9-4 Compositions of Transformations

Triangle *CDE* has vertices *C*(–5, –1), *D*(–2, –5), and *E*(–1, –1). Graph $\triangle CDE$ and its image after the indicated glide reflection.

1. Translation: along $\langle 4, 0\rangle$ Reflection: in *x*-axis

SOLUTION:

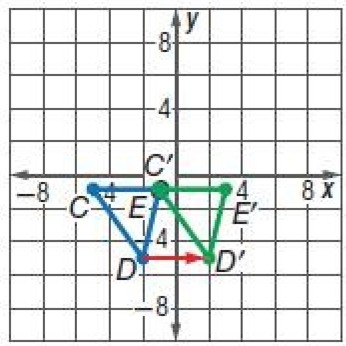
Translation along $\langle 4, 0\rangle$:

$(x, y) \Rightarrow (x+4, y)$

$C(-5, -1) \Rightarrow C'(-1, -1)$

$D(-2, -5) \Rightarrow D'(2, -5)$

$E(-1, -1) \Rightarrow E'(3, -1)$



Reflection in *x*-axis:

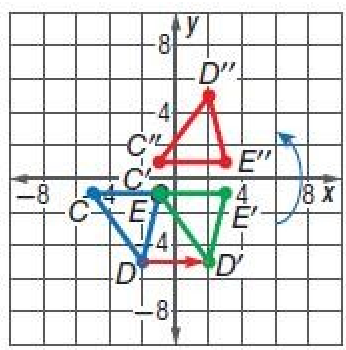
$(x, y) \Rightarrow (x, -y)$

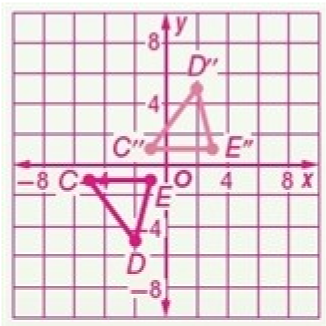
$C'(-1, -1) \Rightarrow C''(-1, 1)$

$E'(3, -1) \Rightarrow E''(3, 1)$

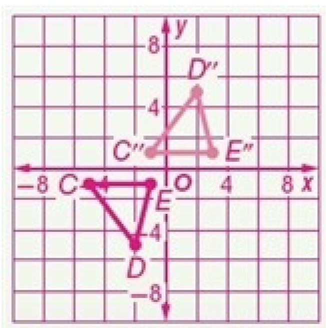
$D'(2, -5) \Rightarrow D''(2, 5)$

Graph $\triangle CDE$ and its image.





ANSWER:



2. Translation: along $\langle 0, 6\rangle$ Reflection: in *y*-axis

SOLUTION:

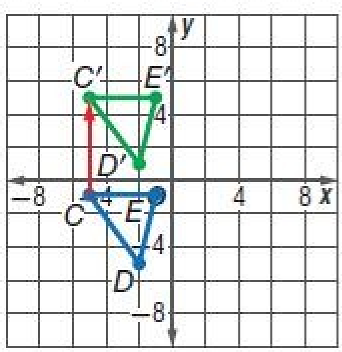
Translation along $\langle 0, 6\rangle$:

$(x, y) \Rightarrow (x, y+6)$

$C(-5, -1) \Rightarrow C'(-5, 5)$

$D(-2, -5) \Rightarrow D'(-2, 1)$

$E(-1, -1) \Rightarrow E'(-1, 5)$



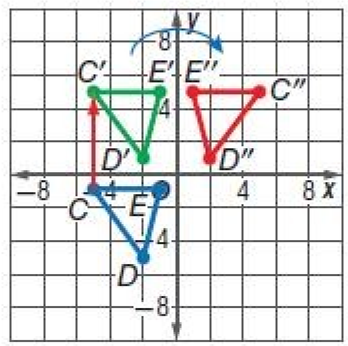
Reflection in *y*-axis:

$(x, y) \Rightarrow (-x, y)$

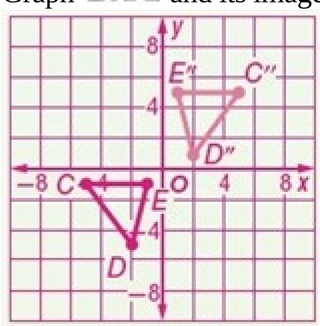
$C'(-5, 5) \Rightarrow C''(5, 5)$

$D'(-2, 1) \Rightarrow D''(2, 1)$

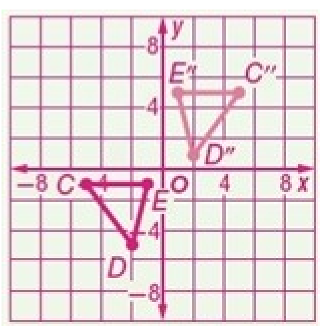
$E'(-1, 5) \Rightarrow E''(1, 5)$



Graph ΔCDE and its image.



ANSWER:



3. The endpoints of $\overline{JK}$ are *J*(2, 5) and *K*(6, 5). Graph $\overline{JK}$ and its image after a reflection in the *x*-axis and a rotation 90° about the origin.

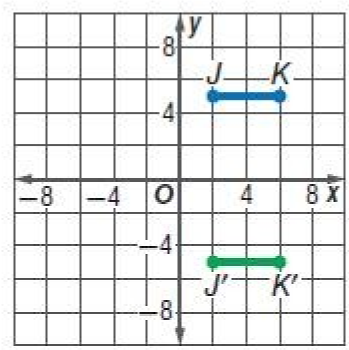
SOLUTION:

Reflection in *x*-axis:

$(x, y) \Rightarrow (x, -y)$

$J(2, 5) \Rightarrow J'(2, -5)$

$K(6, 5) \Rightarrow K'(6, -5)$

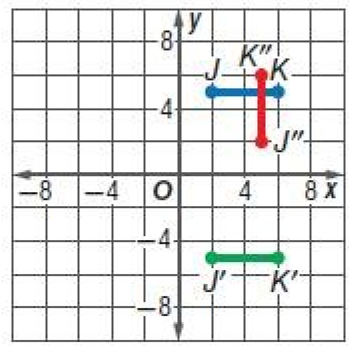


Rotation of 90° about the origin:

