,8)$$
$$B(2,4) \rightarrow B'(4,8)$$
$$C(3,2) \rightarrow C'(6,4)$$
$$D(-2,2) \rightarrow D'(-4,4)$$



ANSWER:



8. $J(-2, 0)$, $K(2, 4)$, $L(8, 0)$, $M(2, -4)$; $k = \frac{3}{4}$

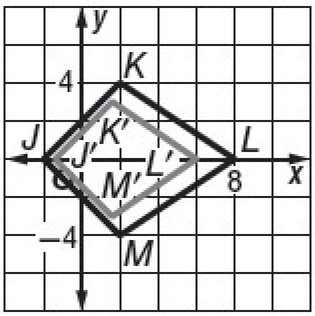
SOLUTION:

Multiply the x- and y-coordinates of each vertex by the scale factor k.

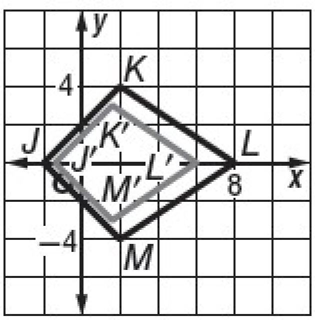
$(x, y) \rightarrow (kx, ky)$.

Multiply the x- and y-coordinates by the scale factor 3/4.

$$J(-2,0) \rightarrow J'(-1.5,0)$$
$$K(2,4) \rightarrow K'(1.5,3)$$
$$L(8,0) \rightarrow L'(6,0)$$
$$M(2,-4) \rightarrow M'(1.5,-3)$$

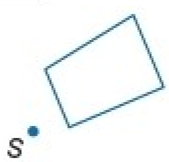


ANSWER:



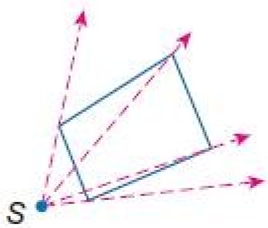
CCSS TOOLS Copy the figure and point *S*. Then use a ruler to draw the image of the figure under a dilation with center *S* and the scale factor *k* indicated.

9. $k = \frac{5}{2}$

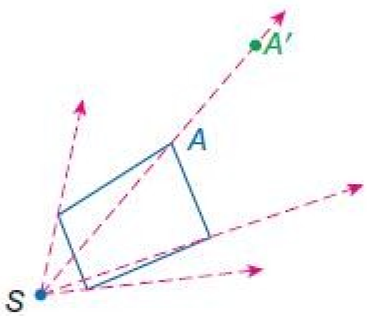


SOLUTION:

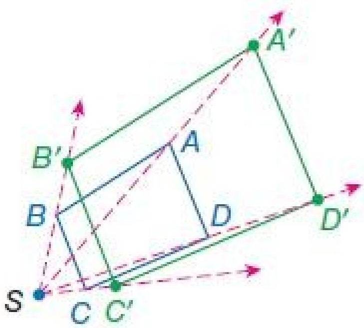
Step 1: Draw rays from *S* though each vertex.



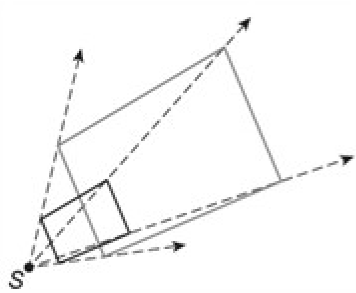
Step 2: Locate A' on $\overrightarrow{SA}$ such that $SA' = \frac{5}{2}SA$.



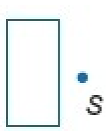
Step 3: Locate B' on $\overrightarrow{SB}$, C' on $\overrightarrow{SC}$, and D' on $\overrightarrow{SD}$ in the same way. Then draw $A'B'C'D'$.



ANSWER:

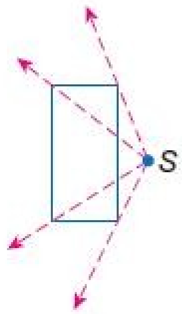


10. $k = 3$

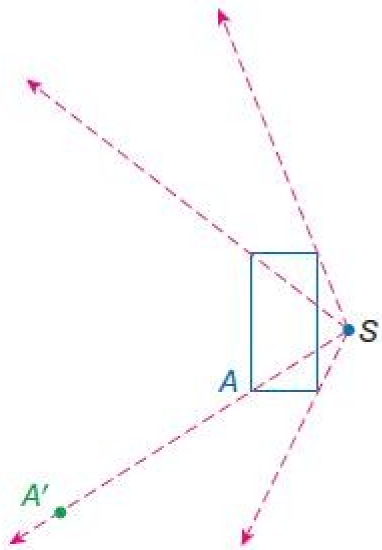


SOLUTION:

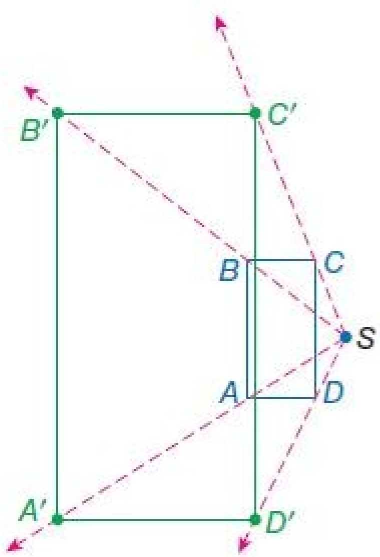
Step 1: Draw rays from S though each vertex.

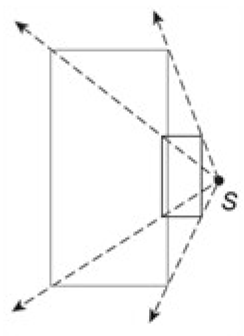


Step 2: Locate A' on $\overrightarrow{SA}$ such that $SA' = 3SA$.

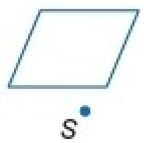


Step 3: Locate B' on $\overrightarrow{SB}$, C' on $\overrightarrow{SC}$,and D' on $\overrightarrow{SD}$ in the same way. Then draw $A'B'C'D'$.



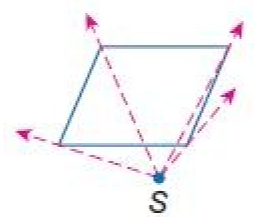


11. $k = 0.8$

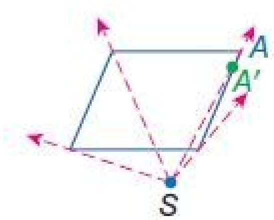


SOLUTION:

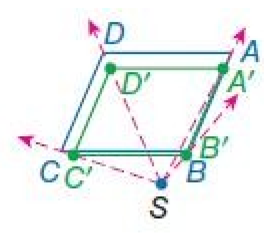
Step 1: Draw rays from *S* though each vertex.



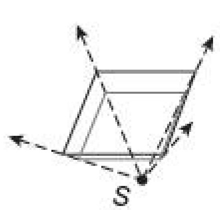
Step 2: Locate *A′* on $\overrightarrow{SA}$ such that $SA' = 0.8SA$.



Step 3: Locate *B′* on $\overrightarrow{SB}$, *C′* on $\overrightarrow{SC}$, and *D′* on $\overrightarrow{SD}$ in the same way. Then draw *A′B′C′D′*.



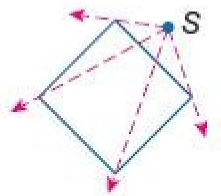
ANSWER:



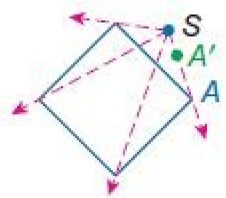
12. $k = \frac{1}{3}$

SOLUTION:

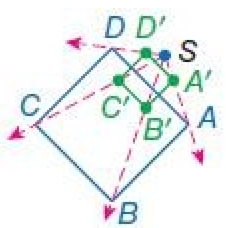
Step 1: Draw rays from *S* though each vertex.



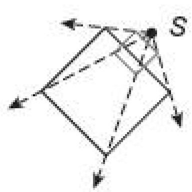
Step 2: Locate *A′* on $\overrightarrow{SA}$ such that $SA' = \frac{1}{3}SA$.



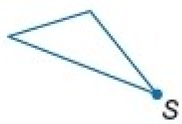
Step 3: Locate *B′* on $\overrightarrow{SB}$, *C′* on $\overrightarrow{SC}$, and *D′* on $\overrightarrow{SD}$ in the same way. Then draw *A′B′C′D′*.



