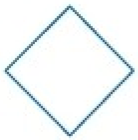
<u>**9-5 Symmetry**</u>

State whether the figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.

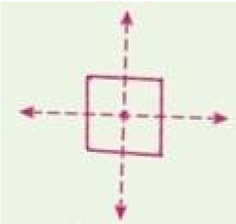
1. 

SOLUTION:
A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

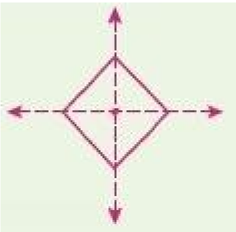
The figure has line symmetry.

In order for the figure to map onto itself, the line of reflection must go through the center point.

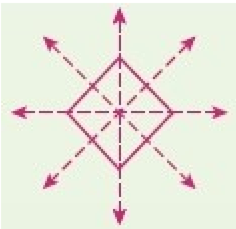
Two lines of reflection go through the sides of the figure.



Two lines of reflection go through the vertices of the figure.

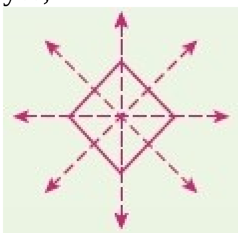


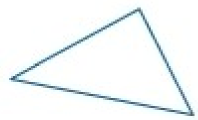
Thus, there are four possible lines that go through the center and are lines of reflections.



Therefore, the figure has four lines of symmetry.

ANSWER:
yes; 4



2. 

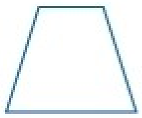
SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

The given figure does not have line symmetry. There is no way to fold or reflect it onto itself.

ANSWER:

no

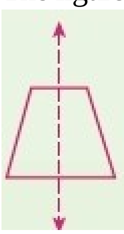
3. 

SOLUTION:

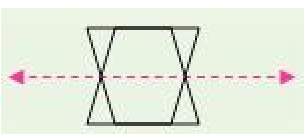
A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

The given figure has line symmetry.

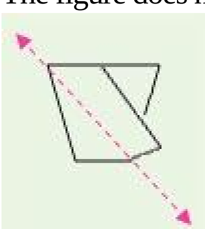
The figure has a vertical line of symmetry.



It does not have a horizontal line of symmetry.



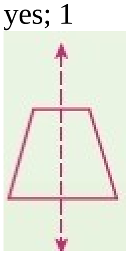
The figure does not have a line of symmetry through the vertices.



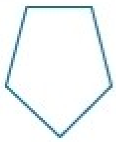
Thus, the figure has only one line of symmetry.

ANSWER:

yes; 1



State whether the figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

4. 

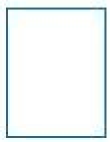
SOLUTION:

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

For the given figure, there is no rotation between 0° and 360° that maps the figure onto itself. If the figure were a regular pentagon, it would have rotational symmetry.

ANSWER:

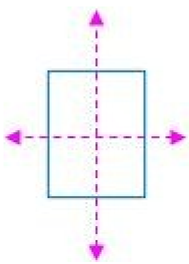
no

5. 

SOLUTION:

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

The given figure has rotational symmetry.



The number of times a figure maps onto itself as it rotates form 0° and 360° is called the order of symmetry.

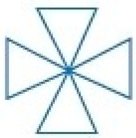
The given figure has order of symmetry of 2, since the figure can be rotated twice in 360°.

The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself.

Since the figure has order 2 rotational symmetry, the magnitude of the symmetry is $360° \div 2$ or $180°$.

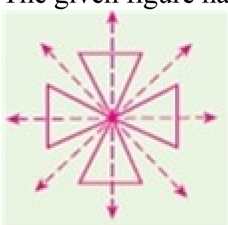
ANSWER:

yes; 2; 180°

6. 

SOLUTION:
A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

The given figure has rotational symmetry.



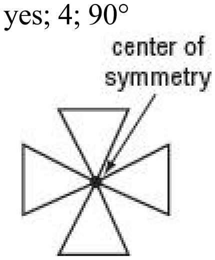
The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry.

Since the figure can be rotated 4 times within 360° , it has order 4 rotational symmetry

The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself.

The figure has magnitude of symmetry of $360° \div 4$ or $90°$.

ANSWER:
yes; 4; 90°



7. **U.S. CAPITOL** Completed in 1863, the dome is one of the most recent additions to the United States Capitol. It is supported by 36 iron ribs and has 108 windows, divided equally among three levels.
Refer to page 666.
a. Excluding the spire of the dome, how many horizontal and vertical planes of symmetry does the dome appear to have?
b. Does the dome have axis symmetry? If so, state the order and magnitude of symmetry.

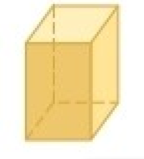
SOLUTION:

a. Each level of the dome will have $\frac{108}{3}$ or 36 windows. Since the dome is not uniform from top to bottom, the figure has no horizontal planes of symmetry. There are 16 vertical planes that would pass through two iron ribs on opposite sides of the dome that would divide the dome into two congruent halves. There are also 16 vertical planes that would pass through 2 windows on opposite sides of the dome that would divide the dome into two congruent halves. Therefore, the dome appears to have 36 vertical planes of symmetry.
b. The dome has axis symmetry as each of the 36 ribs and windows can be mapped onto the rib and window next to it. So, the order of symmetry for the dome is 36 and the magnitude of symmetry is $360° \div 36 \text{ or } 10°$.

ANSWER:

a. no horizontal; 36 vertical
b. yes; 36; 10°

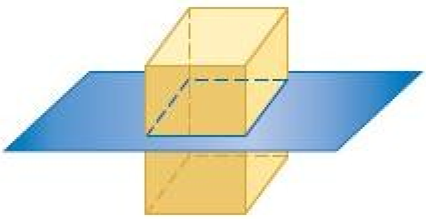
8. State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.



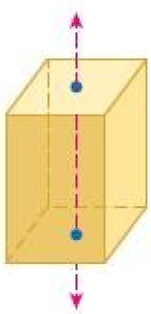
SOLUTION:

A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane.

The figure has plane symmetry.



A three-dimensional figure has axis symmetry if the figure can be mapped onto itself by a rotation between 0° and 360°
in a line.

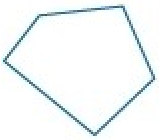


The figure axis symmetry.

ANSWER:

both

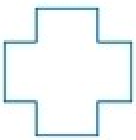
CCSS REGULARITY State whether the figure appears to have line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.

9. 

SOLUTION:
A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

For the given figure, there are no lines of reflection where the figure can map onto itself. Thus, the figure does not have any lines of of symmetry.

ANSWER:
no

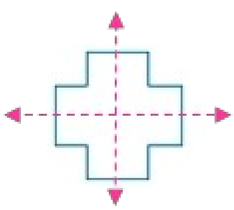
10. 

SOLUTION:
A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

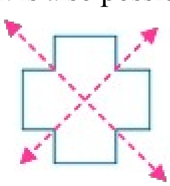
The given figure has line symmetry.

In order for the figure to map onto itself, the line of reflection must go through the center point.

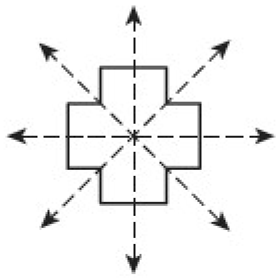
The figure has a vertical and horizontal line of reflection.



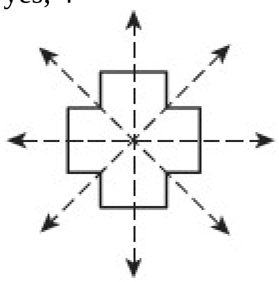
It is also possible to have reflection over the diagonal lines.