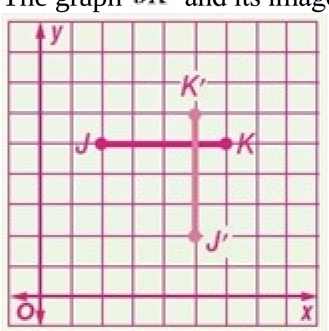
$(x, y) \Rightarrow (-y, x)$

$J'(2, -5) \Rightarrow J''(5, 2)$

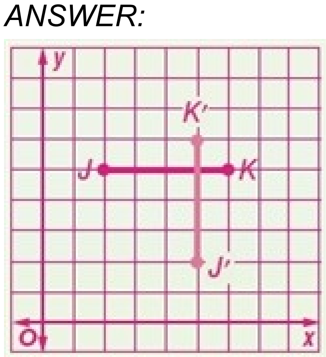
$K'(6, -5) \Rightarrow K''(5, 6)$



The graph $\overline{JK}$ and its image is given below.

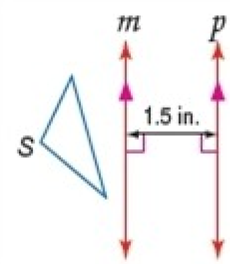


ANSWER:



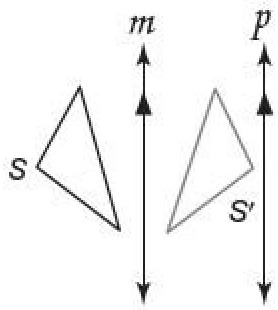
Copy and reflect figure *S* in line m and then line *p*. Then describe a single transformation that maps *S*

onto S''.

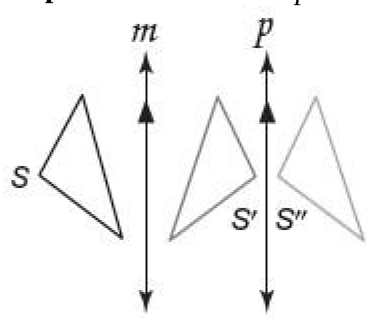
4. 

SOLUTION:

Step 1: Reflect *A* in line *m*.

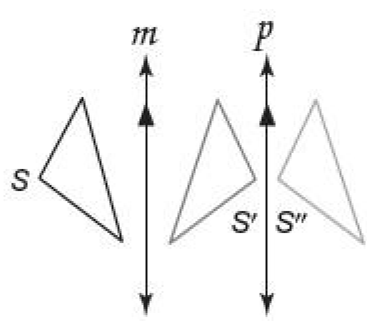


Step 2: Reflect *B* in line *p* .

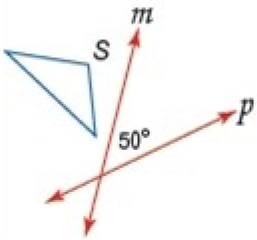


By Theorem 9.2, the composition of two reflections in parallel vertical lines *m* and *p* is equivalent to a horizontal translation right $2 \cdot 1.5$ or 3 inches.

ANSWER:

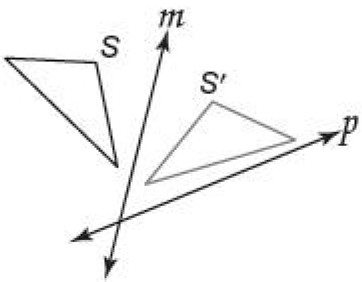


horizontal translation 3 in. to the right

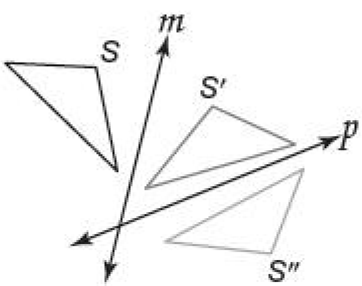
5. 

SOLUTION:

Step 1: Reflect *S* in line *m*.

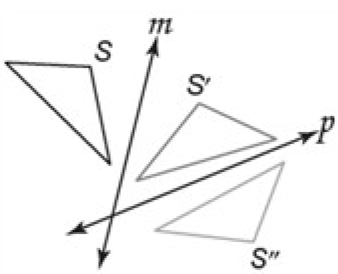


Step 2: Reflect *S* in line *p*.



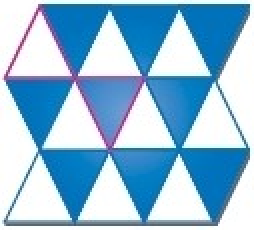
By Theorem 9.3, the composition of two reflections in intersecting lines *m* and *p* is equivalent to a $2 \cdot 50°$ or 100° clockwise rotation about the point where lines *m* and *p* intersect.

ANSWER:



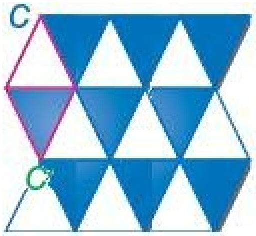
rotation clockwise 100° about point where lines *m* and *p* intersect

6. **TILE PATTERNS** Viviana is creating a pattern for the top of a table with tiles in the shape of isosceles triangles. Describe the transformation combination that was used to transform the white triangle to the blue triangle.

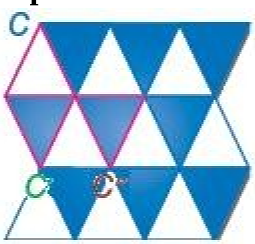


SOLUTION:

Step 1: Reflect the left white triangle with the base as the line of reflection.



Step 2: Translate the triangle to the right.



The resulting transformation can also be called a glide reflection.

ANSWER:

reflection and translation (glide reflection)

Graph each figure with the given vertices and its image after the indicated glide reflection.