ANSWER:

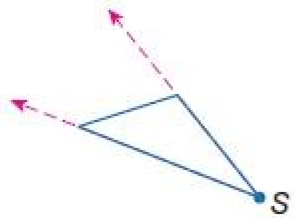


13. k = 2.25

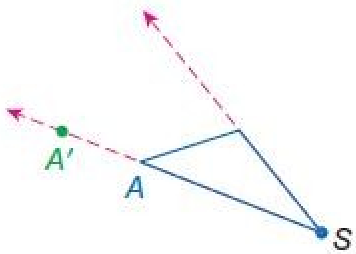


SOLUTION:

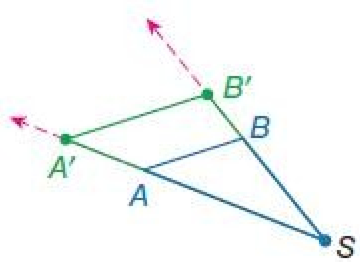
Step 1: Draw rays from S though each vertex.



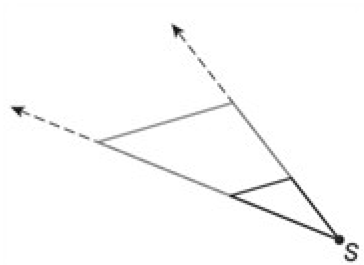
Step 2: Locate A' on $\overrightarrow{SA}$ such that $SA' = 2.25SA$.



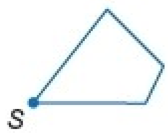
Step 3: Locate B' on $\overrightarrow{SB}$ in the same way. Then draw $\Delta A'B'S$.



ANSWER:

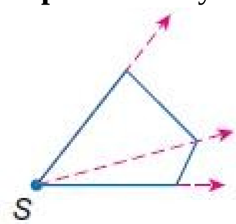


14. $k = \frac{7}{4}$

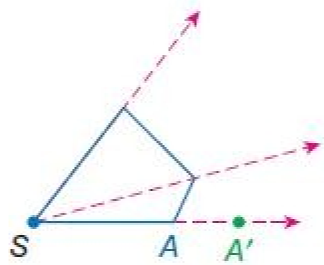


SOLUTION:

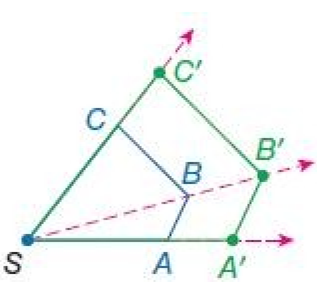
Step 1: Draw rays from *S* though each vertex.



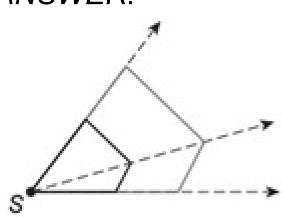
Step 2: Locate *A′* on $\overrightarrow{SA}$ such that $SA' = \frac{7}{4}SA$.



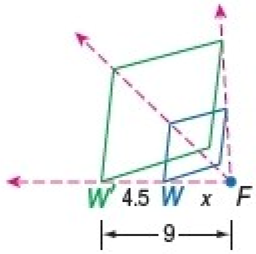
Step 3: Locate *B′* on $\overrightarrow{SB}$ and *C′* on $\overrightarrow{SC}$ in the same way. Then draw *A′B′C′S*.



ANSWER:



Determine whether the dilation from figure *W* to W' is an enlargement or a reduction. Then find the scale factor of the dilation and *x*.

15. 

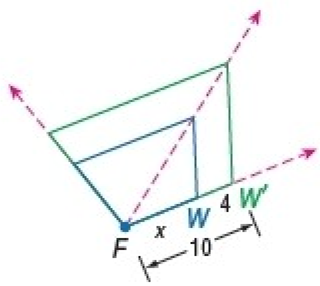
SOLUTION:

The figure W' is larger than the figure *W*, so the dilation an enlargement.
The value of *x* is 9 – 4.5 or 4.5.

The scale factor is $\frac{9}{4.5}$ or 2.

ANSWER:

enlargement;
2; 4.5

16. 

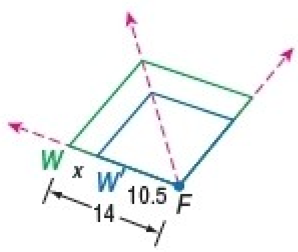
SOLUTION:

The figure W' is larger than the figure *W*, so the dilation an enlargement.
The value of *x* is 10 – 4 or 6.

The scale factor is $\frac{10}{6}$ or $\frac{5}{3}$.

ANSWER:

enlargement; $\frac{5}{3}$; 6



17.

SOLUTION:

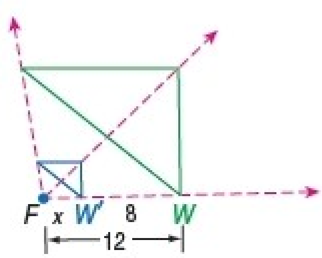
The figure W' is smaller than the figure *W*, so the dilation a reduction.

The scale factor is $\frac{10.5}{14}$ or $\frac{3}{4}$.

The value of *x* is 14 – 10.5 or 3.5.

ANSWER:

reduction; $\frac{3}{4}$; 3.5



18.

SOLUTION:

The figure W' is smaller than the figure *W*, so the dilation a reduction.
The value of *x* is 12 – 8 or 4.

The scale factor is $\frac{4}{12}$ or $\frac{1}{3}$.

ANSWER:

reduction; $\frac{1}{3}$; 4

INSECTS When viewed under a microscope, each insect has the measurement given on the picture. Given the actual measure of each insect, what magnification was used? Explain your reasoning.

19. Refer to the photo on page 664.

SOLUTION:

15×; The insect's image length in millimeters is 3.75×10 or 37.5 mm.

The scale factor of the dilation is $\frac{37.5}{2.5}$ or 15.

ANSWER:

15×; The insect's image length in millimeters is 3.75×10 or 37.5 mm. The scale factor of the dilation is $\frac{37.5}{2.5}$ or 15.

20. Refer to the photo on page 664.

SOLUTION:

96×; The insect's image length in millimeters is $4.8 \cdot 10$ or 48 mm.

The scale factor of the dilation is $\frac{48}{0.5}$ or 96.

ANSWER:

96×; The insect's image length in millimeters is $4.8 \cdot 10$ or 48 mm. The scale factor of the dilation is $\frac{48}{0.5}$ or 96.

CCSS SENSE-MAKING Find the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.

21. $J(–8, 0)$, $K(–4, 4)$, $L(–2, 0)$; $k = 0.5$

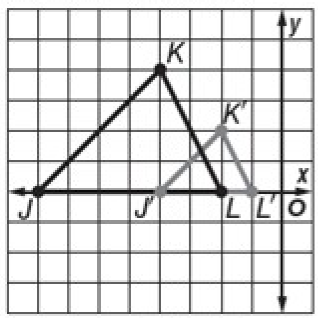
SOLUTION:

Multiply the x- and y-coordinates of each vertex by the scale factor k. That is, $(x, y) \rightarrow (kx, ky)$.
Here multiply the x- and y- coordinates by the scale factor 0.5.

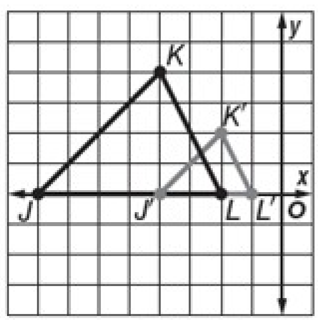
$J(-8,0) \rightarrow J'(-4,0)$

$K(-4,4) \rightarrow K'(-2,2)$

$L(-2,0) \rightarrow L'(-1,0)$



ANSWER:

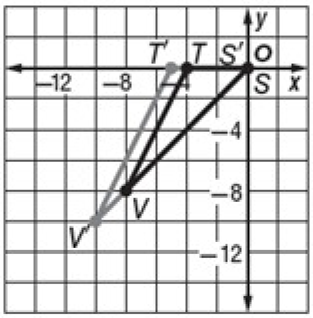


22. $S(0, 0)$, $T(–4, 0)$, $V(–8, –8)$; $k = 1.25$

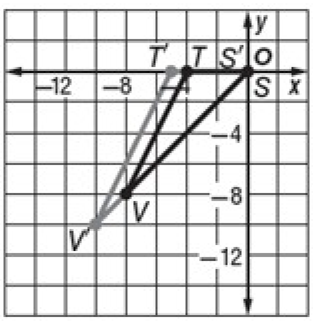
SOLUTION:

Multiply the x- and y-coordinates of each vertex by the scale factor k. That is, $(x, y) \rightarrow (kx, ky)$.
Here multiply the x- and y- coordinates by the scale factor 1.25.

$$S(0,0) \rightarrow S'(0,0)$$
$$T(-4,0) \rightarrow T'(-5,0)$$
$$V(-8,-8) \rightarrow V'(-10,-10)$$



ANSWER:



23. $A(9, 9)$, $B(3, 3)$, $C(6, 0)$; $k = \frac{1}{3}$

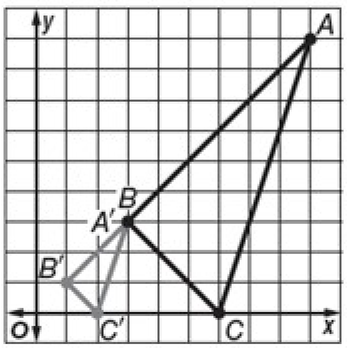
SOLUTION:

Multiply the x- and y-coordinates of each vertex by the scale factor k. That is, $(x, y) \to (kx, ky)$.
Here multiply the x- and y- coordinates by the scale factor 1/3.