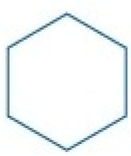


Therefore, the figure has four lines of symmetry



ANSWER:
yes; 4



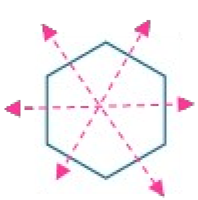
11. 

SOLUTION:
A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

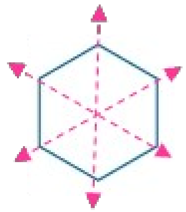
The given hexagon has line symmetry.

In order for the hexagon to map onto itself, the line of reflection must go through the center point.

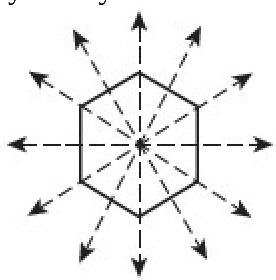
There are three lines of reflection that go though opposites edges.



There are three lines of reflection that go though opposites vertices.

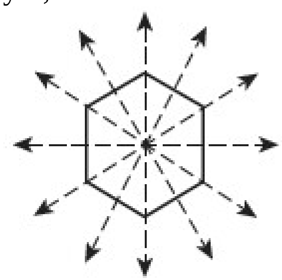


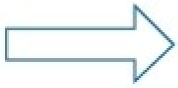
There are six possible lines that go through the center and are lines of reflections. Thus, the hexagon has six lines of symmetry.



ANSWER:

yes; 6

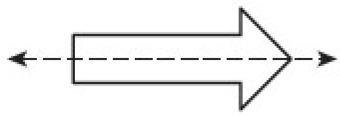


12. 

SOLUTION:
A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

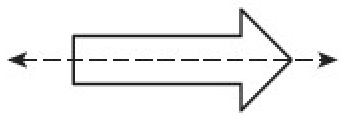
The figure has line symmetry.

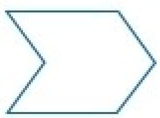
There is only one line of symmetry, a horizontal line through the middle of the figure.



Thus, the figure has one line of symmetry.

ANSWER:
yes; 1

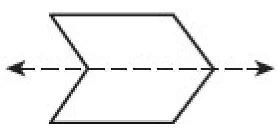


13. 

SOLUTION:
A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

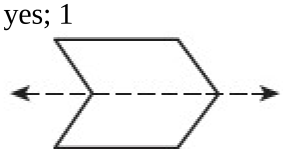
The figure has line symmetry.

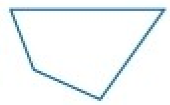
There is only one possible line of reflection, horizontally though the middle of the figure.



Thus, the figure has one line of symmetry.

ANSWER:
yes; 1



14. 

SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

The given figure does not have line symmetry. It is not possible to draw a line of reflection where the figure can map onto itself.

ANSWER:

no

FLAGS State whether each flag design appears to have line symmetry. Write *yes* or *no*. If so, copy the flag, draw all lines of symmetry, and state their number.

15. Refer to page 666.

SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

The flag does not have any line symmetry. If the red lines in the diagonals were in the same location above and below the center horizontal line, the flag would have three lines of symmetry.

ANSWER:

no

16. Refer to the flag on page 666.

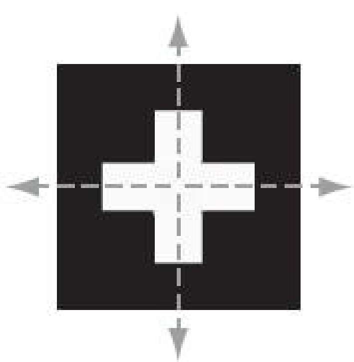
SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

The figure has line symmetry.

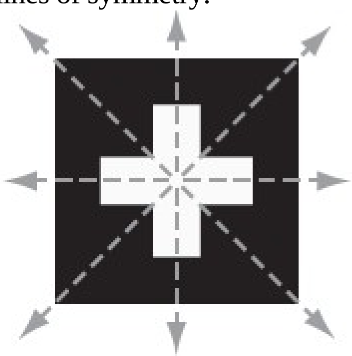
In order for the figure to map onto itself, the line of reflection must go through the center point.

A horizontal and vertical lines of reflection are possible.

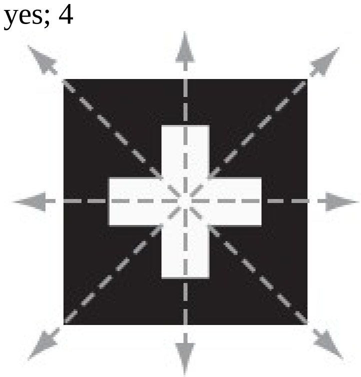


Two diagonal lines of reflection are possible.

There are a total of four possible lines that go through the center and are lines of reflections. Thus, the flag has four lines of symmetry.



ANSWER:
yes; 4



9-5 Symmetry

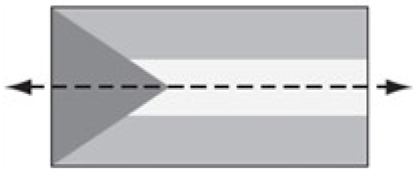
17. Refer to page 666.

SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

The figure has line symmetry.

A horizontal line is a line of reflections for this flag.

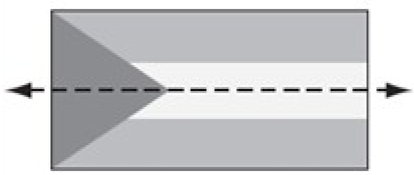


It is not possible to reflect over a vertical or line through the diagonals.

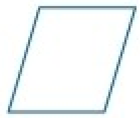
Thus, the figure has one line of symmetry.

ANSWER:

yes; 1

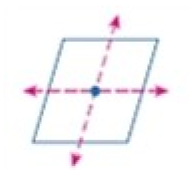


State whether the figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

18. 

SOLUTION:

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.



The figure has rotational symmetry.

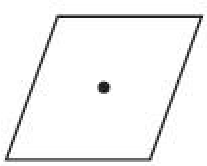
The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry.

This figure has order 2 rotational symmetry, since you have to rotate 180° to get the figure to map onto itself.

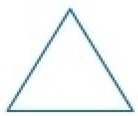
The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself.

The figure has a magnitude of symmetry of $360° \div 2$ or $180°$.

ANSWER:



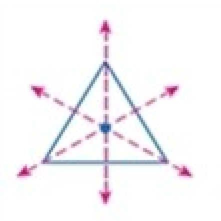
yes; 2; 180°

19. 

SOLUTION:

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

The triangle has rotational symmetry.



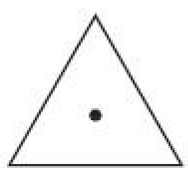
The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry.

The figure has order 3 rotational symmetry.

The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself.

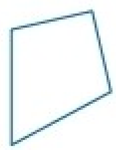
The figure has magnitude of symmetry of $360° \div 3$ or $120°$.

ANSWER:



yes; 3; 120°

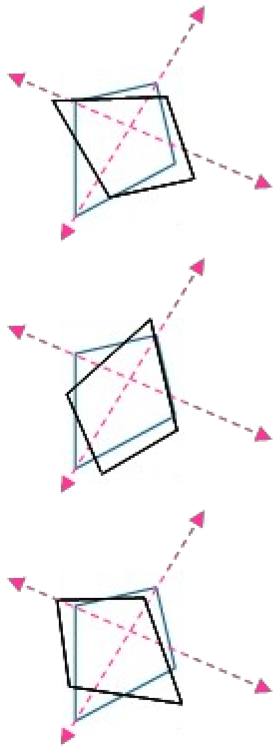
9-5 Symmetry

20. 