7. ΔRST : *R*(1, –4), *S*(6, –4), *T*(5, –1) Translation: along $\langle 2, 0\rangle$ Reflection: in *x*-axis

SOLUTION:

Translation along $\langle 2, 0\rangle$:

$(x, y) \Rightarrow (x + 2, y)$

$R(1, \ -4) \Rightarrow R'(3, \ -4)$

$S(6, \ -4) \Rightarrow S'(8, \ -4)$

$T(5, \ -1) \Rightarrow T'(7, \ -1)$

Reflection in *x*-axis:

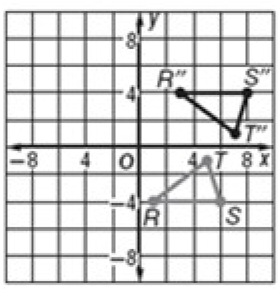
$(x, y) \Rightarrow (x, \ -y)$

$R'(3, \ -4) \Rightarrow R''(3, 4)$

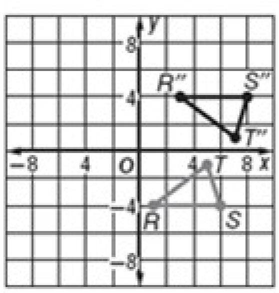
$S'(8, \ -4) \Rightarrow S''(8, 4)$

$T'(7, \ -1) \Rightarrow T''(7, 1)$

Draw a graph.



ANSWER:



8. ΔJKL : $J(1, 3)$, $K(5, 0)$, $L(7, 4)$ Translation: along $\langle -3, 0\rangle$
Reflection: in x-axis

SOLUTION:

Translation along $\langle -3, 0\rangle$:

$(x, y) \Rightarrow (x - 3, y)$

$J(1, 3) \Rightarrow J'(-2, 3)$

$K(5, 0) \Rightarrow K'(2, 0)$

$L(7, 4) \Rightarrow L'(4, 4)$

Reflection in x-axis:

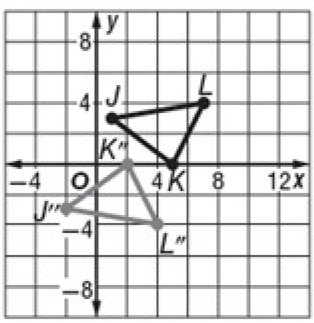
$(x, y) \Rightarrow (x, -y)$

$J'(-2, 3) \Rightarrow J''(-2, -3)$

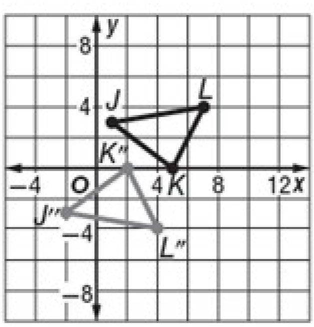
$K'(2, 0) \Rightarrow K''(2, 0)$

$L'(4, 4) \Rightarrow L''(4, -4)$

Draw a graph.



ANSWER:



9. ΔXYZ : X(–7, 2), Y(–5, 6), Z(–2, 4)
Translation: along $\langle 0,-1\rangle$ Reflection: in y-axis

SOLUTION:

Translation along $\langle 0,-1\rangle$:
$(x,y) \Rightarrow (x,y-1)$

$X(-7,2) \Rightarrow X'(-7,1)$

$Y(-5,6) \Rightarrow Y'(-5,5)$

$Z(-2,4) \Rightarrow Z'(-2,3)$

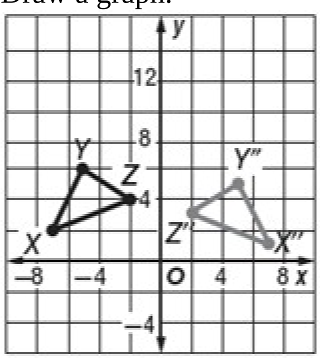
Reflection in y-axis:
$(x,y) \Rightarrow (-x,y)$

$X'(-7,1) \Rightarrow X''(7,1)$

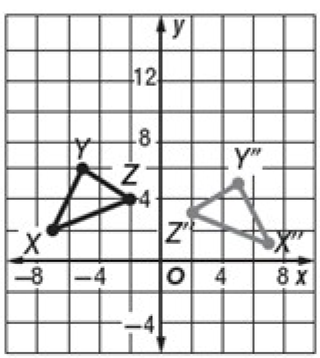
$Y'(-5,5) \Rightarrow Y''(5,5)$

$Z'(-2,3) \Rightarrow Z''(2,3)$

Draw a graph.



ANSWER:



9-4 Compositions of Transformations

10. ΔABC : $A(2, 3)$, $B(4, 7)$, $C(7, 2)$
Translation: along $\langle 0, 4\rangle$ Reflection: in y-axis

SOLUTION:

Translation along $\langle 0, 4\rangle$:

$(x, y) \Rightarrow (x, y + 4)$

$A(2, 3) \Rightarrow A'(2, 7)$

$B(4, 7) \Rightarrow B'(4, 11)$

$C(7, 2) \Rightarrow C'(7, 6)$

Reflection in y-axis:

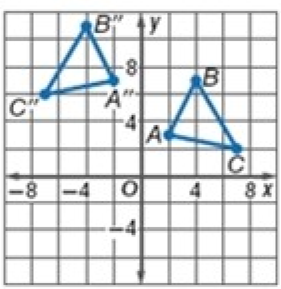
$(x, y) \Rightarrow (-x, y)$

$A'(2, 7) \Rightarrow A''(-2, 7)$

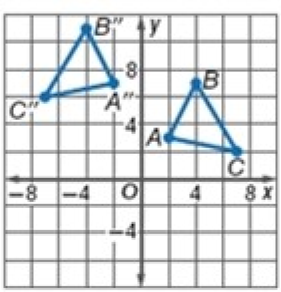
$B'(4, 11) \Rightarrow B''(-4, 11)$

$C'(7, 6) \Rightarrow C''(-7, 6)$

Draw a graph.



ANSWER:



11. $\triangle DFG$: *D*(2, 8), *F*(1, 2), *G*(4, 6)
Translation: along $\langle 3, 3\rangle$ Reflection: in *y* = *x*

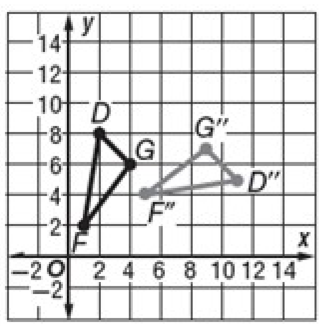
SOLUTION:

Step 1: Translate along $\langle 3, 3\rangle$

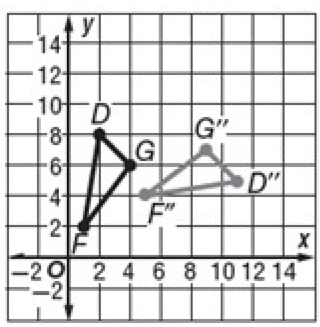
$(x, y) \rightarrow (x+3, y+3)$
$D(2, 8) \rightarrow D'(5, 11)$
$F(1, 2) \rightarrow F'(4, 5)$
$G(4, 6) \rightarrow G'(7, 9)$

Step 2: Reflect in *y* = *x*.