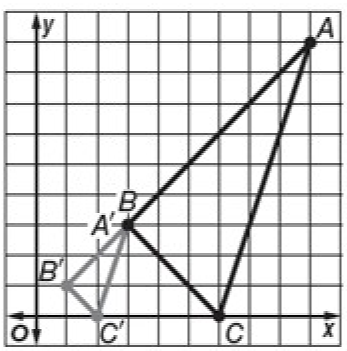
$A(9,9) \to A'(3,3)$

$B(3,3) \to B'(1,1)$

$C(6,0) \to C'(2,0)$



ANSWER:



24. $D(4, 4)$, $F(0, 0)$, $G(8, 0)$; $k = 0.75$

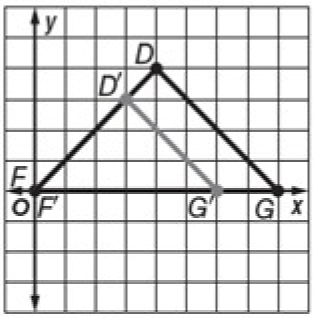
SOLUTION:

Multiply the x- and y-coordinates of each vertex by the scale factor k. That is, $(x, y) \to (kx, ky)$.
Here multiply the x- and y- coordinates by the scale factor 0.75.

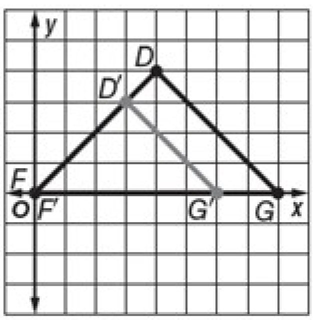
$D(4,4) \to D'(3,3)$

$F(0,0) \to F'(0,0)$

$G(8,0) \to G'(6,0)$



ANSWER:



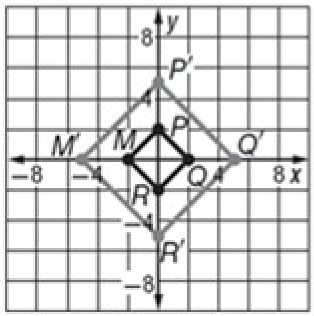
9-6 Dilations

25. $M(-2, 0)$, $P(0, 2)$, $Q(2, 0)$, $R(0, -2)$; $k = 2.5$

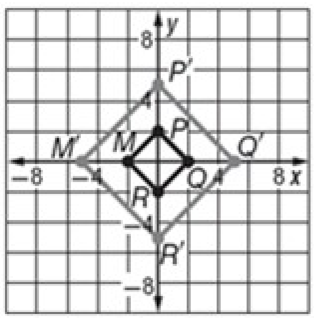
SOLUTION:

Multiply the x- and y-coordinates of each vertex by the scale factor k. That is, $(x, y) \rightarrow (kx, ky)$.
Here multiply the x- and y-coordinates by the scale factor 2.5.

$$M(-2,0) \rightarrow M'(-5,0)$$
$$P(0,2) \rightarrow P'(0,5)$$
$$Q(2,0) \rightarrow Q'(5,0)$$
$$R(0,-2) \rightarrow R'(0,-5)$$



ANSWER:



9-6 Dilations

26. $W(2, 2), X(2, 0), Y(0, 1), Z(1, 2)$; $k = 3$

SOLUTION:

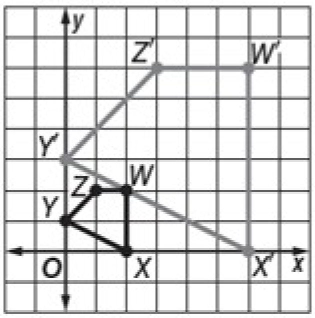
Multiply the x- and y-coordinates of each vertex by the scale factor k. That is, $(x, y) \to (kx, ky)$.
Here multiply the x- and y-coordinates by the scale factor 3.

$W(2,2) \to W'(6,6)$

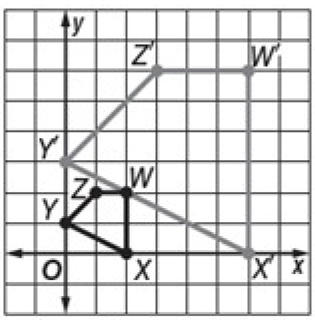
$X(2,0) \to X'(6,0)$

$Y(0,1) \to Y'(0,3)$

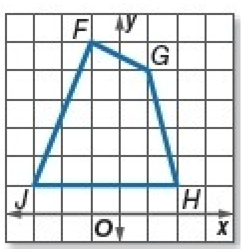
$Z(1,2) \to Z'(3,6)$



ANSWER:



27. **COORDINATE GEOMETRY** Refer to the graph of *FGHJ*.



a. Dilate *FGHJ* by a scale factor of $\frac{1}{2}$ centered at the origin, and then reflect the dilated image in the y-axis.

b. Complete the composition of transformations in part a in reverse order.

c. Does the order of the transformations affect the final image?

d. Will the order of a composition of a dilation and a reflection *sometimes*, *always*, or *never* affect the dilated image? Explain your reasoning.

SOLUTION:

a. Multiply the x- and y-coordinates of each vertex by the scale factor $\frac{1}{2}$.

$$\left(x, y\right) \Rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$$

$$F(-1, 6) \Rightarrow F'\left(-\frac{1}{2}, 3\right)$$

$$G(1, 5) \Rightarrow G'\left(\frac{1}{2}, \frac{5}{2}\right)$$

$$H(2, 1) \Rightarrow H'\left(1, \frac{1}{2}\right)$$

$$J(-3, 1) \Rightarrow J'\left(-\frac{3}{2}, \frac{1}{2}\right)$$

Reflect in the y-axis.

$$\left(x, y\right) \Rightarrow (-x, y)$$

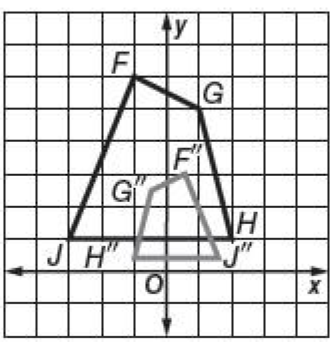
$$F'\left(-\frac{1}{2}, 3\right) \Rightarrow F''\left(\frac{1}{2}, 3\right)$$

$$G'\left(\frac{1}{2}, \frac{5}{2}\right) \Rightarrow G''\left(-\frac{1}{2}, \frac{5}{2}\right)$$

$$H'\left(1, \frac{1}{2}\right) \Rightarrow H''\left(-1, \frac{1}{2}\right)$$

$$J'\left(-\frac{3}{2}, \frac{1}{2}\right) \Rightarrow J''\left(\frac{3}{2}, \frac{1}{2}\right)$$

Then graph the image and the transformed image.



b. First reflect the image in the y-axis.

$$(x, y) \Rightarrow (-x, y)$$

$$F(-1, 6) \Rightarrow F'(1, 6)$$

$$G(1, 5) \Rightarrow G'(-1, 5)$$

$$H(2, 1) \Rightarrow H'(-2, 1)$$

$$J(-3, 1) \Rightarrow J'(3, 1)$$

Then dilate the image by a scale factor of $\frac{1}{2}$.

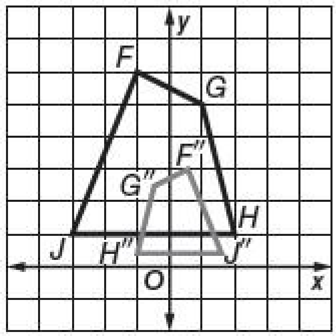
$$(x, y) \Rightarrow \left(\tfrac{1}{2}x, \tfrac{1}{2}y\right)$$

$$F'(1, 6) \Rightarrow F''\left(\tfrac{1}{2}, 3\right)$$

$$G'(-1, 5) \Rightarrow G''\left(-\tfrac{1}{2}, \tfrac{5}{2}\right)$$

$$H'(-2, 1) \Rightarrow H''\left(-1, \tfrac{1}{2}\right)$$

$$J'(3, 1) \Rightarrow J''\left(\tfrac{3}{2}, \tfrac{1}{2}\right)$$

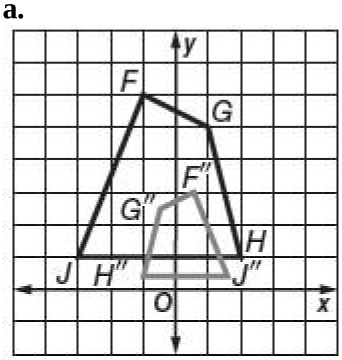


c. No; the coordinates of the final image are the same regardless of which transformation was done first.