SOLUTION:

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

The kite has no rotational symmetry. There is no way to rotate it such that it can be mapped onto itself.



ANSWER:

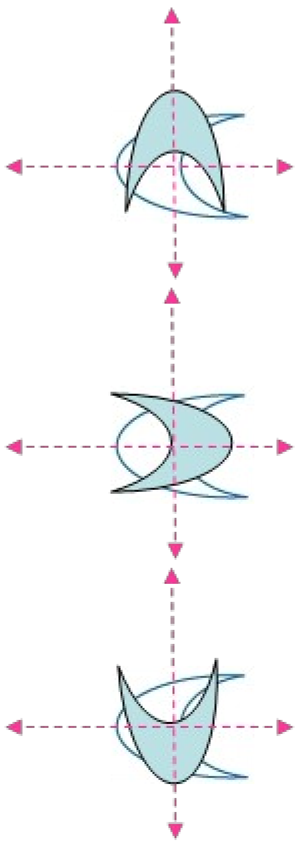
no

21. 

SOLUTION:

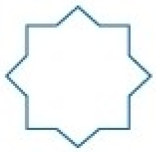
A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

The crescent shaped figure has no rotational symmetry. There is no way to rotate it such that it can be mapped onto itself.



ANSWER:

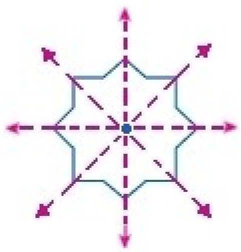
no

22. 

SOLUTION:

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

The figure has rotational symmetry.



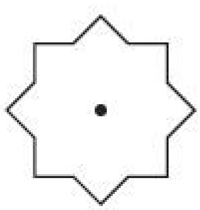
The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry.

The figure has order 8 rotational symmetry. This implies you can rotate the figure 8 times and have it map onto itself within 360°.

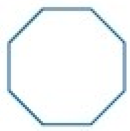
The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself.

The figure has magnitude of symmetry of $360° \div 8 \text{ or } 45°$.

ANSWER:



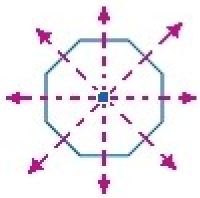
yes; 8; 45°

23. 

SOLUTION:

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

The figure has rotational symmetry.



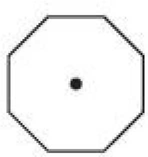
The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry.

The figure has order 8 rotational symmetry. This means that the figure can be rotated 8 times and map onto itself within 360°.

The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself.

The figure has magnitude of symmetry of $360° \div 8$ or $45°$.

ANSWER:



yes; 8; 45°

WHEELS State whether each wheel cover appears to have rotational symmetry. Write *yes* or *no*. If so, state the order and magnitude of symmetry.

24. Refer to page 667.

SOLUTION:

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

The wheel has rotational symmetry.

The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry.

The wheel has order 5 rotational symmetry. There are 5 large spokes and 5 small spokes. You can rotate the wheel 5 times within 360° and map the figure onto itself.

The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself.

The wheel has magnitude of symmetry $360° \div 5$ or $72°$.

ANSWER:

yes; 5; 72°

25. Refer to page 667.

SOLUTION:

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

The wheel has rotational symmetry.

The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry.

The wheel has order 8 rotational symmetry. There are 8 spokes, thus the wheel can be rotated 8 times within 360° and map onto itself.

The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself.

The wheel has order 8 rotational symmetry and magnitude $360° \div 8$ or $45°$.

ANSWER:

yes; 8; 45°

26. Refer to page 667.

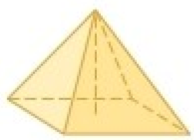
SOLUTION:
A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure. The wheel has rotational symmetry.

The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry. The wheel has order 10 rotational symmetry. There are 10 bolts and the tire can be rotated 10 times within 360° and map onto itself.

The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself. The wheel has magnitude of symmetry of $360° \div 10$ or $36°$.

ANSWER:
yes; 10; 36°

State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

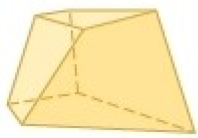
27. 

SOLUTION:
A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane. The square pyramid given has plane symmetry. Any plane that passed through the vertex and is perpendicular to the base will give plane symmetry.

A three-dimensional figure has axis symmetry if the figure can be mapped onto itself by a rotation between 0° and 360°
in a line. An axis of symmetry can be drawn through the vertex and is perpendicular to the base.

Thus the square pyramid has both plane symmetry and axis symmetry.

ANSWER:
both

28. 

SOLUTION:

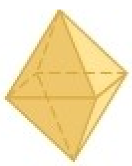
A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane. A three-dimensional figure has axis symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° in a line.

There is no plane that causes the figure to be mapped on to itself, so it does not have plane symmetry. It is not possible to draw an axis of symmetry where the figure can be mapped if rotated through it.

Thus, the figure has neither plane or axis symmetry.

ANSWER:

neither

29. 

SOLUTION:

A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane.

The figure has plane symmetry if you pass a plane horizontally through the center of the figure. It also has plane symmetry for any plane that passes through both vertices and is perpendicular to the base of both pyramids.

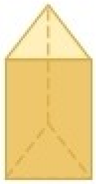
A three-dimensional figure has axis symmetry if the figure can be mapped onto itself by a rotation between 0° and 360°
in a line.

The figure has axis symmetry for an axis drawn through both vertices and is perpendicular to the base of both pyramids.

Thus the figure has both plane symmetry and axis symmetry.

ANSWER:

both

30. 

SOLUTION:

A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane.

A horizontal plane parallel to the bases and in the middle of the figure reflects the figure onto itself.

A three-dimensional figure has axis symmetry if the figure can be mapped onto itself by a rotation between 0° and 360°
in a line.

Draw an axis vertically in the middle of the figure. This would be the axis of symmetry.

Thus, the figure has both plane symmetry and axis symmetry.

ANSWER:

both

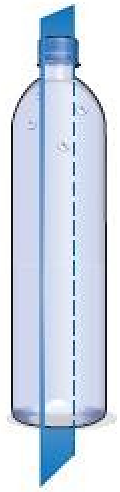
CONTAINERS Determine the number of horizontal and vertical planes of symmetry for each container shown below.

31. 

SOLUTION:

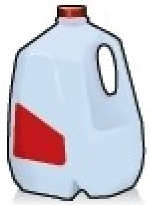
A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane.

The container has no horizontal lines of symmetry and infinitely many vertical lines of symmetry. Any plane through the vertical center of the bottle will be a line of symmetry.



ANSWER:

no horizontal, infinitely many vertical

32. 

SOLUTION:

A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane.

The container has no horizontal lines of symmetry and one vertical line of symmetry.

ANSWER:

no horizontal, 1 vertical

33. 

SOLUTION:

A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane.

The container has one horizontal line of symmetry through the middle of the container.

The container has infinitely many vertical lines of symmetry.

ANSWER:

1 horizontal, infinitely many vertical

34. **CCSS MODELING** Symmetry is an important component of photography. Photographers often use reflection in water to create symmetry in photos. The photo on page 667 is a long exposure shot of the Eiffel tower reflected in a pool.

a. Describe the two-dimensional symmetry created by the photo.
b. Is three-dimensional symmetry applicable? Explain your reasoning.