$(x, y) \rightarrow (y, x)$
$D'(5, 11) \rightarrow D''(11, 5)$
$F'(4, 5) \rightarrow F''(5, 4)$
$G'(7, 9) \rightarrow G''(9, 7)$



ANSWER:



12. ΔMPQ : $M(-4, 3)$, $P(-5, 8)$, $Q(-1, 6)$
Translation: along $\langle -4, -4 \rangle$ Reflection: in $y = x$

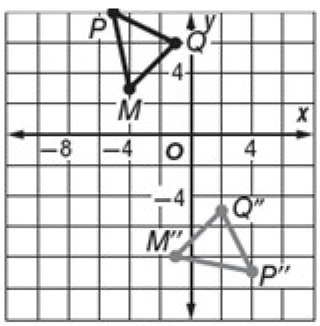
SOLUTION:

Step 1: Translate along $\langle -4, -4 \rangle$.

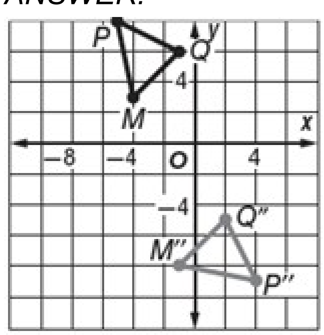
$$(x, y) \to (x-4, y-4)$$
$$M(-4, 3) \to M'(-8, -1)$$
$$P(-5, 8) \to P'(-9, -5)$$
$$Q(-1, 6) \to Q'(-5, 2)$$

Step 2: Reflection: in $y = x$

$$(x, y) \to (y, x)$$
$$M'(-8, -1) \to M''(-1, -8)$$
$$P'(-9, -5) \to P''(-5, -9)$$
$$Q'(-5, 2) \to Q''(2, -5)$$



ANSWER:



CCSS SENSE-MAKING Graph each figure with the given vertices and its image after the indicated composition of transformations.

13. $\overline{WX}$: $W(-4, 6)$ and $X(-4, 1)$
Reflection: in x-axis Rotation: 90° about origin

SOLUTION:

Reflection in x-axis:
$(x, y) \Rightarrow (x, -y)$

$W(-4, 6) \Rightarrow W'(-4, -6)$

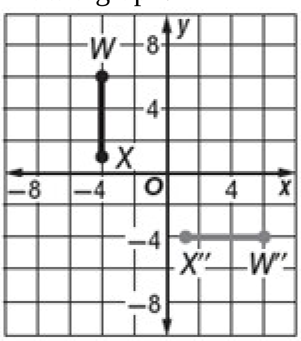
$X(-4, 1) \Rightarrow X'(-4, -1)$

Rotation of 90° about the origin:
$(x, y) \Rightarrow (-y, x)$

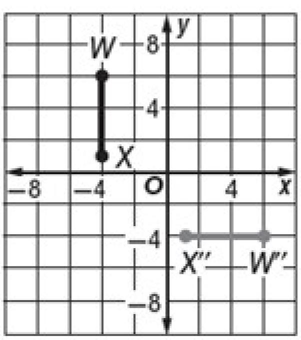
$W'(-4, -6) \Rightarrow W''(6, -4)$

$X'(-4, -1) \Rightarrow X''(1, -4)$

Draw a graph.



ANSWER:



14. $\overline{AB}$: A(–3, 2) and B(3, 8) Rotation: 90° about origin Translation: along $\langle 4, 4\rangle$

SOLUTION:

Rotation of 90° about the origin:

$(x, y) \Rightarrow (-y, x)$

$A(-3, 2) \Rightarrow A'(-2, -3)$

$B(3, 8) \Rightarrow B'(-8, 3)$

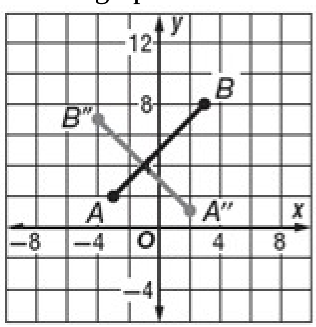
Translation along $\langle 4, 4\rangle$:

$(x, y) \Rightarrow (x+4, y+4)$

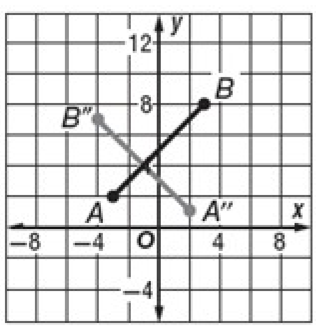
$A'(-2, -3) \Rightarrow A''(2, 1)$

$B'(-8, 3) \Rightarrow B''(-4, 7)$

Draw a graph.



ANSWER:



15. $\overline{FG}$: *F*(1, 1) and *G*(6, 7) Reflection: in *x*-axis Rotation: 180° about origin

SOLUTION:

Reflection in *x*-axis:

$(x, y) \Rightarrow (x, -y)$

$F(1, 1) \Rightarrow F'(1, -1)$

$G(6, 7) \Rightarrow G'(6, -7)$

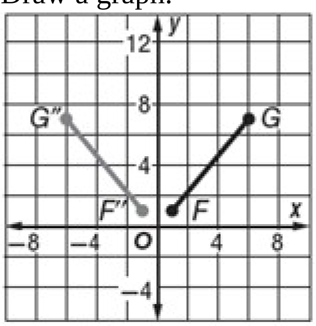
Rotation of 180° about the origin:

$(x, y) \Rightarrow (-x, -y)$

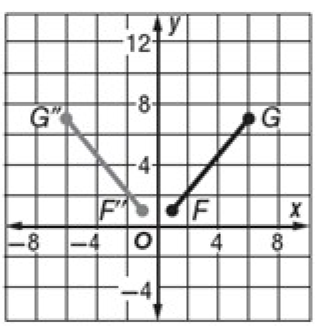
$F'(1, -1) \Rightarrow F''(-1, 1)$

$G'(6, -7) \Rightarrow G''(-6, 7)$

Draw a graph.