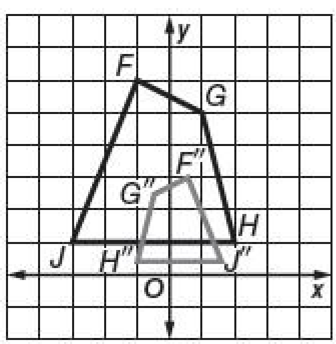
d. Sometimes; sample answer: for the order of a composition of a dilation and a reflection to be unimportant, the dilation must be about the origin or a point on the line of reflection and the line of reflection must contain the origin, or must be of the form $y = mx$.

ANSWER:

a.



b.



c. no

d. Sometimes; sample answer: for the order of a composition of a dilation and a reflection to be unimportant, the dilation must be about the origin or a point on the line of reflection and the line of reflection must contain the origin, or must be of the form $y = mx$.

28. **PHOTOGRAPHY AND ART** To make a grid drawing in the style of Chuck Close, students overlay a $\frac{1}{4}$-inch grid on a 5-inch by 7-inch high contrast photo, overlay a $\frac{1}{2}$ –inch grid on a 10-inch by 14-inch piece of drawing paper, and then sketch the image in each square of the photo to the corresponding square on the drawing paper.

a. What is the scale factor of the dilation?

b. To create an image that is 10 times as large as the original, what size grids are needed?

c. What would be the area of a grid drawing of a 5-inch by 7-inch photo that used 2-inch grids?

SOLUTION:

a. Students overlay a $\frac{1}{4}$-inch grid on a 5-inch by 7-inch high contrast photo and overlay a $\frac{1}{2}$-inch grid on a 10-inch by 14-inch piece of drawing paper.
The drawing paper is twice the contrast photo.
Therefore, the scale factor is 2:1.

b. The original size of the grid is $\frac{1}{4}$-inch. To make it to 10 times larger, multiply by 10.

$$\frac{1}{4} \cdot 10 = 2.5$$

They need 2.5 inch grid.

c. When they use 2-inch grid that would be 8 times larger than the original photo.
The dimension of the photo would be (5)(8) or 40-inch by (7)(8) or 56-inch.
To find the area of the grid drawing multiply the length times the width.
(40)(56) = 2240
Therefore, the area is 2240 in^2.

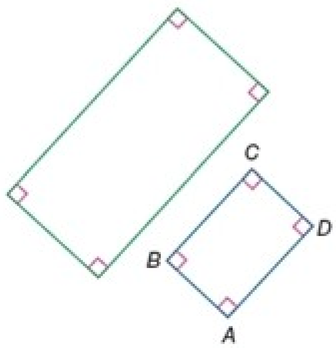
ANSWER:

a. 2 : 1

b. 2.5 in.

c. 2240 in^2

29. **MEASUREMENT** Determine whether the image shown is a dilation of *ABCD*. Explain your reasoning.



SOLUTION:

For the two rectangles to be similar, the ratios of the lengths of each side of the image to that of the original should be the same. Use a ruler to measure the long side of the image and $\overline{BC}$.

$$\frac{3.8\text{ cm}}{1.9\text{ cm}} = 2$$

Next, compare the lengths of the short side of the image and $\overline{BA}$.

$$\frac{1.9\text{ cm}}{1.3\text{ cm}} \approx 1.46$$

Since the measures of the sides of the rectangles are not proportional, so they are not similar and cannot be a dilation.

ANSWER:

No; sample answer: The measures of the sides of the rectangles are not proportional, so they are not similar and cannot be a dilation.

30. **COORDINATE GEOMETRY** *WXYZ* has vertices *W*(6, 2), *X*(3, 7), *Y*(−1, 4), and *Z*(4, −2).

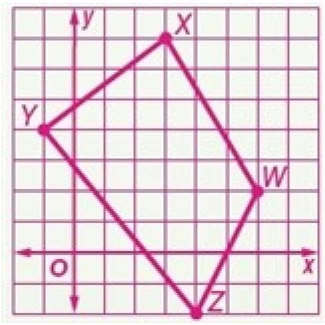
a. Graph *WXYZ* and find the perimeter of the figure. Round to the nearest tenth.

b. Graph the image of *WXYZ* after a dilation of $\frac{1}{2}$ centered at the origin.

c. Find the perimeter of the dilated image. Round to the nearest tenth. How is the perimeter of the dilated image related to the perimeter of *WXYZ*?

SOLUTION:

a. Graph *WXYZ*.



Use the distance formula to find the distance between each vertex.

$$\begin{aligned} WX &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-6)^2 + (7-2)^2} \\ &= \sqrt{(-3)^2 + 5^2} \\ &= \sqrt{34} \\ &\approx 5.8 \end{aligned}$$

$$\begin{aligned} XY &= \sqrt{((-1)-3)^2 + (4-7)^2} \\ &= \sqrt{(-4)^2 + (-3)^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} YZ &= \sqrt{(4-(-1))^2 + ((-2)-4)^2} \\ &= \sqrt{(5)^2 + (-6)^2} \\ &= \sqrt{61} \\ &\approx 7.8 \end{aligned}$$

$$\begin{aligned} ZW &= \sqrt{(4-6)^2 + ((-2)-2)^2} \\ &= \sqrt{(-2)^2 + (-4)^2} \\ &= \sqrt{20} \\ &\approx 4.5 \end{aligned}$$

The perimeter of the figure is 23.1.

b. Multiply the x- and y-coordinates of each vertex by the scale factor k.

$(x, y) \to (kx, ky)$.

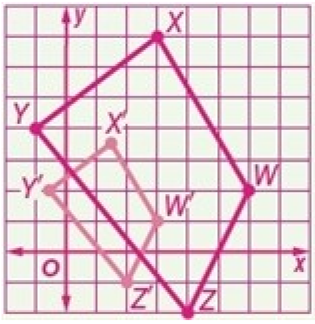
Multiply the x- and y-coordinates by the scale factor 1/2.

$W(6,2) \to W'(3,1)$

$X(3,7) \to X'(1.5,3.5)$

$Y(-1,4) \to Y'(-0.5,2)$

$Z(4,-2) \to Z'(2,-1)$



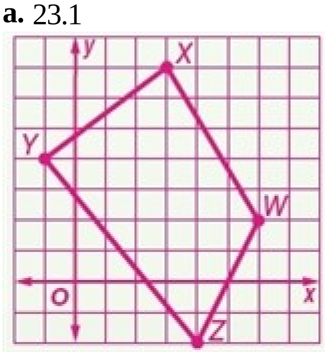
c. Use the distance formula to find the distance between each vertex.

$$W'X' = \sqrt{(1.5-3)^2 + (3.5-1)^2}$$
$$= \sqrt{(-1.5)^2 + (2.5)^2}$$
$$= \sqrt{8.5}$$
$$\approx 2.9$$
$$X'Y' = \sqrt{((-0.5)-1.5)^2 + (2-3.5)^2}$$
$$= \sqrt{(-2)^2 + (-1.5)^2}$$
$$= \sqrt{6.25}$$
$$= 2.5$$
$$Y'Z' = \sqrt{(2-(-0.5))^2 + ((-1)-2)^2}$$
$$= \sqrt{(2.5)^2 + (-3)^2}$$
$$= \sqrt{15.25}$$
$$\approx 3.9$$
$$Z'W' = \sqrt{(3-2)^2 + (1-(-1))^2}$$
$$= \sqrt{(1)^2 + (2)^2}$$
$$= \sqrt{5}$$
$$\approx 2.2$$

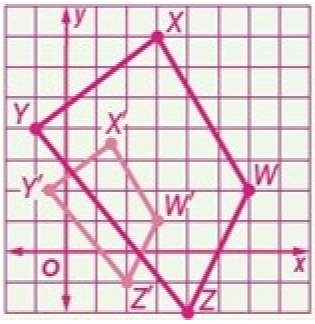
The perimeter of the dilated figure is 11.6.
The perimeter of the dilated figure is half of the perimeter of *WXYZ*.

ANSWER:

a. 23.1

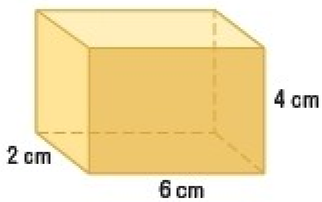


b.



c. 11.6; The perimeter of the dilated figure is half of the perimeter of *WXYZ*.