SOLUTION:

a Sample answer: There is a horizontal line of symmetry between the tower and its reflection. There is a vertical line of symmetry through the center of the photo.
b No; sample answer: Since the tower itself is three-dimensional and the reflection is two-dimensional, three-dimensional symmetry does not apply.

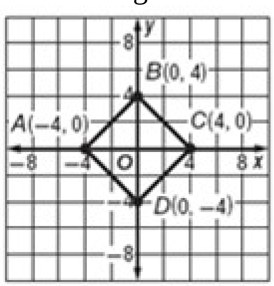
ANSWER:

a. Sample answer: There is a horizontal line of symmetry between the tower and its reflection. There is a vertical line of symmetry through the center of the photo.
b. No; sample answer: Since the tower itself is three-dimensional and the reflection is two-dimensional, three-dimensional symmetry does not apply.

35. **COORDINATE GEOMETRY** Determine whether the figure with the given vertices has *line* symmetry and/or *rotational* symmetry.
A(–4, 0), *B*(0, 4), *C*(4, 0), *D*(0, –4)

SOLUTION:

Draw the figure on a coordinate plane.



A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line. The given figure has 4 line of symmetry. The line of symmetry are though the following pairs of points {(4, 0), (–4, 0)}, {(0, 4) (0, –4)}, {(–2, 2), (–2, –2)}, and {(2, 2), (2, –2)}.

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure. The figure can be rotated from the origin and map onto itself. The order of symmetry is 4.

Thus, the figure has both line symmetry and rotational symmetry.

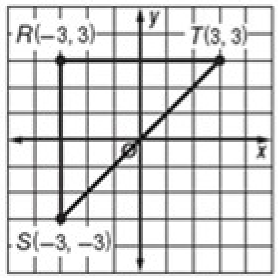
ANSWER:

line and rotational

36. *R*(–3, 3), *S*(–3, –3), *T*(3, 3)

SOLUTION:

Draw the figure on a coordinate plane.



A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

The given triangle has a line of symmetry through points (0, 0) and (–3, 3).

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure. There is not way to rotate the figure and have it map onto itself.

Thus, the figure has only line symmetry.

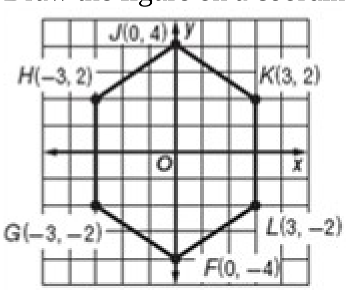
ANSWER:

line

37. *F*(0, –4), *G*(–3, –2), *H*(–3, 2), *J*(0, 4), *K*(3, 2), *L*(3, –2)

SOLUTION:

Draw the figure on a coordinate plane.



A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line. The given hexagon has 2 lines of symmetry. The lines pass through the following pair of points {(0, 4), (0, –4)}, and {(3, 0), (–3, 0)}

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure. The figure has rotational symmetry. You can rotate the figure once within 360° and have it map to itself.

Thus, the figure has both line symmetry and rotational symmetry.

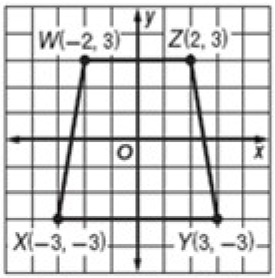
ANSWER:

line and rotational

38. $W(-2, 3)$, $X(-3, -3)$, $Y(3, -3)$, $Z(2, 3)$

SOLUTION:

Draw the figure on a coordinate plane.



A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line. The trapezoid has a line of reflection through points (0,3) and (0, –3).

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure. There is no way to rotate this figure and have it map onto itself. Thus, it does not have rotational symmetry.

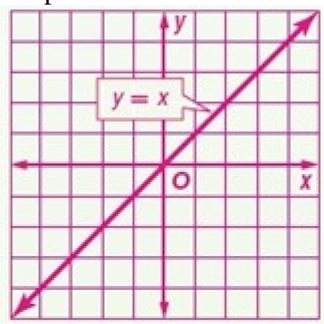
Therefore, the figure has only line symmetry.

ANSWER:

line

39. **ALGEBRA** Graph the function and determine whether the graph has *line* and/or *rotational* symmetry. If so, state the order and magnitude of symmetry, and write the equations of any lines of symmetry.
$y = x$

SOLUTION:
Graph the function.



A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line. The line $y = x$ has line symmetry since any line perpendicular to $y = x$ is a line of reflection. The equation of the line symmetry is $y = -x$.

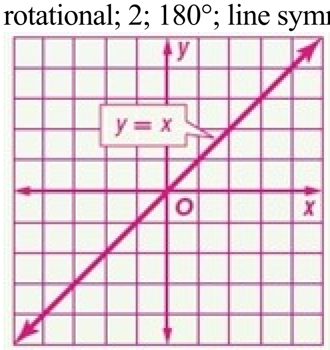
A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure. The line can be rotated twice within 360° and be mapped onto itself.

The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry. The graph has order 2 rotational symmetry.

The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself. The graph has magnitude of symmetry of $360° \div 2$ or $180°$.

Thus, the graph has both line and rotational symmetry.

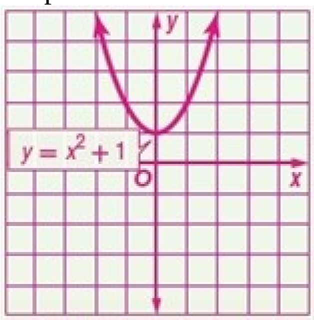
ANSWER:
rotational; 2; 180°; line symmetry; $y = -x$



40. $y = x^2 + 1$

SOLUTION:

Graph the function.



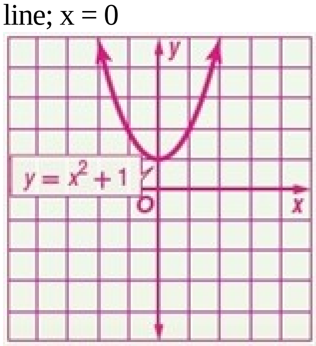
A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line. The graph is reflected through the y-axis. Thus, the equation of the line symmetry is $x = 0$.

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure. There is no way to rotate the graph and have it map onto itself.

Thus, the graph has only line symmetry.

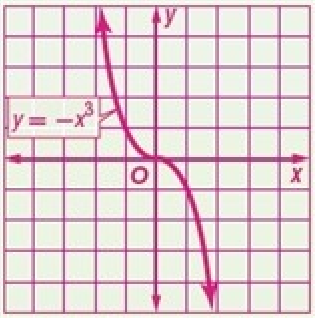
ANSWER:

line; x = 0



41. $y = -x^3$

Graph the function.



A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line. The graph does not have a line of reflections where the graph can be mapped onto itself.

A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure. You can rotate the graph through the origin and have it map onto itself.

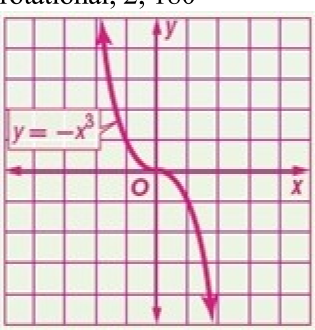
The number of times a figure maps onto itself as it rotates from 0° to 360° is called the order of symmetry. The graph has order 2 rotational symmetry.

The magnitude of symmetry is the smallest angle through which a figure can be rotated so that it maps onto itself. The graph has magnitude of symmetry of $360° \div 2$ or $180°$.

Thus, the graph has only rotational symmetry.