ANSWER:



16. $\overline{RS}$: $R(2, -1)$ and $S(6, -5)$ Translation: along $\langle -2, -2 \rangle$ Reflection: in y-axis

SOLUTION:

Translation along $\langle -2, -2 \rangle$:
$(x, y) \Rightarrow (x - 2, y - 2)$

$R(2, -1) \Rightarrow R'(0, -3)$

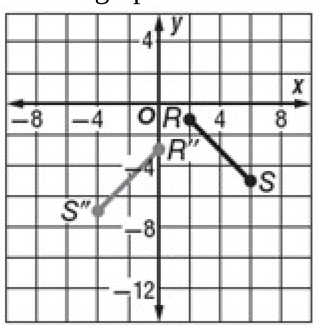
$S(6, -5) \Rightarrow S'(4, -7)$

Reflection in y-axis:
$(x, y) \Rightarrow (-x, y)$

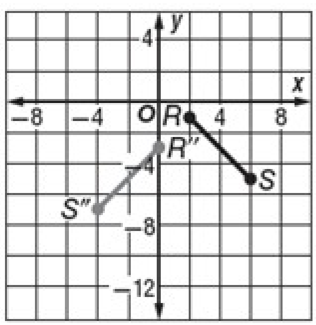
$R'(0, -3) \Rightarrow R''(0, -3)$

$S'(4, -7) \Rightarrow S''(-4, -7)$

Draw a graph.

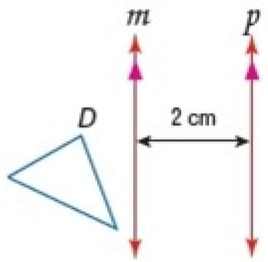


ANSWER:



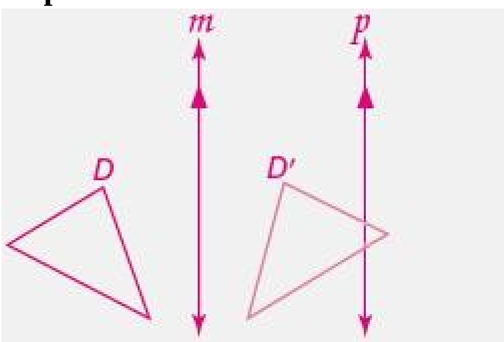
Copy and reflect figure *D* in line *m* and then line *p*. Then describe a single transformation that maps *D* onto D''.

9-4 Compositions of Transformations

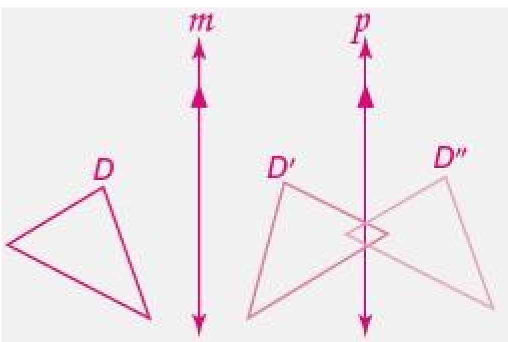
17. 

SOLUTION:

Step 1: Reflect D in line m.

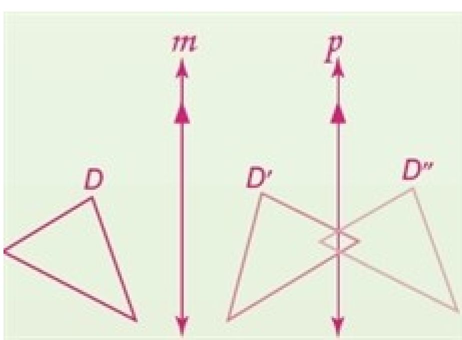


Step 2: Reflect D' in line p.

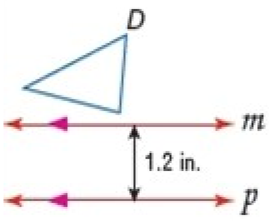


By Theorem 9.2, the composition of two reflections in parallel vertical lines m and p is equivalent to a horizontal translation right $2 \cdot 2$ or 4 centimeters.

ANSWER:

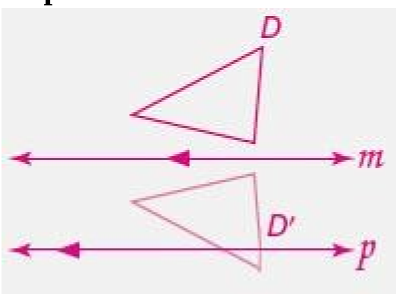


horizontal translation 4 cm to the right

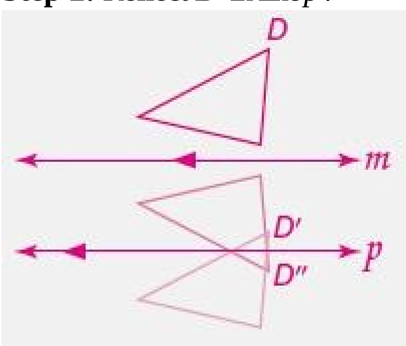
18. 

SOLUTION:

Step 1: Reflect *D* in line *m*.

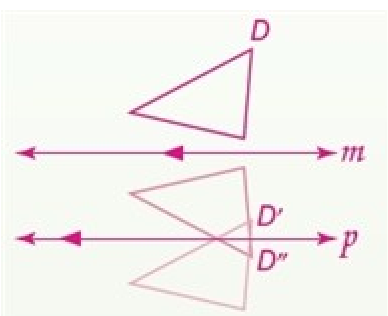


Step 2: Reflect *D′* in line *p* .

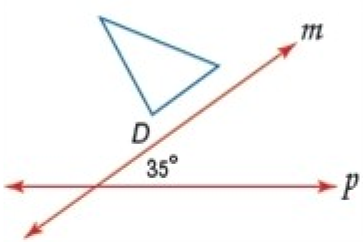


By Theorem 9.2, the composition of two reflections in parallel vertical lines *m* and *p* is equivalent to a vertical translation down 2 • 1.2 or 2.4 inches.