31. **CHANGING DIMENSIONS** A three-dimensional figure can also undergo a dilation. Consider the rectangular prism shown.



a. Find the surface area and volume of the prism.
b. Find the surface area and volume of the prism after a dilation with a scale factor of 2.
c. Find the surface area and volume of the prism after a dilation with a scale factor of $\frac{1}{2}$.
d. How many times as great is the surface area and volume of the image as the preimage after each dilation?
e. Make a conjecture as to the effect a dilation with a positive scale factor r would have on the surface area and volume of a prism.

SOLUTION:

a. The surface area of a rectangular prism is given by $S = 2(lb + bh + hl)$, where l is the length, b is the base, and h is the height.
Substitute 6 for l, 2 for w, and 4 for h in the formula.

$$\begin{aligned} S &= 2(6\cdot 2 + 2\cdot 4 + 4\cdot 6) \\ &= 2(12 + 8 + 24) \\ &= 2(44) \\ &= 88 \end{aligned}$$

The surface area of the rectangular prism is 88 cm^2.
The volume of a rectangular prism is given by $V = lbh$.
Substitute.

$$\begin{aligned} V &= 6\cdot 2\cdot 4 \\ &= 48 \end{aligned}$$

The volume of the rectangular prism is 48 cm^3.

b. When the figure is dilated by a scale factor 2, the surface area and the volume of the dilated figure will be 2^2 or 4 times of the surface area and 2^3 or 8 times of the volume of the original image.
The surface area of the dilated figure would be 88×4 or 352 cm^2.
The volume of the dilated figure would be 48×8 or 384 cm^3.

c. When the figure is dilated by a scale factor $\frac{1}{2}$, the surface area and the volume of the dilated figure will be $\left(\frac{1}{2}\right)^2$ or $\frac{1}{4}$ times of the surface area and $\left(\frac{1}{2}\right)^3$ or $\frac{1}{8}$ times of the volume of the original image.

The surface area of the dilated figure would be $88 \times \frac{1}{4}$ or 22 cm^2.

The volume of the dilated figure would be $48 \times \frac{1}{8}$ or 6 cm^3.

d. surface area:

surface area of preimage: 88 cm^2

surface area of image with scale factor 2: 352 cm^2 or (88 × 4) cm^2

The surface area is 4 times greater after dilation with scale factor 2; $\frac{1}{4}$ as great after dilation with scale factor $\frac{1}{2}$.

volume:

volume of preimage: 48 cm^3

volume of image with scale factor 2: 384 cm^3 or $\left(48 \times \frac{1}{8}\right)$ cm^3

The volume is 8 times greater after dilation with scale factor 2; $\frac{1}{8}$ as great after dilation with scale factor $\frac{1}{2}$.

e. From part **d**, when a scale factor of 2 is applied, the surface area of the preimage would be multiplied by 4. When a scale factor of 1/2 is applied, the surface area of the preimage would be multiplied by 1/4. So, if a positive scale factor r is applied, the surface area of the preimage would be multiplied by r^2. For volume, when the scale factor 2 was applied, the volume of the image was 8 times the volume of the preimage. When the scale factor was 1/2, the volume of the image was 1/8 times the volume of the preimage. So, when a scale factor of r is applied, the volume of the preimage would be multiplied by r^3.

ANSWER:

a. surface area: 88 cm^2 ; volume: 48 cm^3

b. surface area: 352 cm^2 ; volume: 384 cm^3

c. surface area: 22 cm^2 ; volume: 6 cm^3

d. surface area: 4 times greater after dilation with scale factor 2; $\frac{1}{4}$ as great after dilation with scale factor $\frac{1}{2}$.

Volume: 8 times greater after dilation with scale factor 2; $\frac{1}{8}$ as great after dilation with scale factor $\frac{1}{2}$.

e. The surface area of the preimage would be multiplied by r^2. The volume of the preimage would be multiplied by r^3.

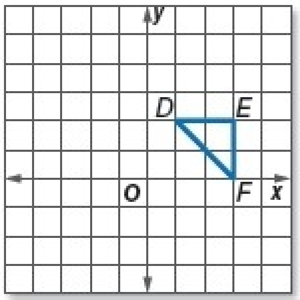
32. **CCSS PERSEVERANCE** Refer to the graph of ΔDEF.

a. Graph the dilation of ΔDEF centered at point D with a scale factor of 3.

b. Describe the dilation as a composition of transformations including a dilation with a scale factor of 3 centered at the origin.

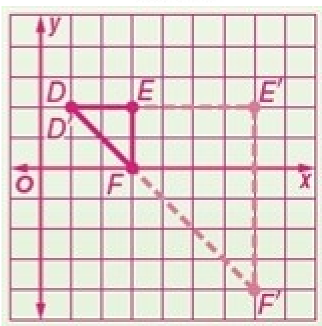
c. If a figure is dilated by a scale factor of 3 with a center of dilation (x, y), what composition of transformations, including a dilation with a scale factor of 3 centered at the origin, will produce the same.

9-6 Dilations

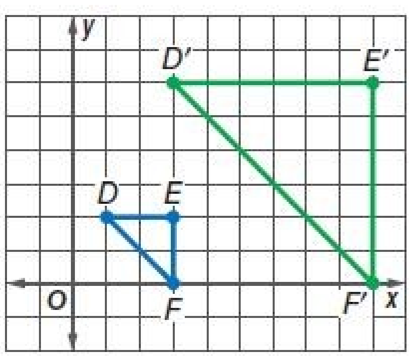


SOLUTION:

a. The point *D* is the center of dilation. The distance between the points *D* and *E* is 2 units. So, make it 3 times larger.
To dilate the point *F* from *D*, first extent the vertical distance of 2 units by 3 times and then extent the horizontal distance of 2 units by 3 times.
Now draw ΔDEF and its image.



b. First, graph triangle *DEF* and the image dilated by a scale factor of 3 centered at the origin.
$(x, y) \rightarrow (3x, 3y)$
$D(1, 2) \rightarrow D'(3, 6)$
$E(3, 2) \rightarrow E'(9, 6)$
$F(3, 0) \rightarrow F'(9, 0)$

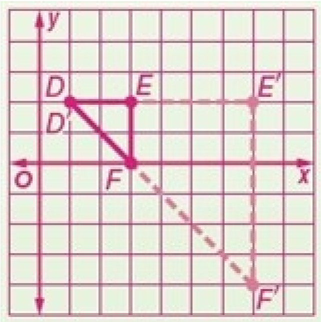


Next, compare this image with the image in part **a**. If this image is translated 4 units down and 2 units to the left, it will coincide with the image in part **a**. So, the composition of a dilation with scale factor 3 centered at the origin and a translation along $\langle -2, -4 \rangle$ is equivalent to a dilation with scale factor 3 centered at point *D*.

c. In part **a**, the center of dilation is at $D(1, 2)$. From part **b**, when $x = 1$ and $y = 2$, the translation vector is <-2, -4>. Next, determine the translation vector in terms of *x* and *y*. The translation vector <-2*x*, -2*y*> yields <-2(1), -2(2)> or <-2, -4> when $(x, y) = (1, 2)$ is substituted.
The composition of a dilation with a scale factor of 3 centered at the origin and a translation along $\langle -2x, -2y \rangle$.

ANSWER:

a.



b. the composition of a dilation with scale factor 3 centered at the origin and a translation along $\langle -2, -4 \rangle$

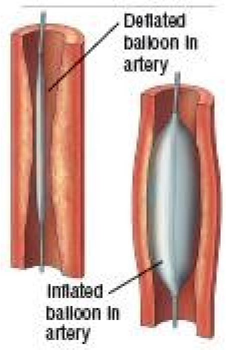
c. the composition of a dilation with a scale factor of 3 centered at the origin and a translation along $\langle -2x, -2y \rangle$

9-6 Dilations

33. **HEALTH** A coronary artery may be dilated with a balloon catheter as shown. The cross section of the middle of the balloon is a circle.

a. A surgeon inflates a balloon catheter in a patient's coronary artery, dilating the balloon as shown. Find the scale factor of this dilation.

b. Find the cross-sectional area of the balloon before and after the dilation.



SOLUTION:

a. The second balloon is larger than the first balloon, so the dilation is an enlargement.

The scale factor of the dilation is $\frac{2}{1.5}$ or $\frac{4}{3}$ or $1\frac{1}{3}$.

b. The cross section of the balloon is a circle.