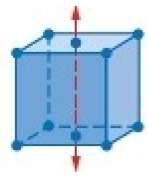
ANSWER:

rotational; 2; 180°



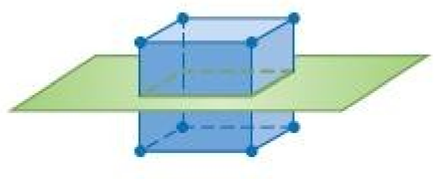
CRYSTALLOGRAPHY Determine whether the crystals below have *plane* symmetry and/or *axis* symmetry. If so, state the magnitude of symmetry.

42. cubic

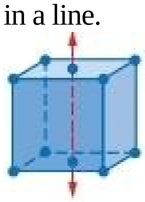


SOLUTION:

A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane. A horizontal plane parallel to the base intersecting the middle of the prism is the plane of reflection for this figure.



A three-dimensional figure has axis symmetry if the figure can be mapped onto itself by a rotation between 0° and 360°
in a line.



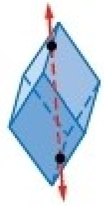
A vertical line through the center is the axis of symmetry. Since the order of rotational symmetry is infinite, the magnitude of symmetry is 0.

Thus, the figure has both plane and axis symmetry.

ANSWER:

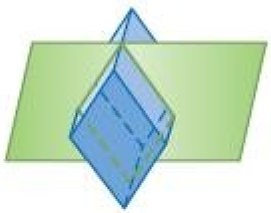
plane and axis; 0

43. rhombohedral



SOLUTION:

A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane. A plane that is parallel to the front and back rhombus is the plane of reflection.



A three-dimensional figure has axis symmetry if the figure can be mapped onto itself by a rotation between 0° and 360°
in a line. The vertical line shown is the axis of symmetry.

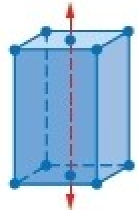
Since the order of rotational symmetry is 2, the magnitude of symmetry is $360 \div 2$ or 180.

Thus, the figure has both plane and axis symmetry.

ANSWER:

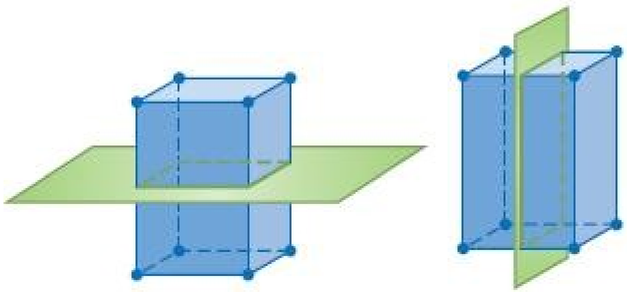
plane and axis; 180

44. orthorhombic

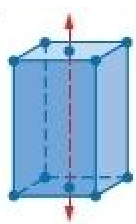


SOLUTION:

A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane. There is a plane of symmetry parallel to the bases and one parallel to the sides.



A three-dimensional figure has axis symmetry if the figure can be mapped onto itself by a rotation between 0° and 360°
in a line. A vertical line through the center is the axis of symmetry.



Since the order of rotational symmetry is 2, the magnitude of symmetry is $360 \div 2$ or 180.

Thus, the figure has both plane and axis symmetry.

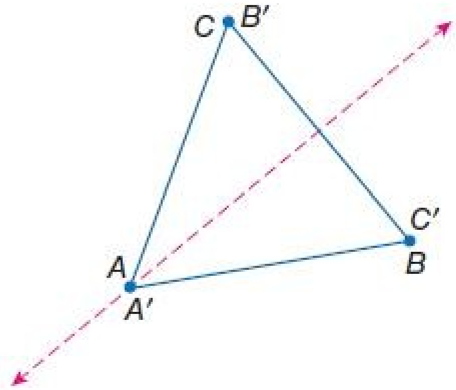
ANSWER:
plane and axis; 180

45. **MULTIPLE REPRESENTATIONS** In this problem, you will use dynamic geometric software to investigate line and rotational symmetry in regular polygons.

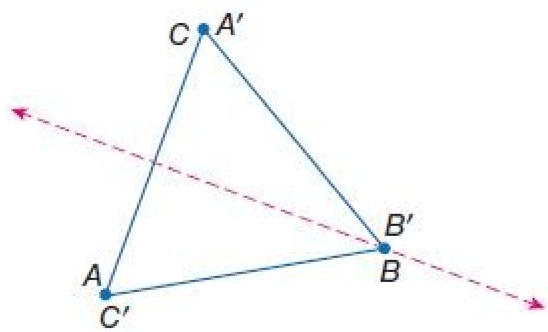
a. Geometric Use The Geometer's Sketchpad to draw an equilateral triangle. Use the reflection tool under the transformation menu to investigate and determine all possible lines of symmetry. Then record their number.
b. Geometric Use the rotation tool under the transformation menu to investigate the rotational symmetry of the figure in part **a**. Then record its order of symmetry.
c. Tabular Repeat the process in parts **a** and **b** for a square, regular pentagon, and regular hexagon. Record the number of lines of symmetry and the order of symmetry for each polygon.
d. Verbal Make a conjecture about the number of lines of symmetry and the order of symmetry for a regular polygon with *n* sides.

SOLUTION:

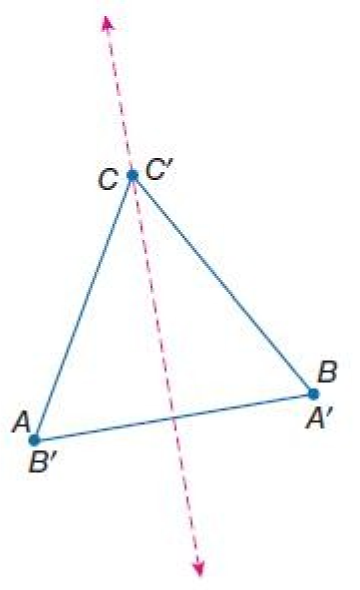
a. Construct an equilateral triangle and label the vertices *A*, *B*, and *C*. Draw a line through *A* perpendicular to $\overline{BC}$. Reflect the triangle in the line. Show the labels of the reflected image. If the image maps to the original, then this line is a line of reflection.



Next, draw a line through *B* perpendicular to $\overline{AC}$. Reflect the triangle in the line. Show the labels of the reflected image. If the image maps to the original, then this line is a line of reflection.

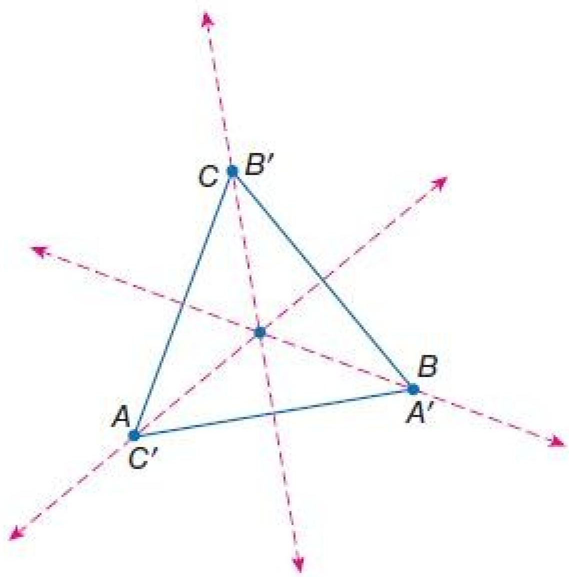


Lastly, draw a line through *C* perpendicular to $\overline{AB}$. Reflect the triangle in the line. Show the labels of the reflected image. If the image maps to the original, then this line is a line of reflection.