ANSWER:

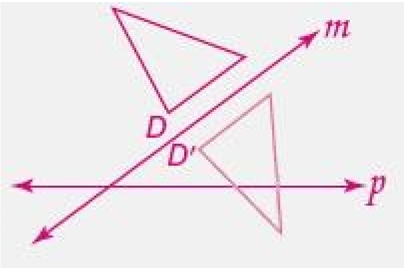


vertical translation 2.4 in. down

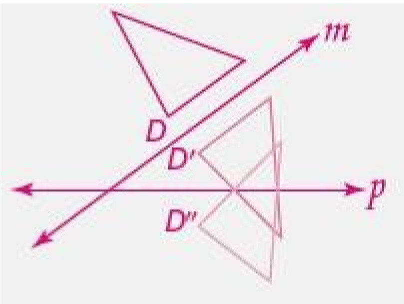
19. 

SOLUTION:

Step 1: Reflect *D* in line *m*.

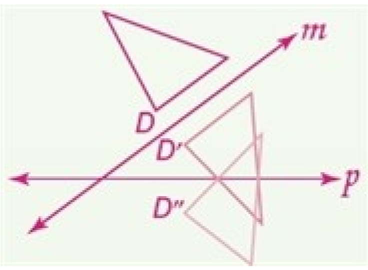


Step 2: Reflect *D′* in line *p*.

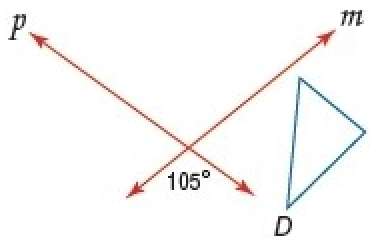


By Theorem 9.3, the composition of two reflections in intersecting lines *m* and *p* is equivalent to a $2 \cdot 35°$ or 70° clockwise rotation about the point where lines *m* and *p* intersect.

ANSWER:

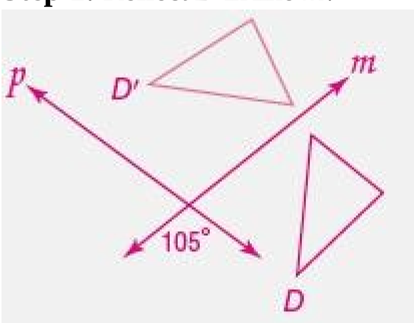


70° rotation about the point where lines *m* and *p* intersect

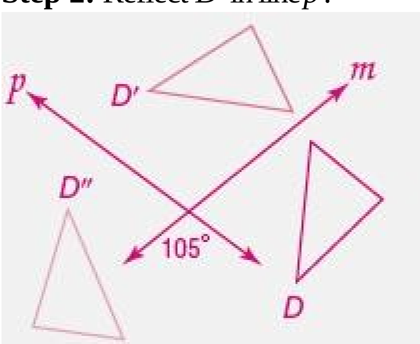
20. 

SOLUTION:

Step 1: Reflect *D* in line *m*.

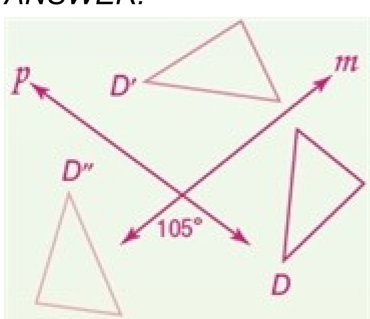


Step 2: Reflect *D′* in line *p*.



By Theorem 9.3, the composition of two reflections in intersecting lines *m* and *p* is equivalent to a 2 • 105° or 210° counterclockwise rotation about the point where lines *m* and *p* intersect.

ANSWER:



210° rotation about the point where lines *m* and *p* intersect

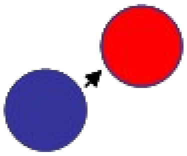
9-4 Compositions of Transformations

CCSS MODELING Describe the transformations combined to create the outlined kimono pattern.

21. Refer to page 656.

SOLUTION:

The blue flower is translated northeast to the red flower.



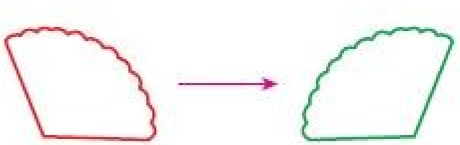
ANSWER:

translation

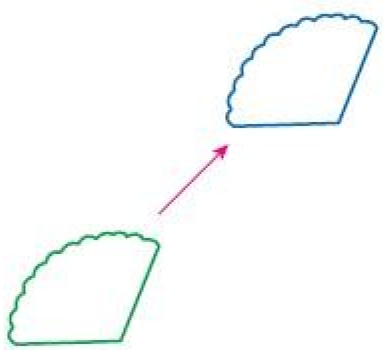
22. Refer to page 656.

SOLUTION:

First, the figure is rotated.



Next, the figure is translated up and right.

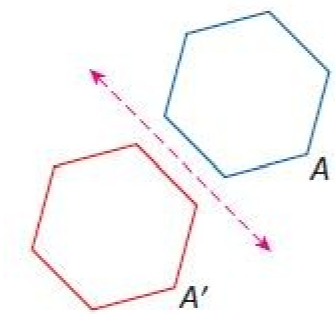


ANSWER:

rotation, then translation

23. Refer to page 656.

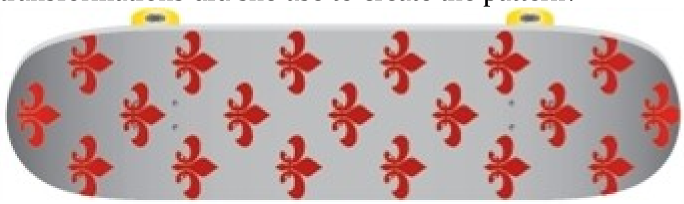
SOLUTION:



This is a reflection about the imaginary green line.

ANSWER:
reflection

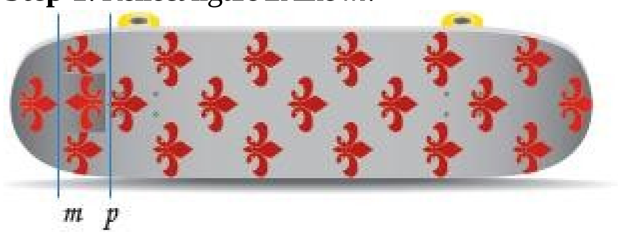
24. **SKATEBOARDS** Elizabeth has airbrushed the pattern shown onto her skateboard. What combination of transformations did she use to create the pattern?