The area of a circle is given by the formula $A = \pi r^2$, where r is the radius of the circle.
Since the diameter of the original balloon is 1.5 mm, the radius is 0.75.

$$\begin{aligned} A &= \pi(0.75)^2 \\ &\approx 1.77 \end{aligned}$$

The cross sectional area of the balloon before dilation is about 1.77 mm^2.
Since the diameter of the dilated balloon is 2 mm, the radius is 1 mm.

$$\begin{aligned} A &= \pi(1)^2 \\ &\approx 3.14 \end{aligned}$$

Therefore, the cross sectional area of the balloon after dilation is about 3.14 mm^2.

ANSWER:

a. $1\frac{1}{3}$

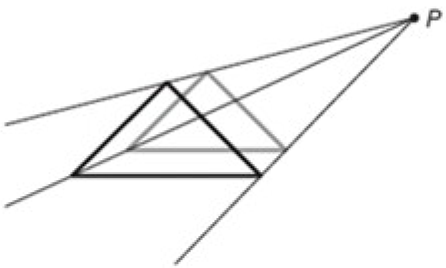
b. 1.77 mm ^{2} ; 3.14 mm ^{2}

Each figure shows a preimage and its image after a dilation centered at point *P*. Copy each figure, locate point *P*, and estimate the scale factor.

34. 

SOLUTION:

Join each vertex of the image with its preimage and extend the line. The point at which all the three lines meet is *P*. The preimage is smaller than the image, so the dilation is a reduction.

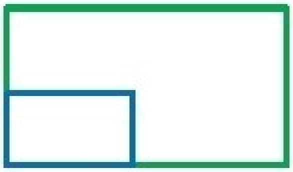


The scale factor is the ratio of a length on the image to a corresponding length of the preimage.

The scale factor is $\frac{4}{5}$.

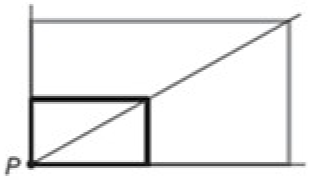
ANSWER:

The scale factor is $\frac{4}{5}$.

35. 

SOLUTION:

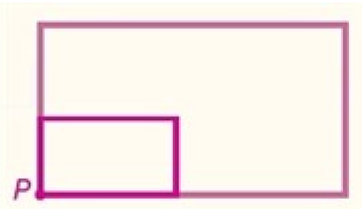
Join each vertex of the image with its preimage and extend the line. The point at which all the three lines meet is *P*. The preimage is larger than the image, so the dilation is an enlargement.



The scale factor is the ratio of a length on the image to a corresponding length of the preimage.

The scale factor is $\frac{11}{5}$.

ANSWER:



The scale factor is $\frac{11}{5}$.

36. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate dilations centered at the origin with negative scale factors.

a. GEOMETRIC Draw ΔABC with points $A(-2, 0)$, $B(2, -4)$, and $C(4, 2)$. Then draw the image of ΔABC after a dilation centered at the origin with a scale factor of -2. Repeat the dilation with scale factors of $-\frac{1}{2}$ and -3. Record the coordinates for each dilation in a table.

d. VERBAL Make a conjecture about the function relationship for a dilation centered at the origin with a negative scale factor.

e. ANALYTICAL Write the function rule for a dilation centered at the origin with a scale factor of $-k$.

f. VERBAL Describe a dilation centered at the origin with a negative scale factor as a composition of transformations.

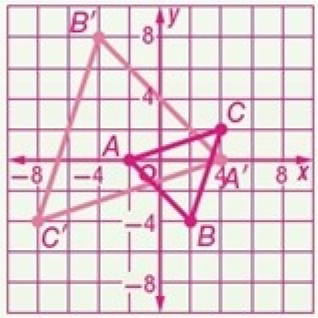
SOLUTION:

a. Multiply the x- and y-coordinates of each vertex by the scale factor k. That is, $(x, y) \to (kx, ky)$.
Here multiply the x- and y- coordinates by the scale factor –2.

$A(-2,0) \to A'(4,0)$

$B(2,-4) \to B'(-4,8)$

$C(4,2) \to C'(-8,-4)$

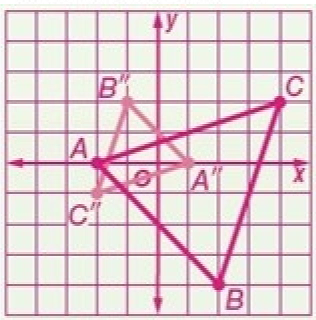


First multiply the x- and y- coordinates by the scale factor $\frac{-1}{2}$.

$A(-2,0) \rightarrow A''(1,0)$

$B(2,-4) \rightarrow B''(-1,2)$

$C(4,2) \rightarrow C''(-2,-1)$

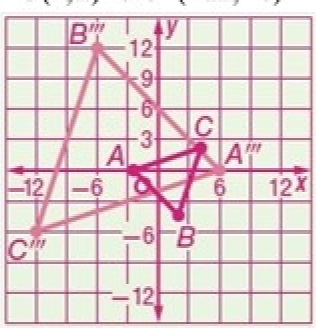


Now multiply the x- and y-coordinates by the scale factor –3.

$A(-2,0) \rightarrow A'''(6,0)$

$B(2,-4) \rightarrow B'''(-6,12)$

$C(4,2) \rightarrow C'''(-12,-6)$



Record the coordinates of each vertex after dilations with each scale factor were applied.

	Coordinates		
Scale Factor	A	B	C
−2	(4, 0)	(−4, 8)	(−8, −4)
$-\frac{1}{2}$	(1, 0)	(−1, 2)	(−2, −1)
−3	(6, 0)	(−6, 12)	(−12, −6)

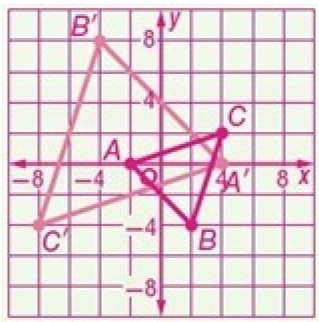
b. Sample answer: The coordinates of the original image are A(-2, 0), B(2, -4), and C(4, 2). Compare these coordinates to each listed in the table. When a scale factor of -2 was applied, the coordinates were each multiplied by -2. Each of the coordinates is multiplied by the negative scale factor.

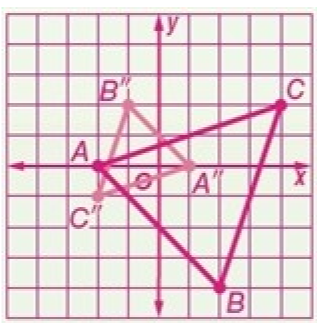
c. The function rule for a dilation with a scale factor k is $(x, y) \rightarrow (kx, ky)$. If k is a negative value, a dilation can be represented by the function rule $(x, y) \rightarrow (-kx, -ky)$.

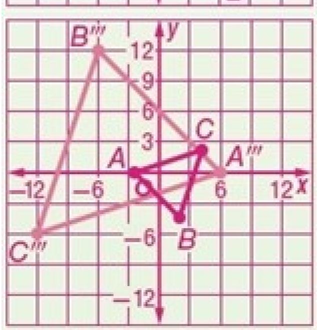
d. Sample answer: From the dilations graphed in part **a** and part **b**, the images are dilated but the orientation of the image is not the same as the original. The function rule for a 180º rotation about the origin is $(x, y) \rightarrow (-x, -y)$. The graphs above show a dilation centered at the origin and a rotation 180º degrees about the origin. A dilation centered at the origin with a scale factor of $-k$ can be described as a dilation centered at the origin with a scale factor of k and a rotation 180° about the origin.

ANSWER:

a.







Scale Factor	Coordinates A	B	C
−2	(4, 0)	(−4, 8)	(−8, −4)
$-\frac{1}{2}$	(1, 0)	(−1, 2)	(−2, −1)
−3	(6, 0)	(−6, 12)	(−12, −6)

b. Sample answer: Each of the coordinates is multiplied by the negative scale factor.

c. $(x, y) \rightarrow (-kx, -ky)$

d. Sample answer: A dilation centered at the origin with a scale factor of $-k$ can be described as a dilation centered at the origin with a scale factor of k and a rotation 180° about the origin.

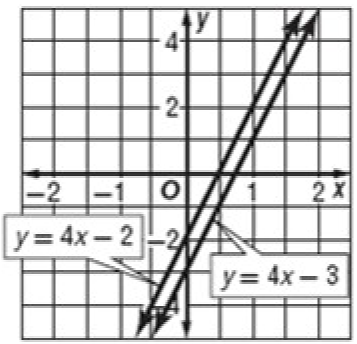
37. **CHALLENGE** Find the equation for the dilated image of the line $y = 4x - 2$ if the dilation is centered at the origin with a scale factor of 1.5.

SOLUTION:

The x-intercept is 0.5 and the y-intercept is –2.
The distance between the vertices at (0, 0) and (0.5, 0) is 0.5 and from the vertices (0, 0) and (0, –2) is 2.
Dilate the distance by 1.5 units.