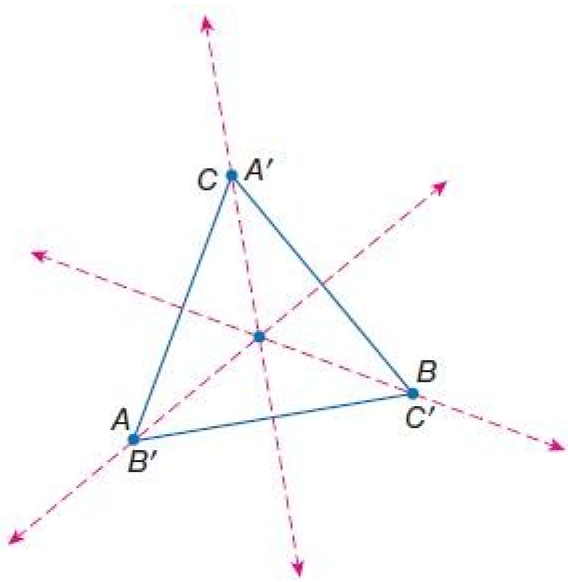


There are 3 lines of symmetry.

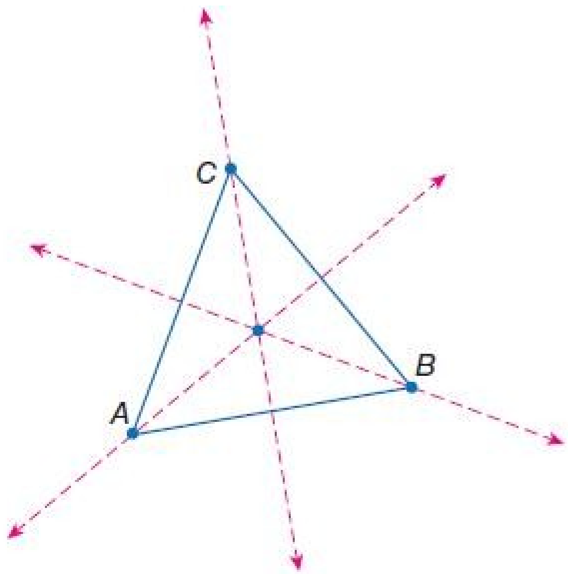
b. Construct an equilateral triangle and show the labels of the vertices. Next, find the center of the triangle. Since this is an equilateral triangle, the circumcenter, incenter, centroid, and orthocenter are the same point. Construct altitudes through each vertex and label the intersection.
Rotate the triangle about point *D*. A 120 degree rotation will map the image to the original. Show the labels of the image.



Rotate the triangle again about point *D*. A 240 degree rotation will map the image to the original. Show the labels of the image



The triangle can be rotated a third time about *D*. A 360 degree rotation maps the image to the original.



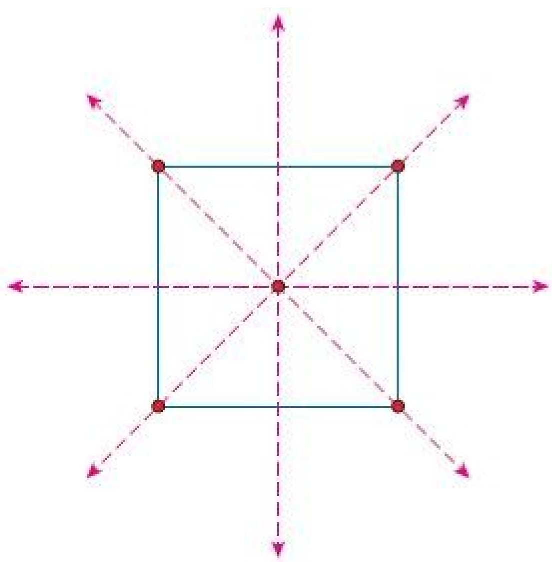
Since the figure maps onto itself 3 times as it is rotated, the order of symmetry is 3.

c.

Square

Construct a square and then construct lines through the midpoints of each side and diagonals. Use the reflection tool first to find that the image maps onto the original when reflected in each of the 4 lines constructed. So there are 4 lines of symmetry.

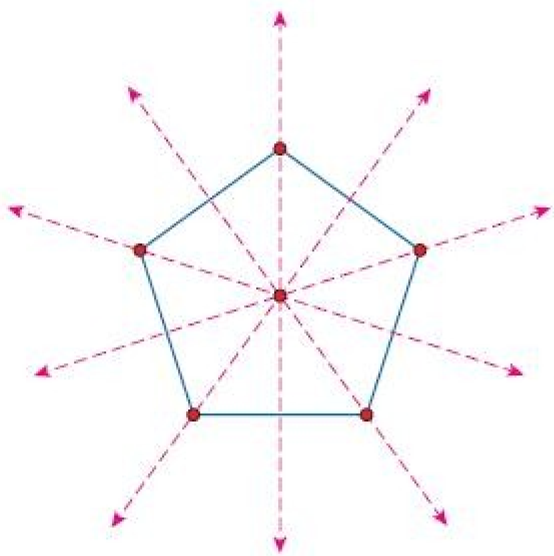
Next, rotate the square about the center point. The image maps to the original at 90, 180, 270, and 360 degree rotations. So the order of symmetry is 4.



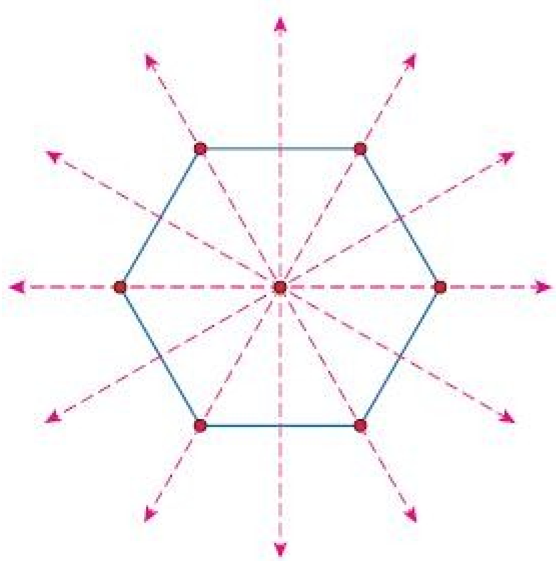
Regular Pentagon

Construct a regular pentagon and then construct lines through each vertex perpendicular to the sides. Use the reflection tool first to find that the image maps onto the original when reflected in each of the 5 lines constructed. So there are 5 lines of symmetry.

Next, rotate the square about the center point. The image maps to the original at 72, 144, 216, 288, and 360 degree rotations. So the order of symmetry is 5.



Regular Hexagon
Construct a regular hexagon and then construct lines through each vertex perpendicular to the sides. Use the reflection tool first to find that the image maps onto the original when reflected in each of the 6 lines constructed. So there are 6 lines of symmetry.
Next, rotate the square about the center point. The image maps to the original at 60, 120, 180, 240, 300, and 360 degree rotations. So the order of symmetry is 6.



Polygon	Lines of Symmetry	Order of Symmetry
equilateral triangle	3	3
square	4	4
regular pentagon	5	5
regular hexagon	6	6

d. Sample answer: for each figure studied, the number of sides of the figure is the same as the lines of symmetry and the order of symmetry. A regular polygon with *n* sides has *n* lines of symmetry and order of symmetry *n*.

ANSWER:

a. 3
b. 3
c.

Polygon	Lines of Symmetry	Order of Symmetry
equilateral triangle	3	3
square	4	4
regular pentagon	5	5
regular hexagon	6	6

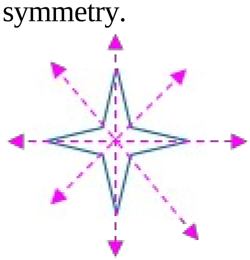
d. Sample answer: A regular polygon with n sides has n lines of symmetry and order of symmetry n.