SOLUTION:

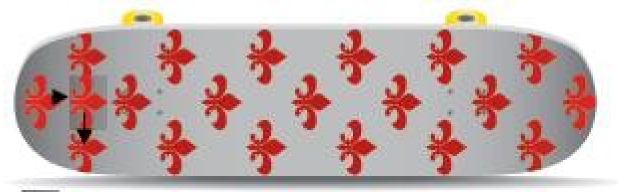
Method 1:

Step 1: Reflect figure in line *m*.



Step 2: Reflect figure in line *p* .

To get the side figures, translate the back figure right the translate it up and down.

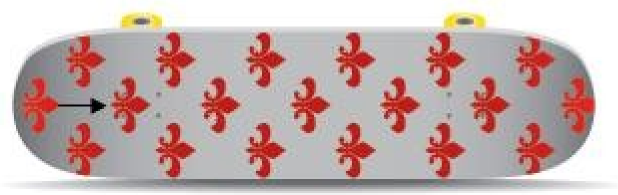


This is also know as a glide reflection.

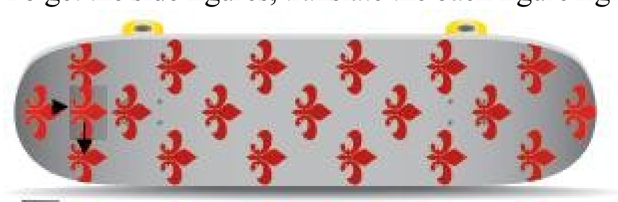
Method 2:

You can create the patterns with translations only.

Translate the middle figure to the right.



To get the side figures, translate the back figure right the translate it up and down.

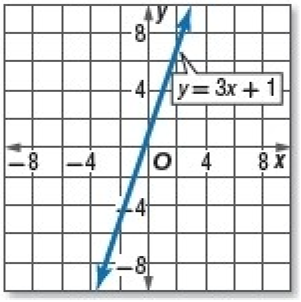


ANSWER:

glide reflection or two translations

ALGEBRA Graph each figure and its image after the indicated transformations.

25. Rotation: 90° about the origin Reflection: in x-axis



SOLUTION:

Identify two points on the line $y = 3x + 1$. Let A be at (0, 1) and B at (–2, –5).

Step 1: Rotate 90° about the origin.

$(x, y) \Rightarrow (-y, x)$

$A(0, 1) \Rightarrow A'(-1, 0)$

$B(-2, -5) \Rightarrow B'(5, -2)$

Find the equations using the two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-2 - 0}{5 - (-1)}$$

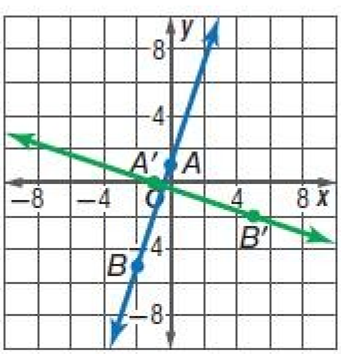
$$m = \frac{-2}{6}$$

$$m = -\frac{1}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - (0) = -\frac{1}{3}(x - (-1))$$

$$y = -\frac{1}{3}x - \frac{1}{3}$$



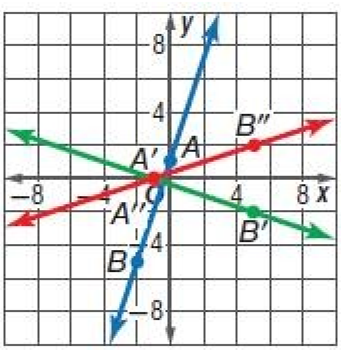
Step 2: Reflect line in x-axis.

$$(x, y) \Rightarrow (x, -y)$$
$$A'(-1, 0) \Rightarrow A''(-1, 0)$$
$$B'(5, -2) \Rightarrow B''(5, 2)$$

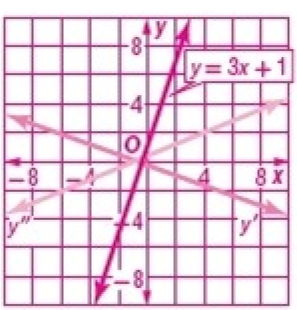
Find the equations using the two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{2 - 0}{5 - (-1)}$$
$$m = \frac{2}{6}$$
$$m = \frac{1}{3}$$

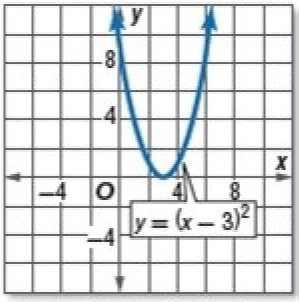
$$y - y_1 = m(x - x_1)$$
$$y - (0) = \frac{1}{3}(x - (-1))$$
$$y = \frac{1}{3}x + \frac{1}{3}$$



ANSWER:



26. Reflection: in *x*-axis Reflection: in *y*-axis



SOLUTION:

Select several points on the quadratic $y = (x-3)^2$.
A(3,0), *B*(4,1), *C*(5,4), *D*(6,9), *E*(0,9), *F*((1,4), *G*(2,1)

Step 1: Reflect the quadratic in the *x*-axis.

$(x, y) \Rightarrow (x, -y)$

$A(3, 0) \Rightarrow A'(3, 0)$

$B(4, 1) \Rightarrow B'(4, -1)$

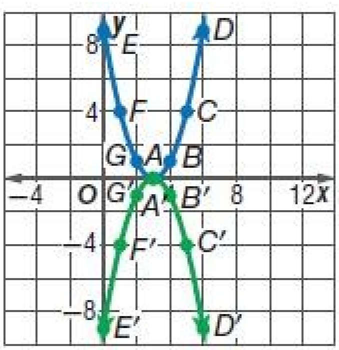
$C(5, 4) \Rightarrow C'(4, -5)$

$D(6, 9) \Rightarrow D'(6, -9)$

$E(0, 9) \Rightarrow E'(0, -9)$

$F(1, 4) \Rightarrow F'(1, -4)$

$G(2, 1) \Rightarrow G'(2, -1)$



The equation of the reflected quadratic is $y = -(x-3)^2$.