The x-intercept of the dilated image would be (0.5)(1.5) or 0.75 and the y-intercept of the dilated image would be (1.5)(2) or 3 units down form the origin.

Therefore, the equation of the dilated image of the line $y = 4x - 2$ is $y = 4x - 3$.



ANSWER:

$y = 4x - 3$

38. **WRITING IN MATH** Are parallel lines (parallelism) and collinear points (collinearity) preserved under all transformations? Explain.

SOLUTION:

Sample answer: When a figure is translated, all the points are moved the same distance horizontally and vertically. The image will be congruent to the preimage. Corresponding angles and sides will have the same measures. When a figure is reflected in a line, all points are moved perpendicular to the line so that they are the same distance from the line. The image will be congruent to the preimage. Corresponding angles and sides will have the same measures. When a figure is rotated, each point of the image and preimage is the same distance from the center of rotation and the angle of rotation is the same. The image will be congruent to the preimage. Corresponding angles and sides will have the same measures. When figures are congruent, the sides that were parallel in one figure must be parallel in the other since the angle measures are the same. Points that were collinear in one figure must be collinear in the other since line segments are congruent. Therefore, in translations, reflections, and rotations, congruent figures are formed, which means that sides that were parallel before transformation will be parallel after transformation and points that were collinear before transformation will still be collinear after transformation. When a figure is dilated, all the points are multiplied by the same scale factor. All the corresponding segments will be in the ration of the scale factor, so the image will be similar to the preimage. Similar figures will have congruent corresponding angles. Sides that were parallel in one of the figures will be parallel in the other since the angles have the same measures. Since the figures correspond, the segments correspond and points collinear in one are collinear in the other. So, both parallel sides and collinear points are also preserved under dilations.

ANSWER:

Sample answer: Yes; in translations, reflections, and rotations, congruent figures are formed, which means that sides that were parallel before transformation will be parallel after transformation and points that were collinear before transformation will still be collinear after transformation. Both parallel sides and collinear points are also preserved under dilations because a similar figure is formed, which has the same shape, but in a different proportion.

39. **CCSS ARGUMENTS** Determine whether invariant points are *sometimes, always,* or *never* maintained for the transformations described below. If so, describe the invariant point(s). If not, explain why invariant points are not possible.

a. a dilation of $ABCD$ with a scale factor of 1

b. a rotation of $\overline{AB}$ 74° about B

c. a reflection of ΔMNP in the x-axis

d. a translation of $PQRS$ along $\langle 7,3 \rangle$

e. a dilation of ΔXYZ centered at the origin with a scale factor of 2

SOLUTION:

a. Sample answer: A dilation of 1 maps (x, y) onto $(1x, 1y)$ or (x, y). Since a dilation of 1 maps an image onto itself, all four vertices will remain invariant under the dilation. So, invariant points are *always* maintained under this dilation.

b. Sample answer: In any rotation, the image and preimage of the center of rotation is the same point. Since the rotation is centered about point B, B and B' will be the same point. So, point B will *always* remain invariant under this rotation.

c. Sample answer: If a point is on the line of reflection, then the image and preimage are the same point. If a vertex or any other point of ΔMNP lies on the x-axis, then that point will remain invariant under reflection. If two vertices or other points of ΔMNP are on the x-axis, then both points located on the x-axis will remain invariant under reflection. So, *sometimes* invariant points are maintained under this reflection.

d. Sample answer: When a figure is translated, all points move an equal distance. When $PQRS$ is translated along <7, 3>, each point (x, y) is mapped onto $<x + 7, y + 3>$. None of the points can remain the same. So, points are *never* invariant under this translation.

e. Sample answer: When ΔXYZ is dilated by a scale factor of 2 with the center at the origin, each point (x, y) is mapped onto $(2x, 2y)$. If one of the vertices of the triangle is located at the origin, then (x, y) and $(2x, 2y)$ both equal (0, 0). The preimage and image are the same point so that vertex would remain invariant under the dilation.(This would be true for any point of the triangle located at the origin.) If none of the points of ΔXYZ are located at the origin, then no points will remain invariant under the dilation. So, *sometimes* invariant points are maintained under this dilation.

ANSWER:

a. Always; sample answer: Since a dilation of 1 maps an image onto itself, all four vertices will remain invariant under the dilation.

b. Always; sample answer: Since the rotation is centered at B, point B will always remain invariant under the rotation.

c. Sometimes: sample answer: If one of the vertices is on the x-axis, then that point will remain invariant under reflection. If two vertices are on the x-axis, then the two vertices located on the x-axis will remain invariant under reflection.

d. Never; when a figure is translated, all points move an equal distance. Therefore, no points can remain invariant under translation.

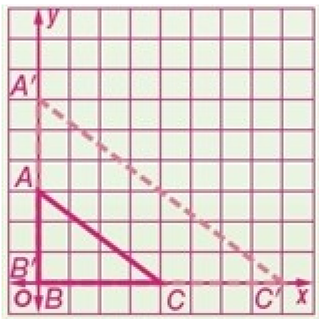
e. Sometimes; sample answer: If one of the vertices of the triangle is located at the origin, then that vertex would remain invariant under the dilation. If none of the points of ΔXYZ are located at the origin, then no points will remain invariant under the dilation.

40. **OPEN ENDED** Graph a triangle on the coordinate plane. Dilate the triangle so that the area of the dilation is four times the area of the original triangle. State the scale factor and center of your dilation.

SOLUTION:

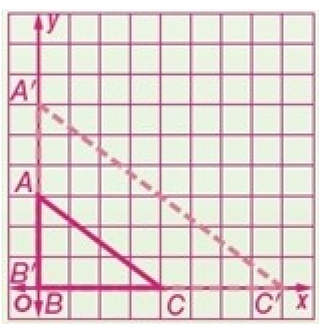
Graph $\triangle ABC$ with vertices $A(0, 3)$, $B(0, 0)$, and $C(4, 0)$. $AB = 3$ and $BC = 4$, so the area of $\triangle ABC = \frac{3(4)}{2}$ or 6 units^2. Dilate the triangle using a scale factor of 2 and the origin as the center of dilation.

$(x, y) \rightarrow (2x, 2y)$
$A(0, 3) \rightarrow A'(0, 6)$
$B(0, 0) \rightarrow B'(0, 0)$
$C(4, 0) \rightarrow C'(8, 0)$



$A'B' = 6$ and $B'C' = 8$, so the area of $\triangle A'B'C' = \frac{6(8)}{2}$ or 24 units^2. The area of $\triangle A'B'C'$ is four times the area of $\triangle ABC$ when $\triangle ABC$ is dilated by a factor of 2 with point B as the center of dilation.

ANSWER:



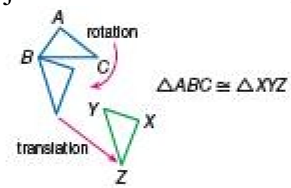
$k = 2$, center of dilation is B.

9-6 Dilations

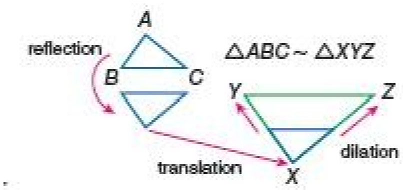
41. **WRITING IN MATH** Can you use transformations to create congruent figures, similar figures, and equal figures? Explain.

SOLUTION:

Draw a triangle and observe how it changes under different transformations. Rotations, translations, and reflections just move the same triangle to a different location with a different orientation.



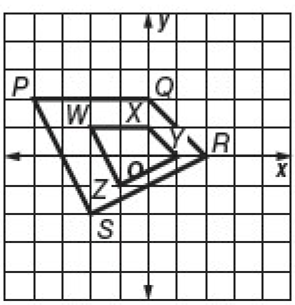
Dilations cause the triangle to shrink or grow, while preserving the angles.



ANSWER:

Sample answer: Translations, reflections, and rotations produce congruent figures because the sides and angles of the preimage are congruent to the corresponding sides and angles of the image. Dilations produce similar figures, because the angles of the preimage and the
image are congruent and the sides of the preimage are proportional to the corresponding sides of the image. A dilation with a scale factor of 1 produces an equal figure because the image is mapped onto its corresponding parts in the preimage.

42. **EXTENDED RESPONSE** Quadrilateral *PQRS* was dilated to form quadrilateral *WXYZ*.



a. Is the dilation from *PQRS* to *WXYZ* an enlargement or reduction?
b. Which number best represents the scale factor for this dilation?

SOLUTION:

a. The quadrilateral *WXYZ* is smaller than the quadrilateral *PQRS*, so the dilation is a reduction.

b. The distance between the vertices at (1, 0) and (–1, –1) for *WXYZ* is $\sqrt{5}$ and from the vertices (2, 0) and (–2, –2) for *PQRS* is $2\sqrt{5}$. So, the scale factor is $\frac{\sqrt{5}}{2\sqrt{5}}$ or $\frac{1}{2}$.