46. **CCSS CRITIQUE** Jaime says that Figure A has only line symmetry, and Jewel says that Figure A has only rotational symmetry. Is either of them correct? Explain your reasoning.

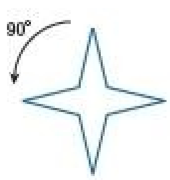
Figure *A*

SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line. This figure has 4 lines of symmetry.



A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.



The figure also has rotational symmetry.

Therefore, neither of them are correct. Figure A has both line and rotational symmetry.

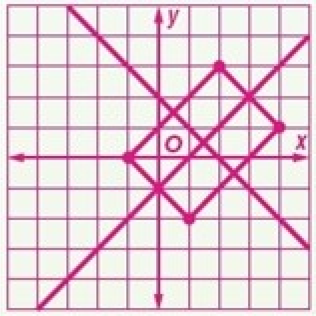
ANSWER:

Neither; Figure A has both line and rotational symmetry.

47. **CHALLENGE** A quadrilateral in the coordinate plane has exactly two lines of symmetry, $y = x - 1$ and $y = -x + 2$. Find a set of possible vertices for the figure. Graph the figure and the lines of symmetry.

SOLUTION:

Graph the figure and the lines of symmetry.

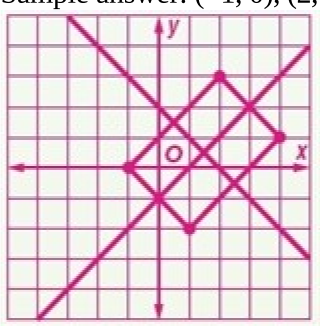


Pick points that are the same distance *a* from one line and the same distance *b* from the other line. In the same answer, the quadrilateral is a rectangle with sides which are parallel to the lines of symmetry. This guarantees that the vertices of the quadrilateral are the same distance *a* from one line and the same distance *b* from the other line. In this case, $a = \sqrt{2}$ and $b = \frac{3\sqrt{2}}{2}$.

Sample answer: A set of possible vertices for the figure are, (–1, 0), (2, 3), (4, 1), and (1, 2).

ANSWER:

Sample answer: (–1, 0), (2, 3), (4, 1), and (1, –2)



48. **REASONING** A regular polyhedron has axis symmetry of order 3, but does not have plane symmetry. What is the figure? Explain.

SOLUTION:

A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane. A three-dimensional figure has axis symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° in a line.

We need to identify a three-dimensional figure that is not symmetric so it does not have plane symmetry. We also need a three-dimensional figure that has order 3 of axis symmetry. This implies that the figure can be rotated three time and map onto itself.

Since the figure has axis symmetry of order 3, the base has to be an equilateral triangle. Because it does not have plane symmetry, you know that it is a pyramid instead of a prism. Therefore, the figure must be an equilateral triangular pyramid.

ANSWER:

Equilateral triangular pyramid; sample answer: Since the figure has axis symmetry of order 3, the base has to be an equilateral triangle. Because it does not have plane symmetry, you know that it is a pyramid instead of a prism. Therefore, the figure must be an equilateral triangular pyramid.

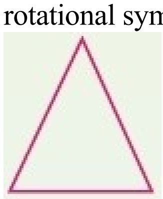
49. **OPEN ENDED** Draw a figure that has line symmetry but not rotational symmetry. Explain.

SOLUTION:

A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line. A figure in the plane has rotational symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure.

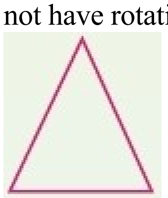
Identify a figure that has line symmetry but does not have rotational symmetry.

An isosceles triangle has line symmetry from the vertex angle to the base of the triangle, but it does not have rotational symmetry because it cannot be rotated from 0° to 360° and map onto itself.



ANSWER:

Sample answer: An isosceles triangle has line symmetry from the vertex angle to the base of the triangle, but it does not have rotational symmetry because it cannot be rotated from 0° to 360° and map onto itself.

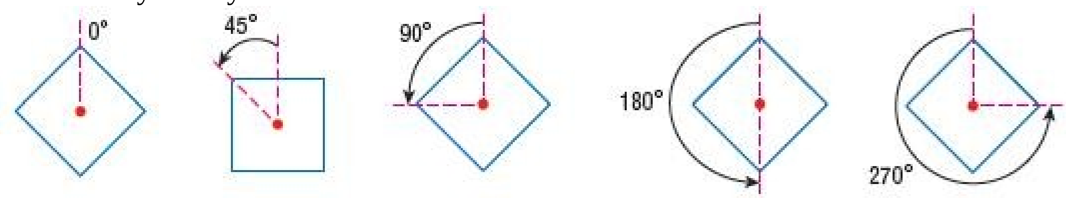


50. **WRITING IN MATH** How are line symmetry and rotational symmetry related?

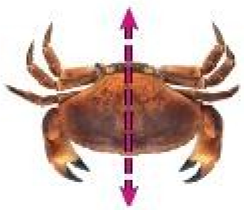
SOLUTION:

In both types of symmetries the figure is mapped onto itself.

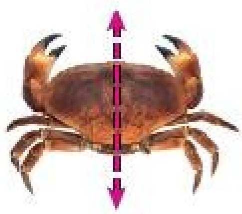
Rotational symmetry.



Reflection symmetry:



In some cases an object can have both rotational and line symmetry, such as the diamond, however some objects do not have both such as the crab.



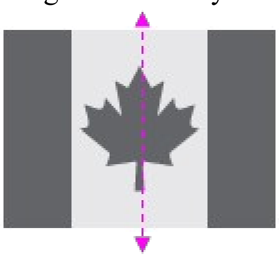
ANSWER:

Sample answer: In both rotational and line symmetry a figure is mapped onto itself. However, in line symmetry the figure is mapped onto itself by a reflection, and in rotational symmetry a figure is mapped onto itself by a rotation. A figure can have line symmetry and rotational symmetry.

51. How many lines of symmetry can be drawn on the picture of the Canadian flag below?

A 0
B 1
C 2
D 4

SOLUTION:
A figure has line symmetry if the figure can be mapped onto itself by a reflection in a line.

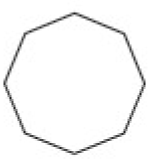


Because the flag is rectangular and has a leaf in the center, there can be only one possible line symmetry.

Therefore, the correct choice is B.

ANSWER:
B

52. **GRIDDED RESPONSE** What is the order of symmetry for the figure below?



SOLUTION:
Since the regular polygon has 8 sides, the order of symmetry is 8.

ANSWER:
8

53. **ALGEBRA** A computer company ships computers in wooden crates that each weigh 45 pounds when empty. If each computer weighs no more than 13 pounds, which inequality best describes the total weight in pounds w of a crate of computers that contains c computers?
F $c \le 13 + 45w$
G $c \ge 13 + 45w$
H $w \le 13c + 45$
J $w \ge 13c + 45$

SOLUTION:

The empty wooden crate weighs 45 pounds. Each computer weighs no more than 13 pounds. So, the weight w for c number of computers is $w \le 13c + 45$.
Therefore, the correct choice is H.

ANSWER:
H

54. **SAT/ACT** What is the slope of the line determined by the linear equation $5x - 2y = 10$?
A -5
B $-\frac{5}{2}$
C $-\frac{2}{5}$
D $\frac{2}{5}$
E $\frac{5}{2}$

SOLUTION:
Write the equation in slope-intercept form.

$$5x - 2y = 10$$
$$-2y = 10 - 5x$$
$$y = \frac{10}{-2} - \frac{5x}{-2}$$
$$y = \frac{5x}{2} - 5$$

The slope of the line is 5/2.
Therefore, the correct choice is E.

ANSWER:
E

Triangle *JKL* has vertices *J*(1, 5), *K*(3, 1), and *L*(5, 7). Graph $\triangle JKL$ and its image after the indicated transformation.

55. Translation: along $\langle -7, -1 \rangle$ Reflection: in *x*-axis

SOLUTION:

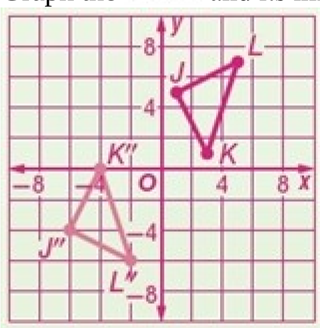
Translate along $\langle -7, -1 \rangle$.

$$(x, y) \to (x - 7, y - 1)$$
$$J(1,5) \to J'(-6,4)$$
$$K(3,1) \to K'(-4,0)$$
$$L(5,7) \to L'(-2,6)$$

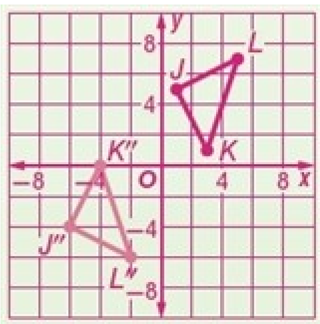
Reflect in the *x*-axis.

$$(x, y) \to (x, -y)$$
$$J'(-6,4) \to J''(-6,-4)$$
$$K'(-4,0) \to K''(-4,0)$$
$$L'(-2,6) \to L''(-2,-6)$$

Graph the $\triangle JKL$ and its image $\triangle J''K''L''$.



ANSWER:



56. Translation: along $\langle 1, 2 \rangle$ Reflection: in y-axis

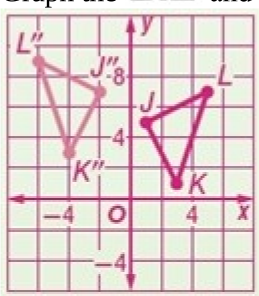
SOLUTION:

Translate along $\langle 1, 2 \rangle$.

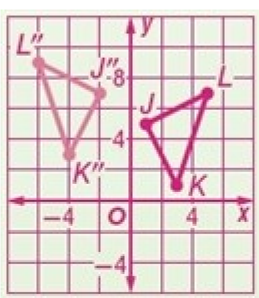
$(x, y) \rightarrow (x + 1, y + 2)$
$J(1,5) \rightarrow J'(2,7)$
$K(3,1) \rightarrow K'(4,3)$
$L(5,7) \rightarrow L'(6,9)$

Reflect in the y-axis.
$(x, y) \rightarrow (-x, y)$
$J'(2,7) \rightarrow J''(-2,7)$
$K'(4,3) \rightarrow K''(-4,3)$
$L'(6,9) \rightarrow L''(-6,9)$

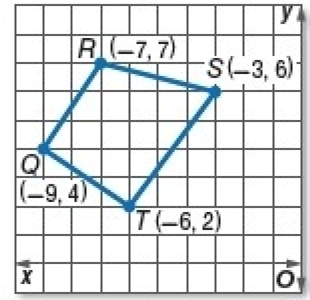
Graph the ΔJKL and its image $\Delta J''K''L''$.



ANSWER:



57. Quadrilateral *QRST* is shown. What is the image of point *R* after a rotation 180° counter clockwise about the origin?



SOLUTION:

To rotate a point $180°$ counter clockwise about the origin, multiply the *x*- and *y*-coordinates by 1.

$$(x, y) \to (-x, -y)$$
$$R(-7, 7) \to R'(7, -7)$$

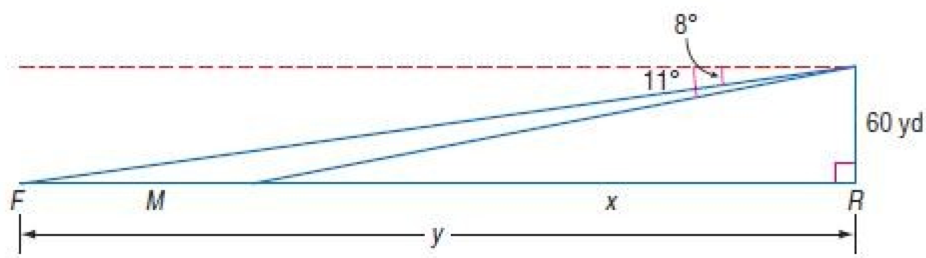
ANSWER:

$(7, -7)$

58. **AMUSEMENT PARKS** From the top of a roller coaster, 60 yards above the ground, a rider looks down and sees the merry-go-round and the Ferris wheel. If the angles of depression are 11° and 8° respectively, how far apart are the merry-go-round and the Ferris wheel?

SOLUTION:

Let x be the horizontal distance between the rider and the merry-go-round and let y be the horizontal distance between the rider and the Ferris wheel. Draw a diagram. Let R represent the roller coaster, F is the Ferris wheel, and M is the merry-go-round.



Use tangent ratio to find the horizontal distance.

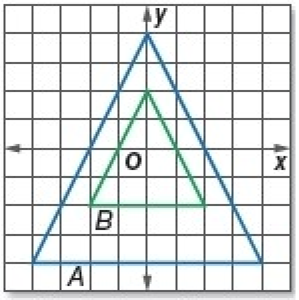
$$\tan 11° = \frac{60}{x}$$
$$x = \frac{60}{\tan 11°} \approx 308.7$$
$$\tan 8° = \frac{60}{y}$$
$$y = \frac{60}{\tan 8°} \approx 426.9$$

The merry-go-round is about 426.9 – 308.7 or 118.2 yd apart from the Ferris wheel.

ANSWER:

about 118.2 yd

Determine whether the dilation from Figure A to Figure B is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.

59. 

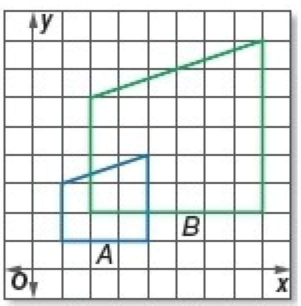
SOLUTION:

B is smaller than A, so the dilation is a reduction.
The distance between the vertices at

(–4, –4) and (4, –4) for A is 8 and from the vertices (–2, –2) and (2, –2) for B is 4. So, the scale factor is $\frac{4}{8}$ or $\frac{1}{2}$.

ANSWER:

reduction; $\frac{1}{2}$

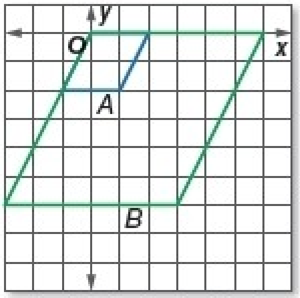
60. 

SOLUTION:

B is larger than A, so the dilation is an enlargement.
The distance between the vertices at (1, 1) and (1, 3) for A is 2 and from the vertices (2, 2) and (2, 6) for B is 4. So, the scale factor is $\frac{4}{2}$ or 2.

ANSWER:

enlargement; 2

61. 

SOLUTION:

B is larger than A, so the dilation is an enlargement.
The distance between the vertices at

(0, 0) and (2, 0) for A is 2 and from the vertices (0, 0) and (0, 6) for B is 6. So, the scale factor is $\frac{6}{2}$ or 3 .

ANSWER:

enlargement; 3