ANSWER:

a. reduction

b. $\frac{1}{2}$

9-6 Dilations

43. **ALGEBRA** How many ounces of pure water must a pharmacist add to 50 ounces of a 15% saline solution to make a solution that is 10% saline?
A 25
B 20
C 15
D 5

SOLUTION:

Let x be the amount of pure water added. Pure water is 0% saline. Write and solve an equation comparing the concentrations.

$$\begin{aligned} 50(0.15) + x(0.00) &= (50 + x)(0.10) \\ 7.5 &= 5 + 0.10x \\ 2.5 &= 0.10x \\ x &= \frac{2.5}{0.10} \\ &= 25 \end{aligned}$$

Therefore, the correct choice is A.

ANSWER:

A

44. Tionna wants to replicate a painting in an art museum. The painting is 3 feet wide and 6 feet long. She decides on a dilation reduction factor of 0.25. What size paper should she use?

F 4 in. × 8 in.
G 6 in. × 12 in.
H 8 in. × 16 in.
J 10 in. × 20 in.

SOLUTION:

Multiply the dimensions by the scale factor 0.25.
$D_{0.25}(3,6) \rightarrow D(0.75,1.5)$

The dilated painting is 0.75 feet wide and 1.5 feet long.

Convert feet into inches. Since 1 inch is 12 feet, multiply by 12.
The dimensions of the dilated painting would be (0.75)(12) or 9 in wide and (1.5)(12) or 18 in long.
So, Tionna should use the paper side of $10 \text{ in} \times 20 \text{ in}$.

The correct choice is J.

ANSWER:

J

45. **SAT/ACT** For all x, $(x - 7)^2$ =?

A $x^2 - 49$

B $x^2 + 49$

C $x^2 - 14x - 49$

D $x^2 - 14x + 49$

E $x^2 + 14x - 49$

SOLUTION:

Use the identify $(a-b)^2 = a^2 - 2ab + b^2$.

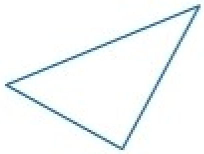
$$\begin{aligned}(x-7)^2 &= (x)^2 - 2(x)(7) + (7)^2 \\ &= x^2 - 14x + 49\end{aligned}$$

Therefore, the correct choice is D.

ANSWER:

D

State whether the figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.

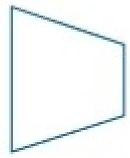
46. 

SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.
The given figure does not have line symmetry. There is no way to fold, or reflect it onto itself.

ANSWER:

no

47. 

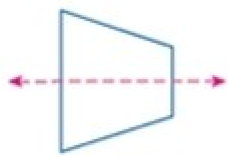
SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

The figure has line symmetry.

In order for the figure to map onto itself, the line of reflection must go through the center point.

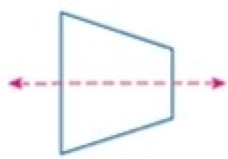
One line of reflection goes through the bases of the trapezoid.

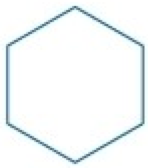


The figure has one line of symmetry.

ANSWER:

yes; 1



48. 

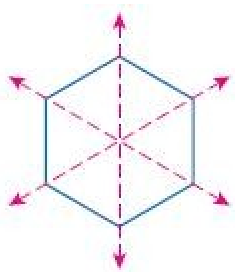
SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

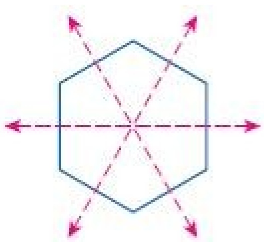
The figure has line symmetry.

In order for the figure to map onto itself, the line of reflection must go through the center point.

Three lines of reflection go through the vertices.



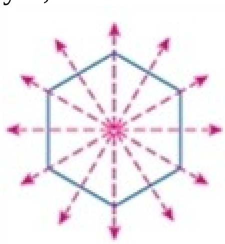
Three lines of reflection go through the midpoints of the sides.



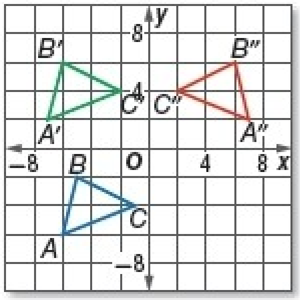
The figure has 6 lines of symmetry.

ANSWER:

yes; 6



Describe the transformations that combined to map each figure.

49. 

SOLUTION:

Triangle ABC is translated along <-1, 8> first.

$(x, y) \to (x - 1, y + 8)$
$A(-6, -6) \to A'(-7, 2)$
$B(-5, -2) \to B'(-6, 6)$
$C(-1, -4) \to C'(-2, 4)$

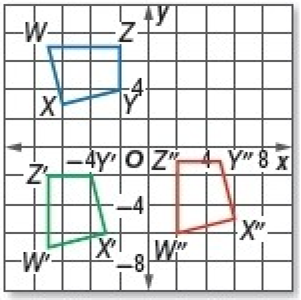
Triangle $A'B'C'$ is reflected in the y-axis.

$(x, y) \to (-x, y)$
$A'(-7, 2) \to A''(7, 2)$
$B'(-6, 6) \to B''(6, 6)$
$C'(-2, 4) \to C''(2, 4)$

The image ABC is translated along $\langle -1, 8 \rangle$ and then reflected in the y-axis.

ANSWER:

translation along $\langle -1, 8 \rangle$ and reflection in the y-axis

50. 

SOLUTION:

The image *WXYZ* is rotated $90°$ about the origin.

$(x, y) \to (-y, x)$
$W(-7, 7) \to W'(-7, -7)$
$X(-6, 3) \to X'(-3, -6)$
$Y(-2, 4) \to Y'(-4, -2)$
$Z(-2, 7) \to Z'(-7, -2)$

Quadrilateral *W'X'Y'Z'* is translated along <9, 1>.

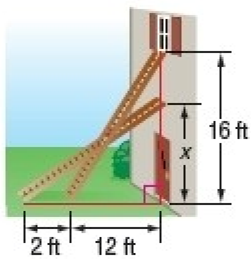
$(x, y) \to (x + 9, y + 1)$
$W'(-7, -7) \to W''(2, -6)$
$X'(-3, -6) \to X''(6, -5)$
$Y'(-4, -2) \to Y''(5, -1)$
$Z'(-7, -2) \to Z''(2, -1)$

The image *WXYZ* is rotated $90°$ about the origin and then translated along $\langle 9,1 \rangle$.

ANSWER:

rotation 90° about the origin and translation along $\langle 9,1 \rangle$

51. **PAINTING** A painter sets a ladder up to reach the bottom of a second-story window 16 feet above the ground. The base of the ladder is 12 feet from the house. While the painter mixes the paint, a neighbor's dog bumps the ladder, which moves the base 2 feet farther away from the house. How far up the side of the house does the ladder reach?



SOLUTION:
Use the Pythagorean theorem to find the length of the ladder.

The ladder is the hypotenuse of the right triangle.

$$\begin{aligned} l^2 &= 16^2 + 12^2 \\ &= 256 + 144 \\ &= 400 \\ l &= 20 \end{aligned}$$

The length of the ladder is 20 feet.

Use the Pythagorean theorem again to find the value of x.

$$\begin{aligned} 20^2 &= (12+2)^2 + x^2 \\ x^2 &= 20^2 - 14^2 \\ &= 400 - 196 \\ &= 204 \\ x &= \sqrt{204} \\ &= 2\sqrt{51} \\ &\approx 14.3 \end{aligned}$$

The ladder is about 14.3 feet up from the ground on the side of the wall.

ANSWER:
$2\sqrt{51}\ ft \approx 14.3\ ft$

Find the value of *x*.

52. $58.9 = 2x$

SOLUTION:

$$\begin{aligned} 58.9 &= 2x \\ x &= \frac{58.9}{2} \\ &\approx 29.5 \end{aligned}$$

ANSWER:

29.5

53. $\frac{108.6}{\pi} = x$

SOLUTION:

$$\begin{aligned} \frac{108.6}{\pi} &= x \\ x &\approx 34.6 \end{aligned}$$

ANSWER:

34.6

54. $228.4 = \pi x$

SOLUTION:

$$\begin{aligned} 228.4 &= \pi x \\ x &= \frac{228.4}{\pi} \\ &\approx 72.7 \end{aligned}$$

ANSWER:

72.7

55. $\frac{336.4}{x} = \pi$

SOLUTION:

$$\begin{aligned} \frac{336.4}{x} &= \pi \\ 336.4 &= \pi x \\ \frac{336.4}{\pi} &= x \\ x &\approx 107.1 \end{aligned}$$

ANSWER:

107.1