Step 2: Reflect the quadratic in the *y*-axis

$(x, y) \Rightarrow (-x, y)$

$A'(3, 0) \Rightarrow A''(-3, 0)$

$B'(4, -1) \Rightarrow B''(-4, -1)$

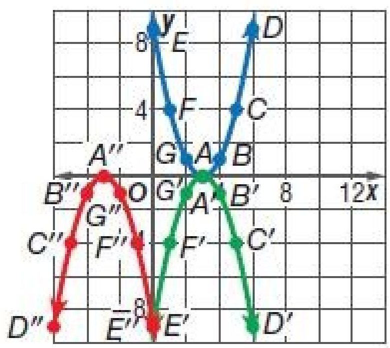
$C'(5, -4) \Rightarrow C''(-4, -5)$

$D'(6, -9) \Rightarrow D''(-6, -9)$

$E'(0, -9) \Rightarrow E''(-0, -9)$

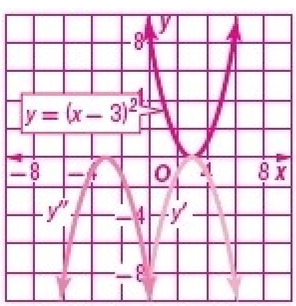
$F'(1, -4) \Rightarrow F''(-1, -4)$

$G'(2, -1) \Rightarrow G''(-2, -1)$



The equation of the reflected quadratic is $y = -(x + 3)^2$.

ANSWER:



27. Find the coordinates of $\Delta A''B''C''$ after a reflection in the *x*-axis and a rotation of 180° about the origin if ΔABC has vertices *A*(–3, 1), *B*(–2, 3), and *C*(–1, 0).

SOLUTION:

reflection in the *x*-axis

$(x, y) \Rightarrow (x, -y)$

$A(-3, 1) \Rightarrow A'(-3, -1)$

$B(-2, 3) \Rightarrow B'(-2, -3)$

$C(-1, 0) \Rightarrow C'(-1, 0)$

To rotate a point 180° clockwise about the origin, multiply the *x*- and *y*-coordinate of each vertex by –1.

$(x, y) \Rightarrow (-x, -y)$

$A'(-3, -1) \Rightarrow A''(3, 1)$

$B'(-2, -3) \Rightarrow B''(2, 3)$

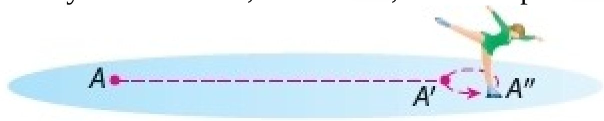
$C'(-1, 0) \Rightarrow C''(1, 0)$

Then A'' (3, 1), B'' (2, 3), and C'' (1, 0).

ANSWER:

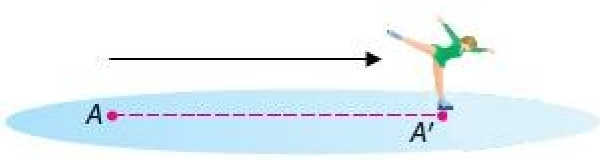
A'' (3, 1), B'' (2, 3),
C'' (1, 0)

28. **FIGURE SKATING** Kayla is practicing her figure skating routine. What combination of transformations is needed for Kayla to start at A, skate to A', and end up at A''?



SOLUTION:

Step 1: Translate right from A to A'



Step 2: Rotate from A' to A''

Kayla starts to rotate at A' does one complete rotation followed by a quarter rotation. Thus she rotation $360° + 90°$ or $450°$.

ANSWER:

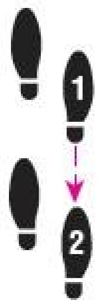
Sample answer: translation and a 450° rotation

9-4 Compositions of Transformations

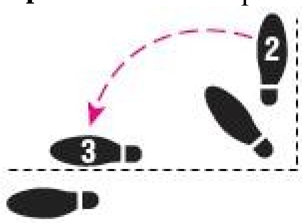
29. **DANCING** Describe the transformations combined to go from Step 1 to Step 3.

SOLUTION:

Step 1: Translate the step from position 1 to position 2.



Step 2: Rotate the step from position 2 to position 3.

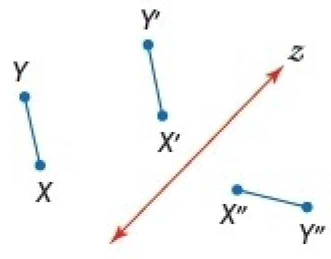


The rotation is a 90° rotation.

ANSWER:

translation and a 90° rotation

30. **PROOF** Write a paragraph proof for one case of the Composition of Isometries Theorem.
Given: A translation along $\langle a, b\rangle$ maps X to X' and Y to Y'.
A reflection in z maps X' to X'' and Y' to Y''.
Prove: $\overline{XY} \cong \overline{X''Y''}$



SOLUTION:
Walk through the proof step by step. Review what is given and what needs to be proved. Here, you are given two translations and two reflections. Show that the image and preimage are congruent. Use the properties that you have learned about reflections and translations to walk through the proof.

Proof: It is given that a translation along $\langle a, b\rangle$ maps X to X' and Y to Y'. Using the definition of a translation, points X and Y move the same distance in the same direction, therefore $\overline{XY} \cong \overline{X'Y'}$. It is also given that a reflection in z maps X' to X'' and Y' to Y''. Using the definition of a reflection, points X and Y are the same distance from line z, so $\overline{X'Y'} \cong \overline{X''Y''}$. By the Transitive Property of Congruence, $\overline{XY} \cong \overline{X''Y''}$.

ANSWER:
Proof: It is given that a translation along $\langle a, b\rangle$ maps X to X' and Y to Y'. Using the definition of a translation, points X and Y move the same distance in the same direction, therefore $\overline{XY} \cong \overline{X'Y'}$. It is also given that a reflection in z maps X' to X'' and Y' to Y''. Using the definition of a reflection, points X'' and Y'' are the same distance from line z as X' and Y', so $\overline{X'Y'} \cong \overline{X''Y''}$. By the Transitive Property of Congruence, $\overline{XY} \cong \overline{X''Y''}$.

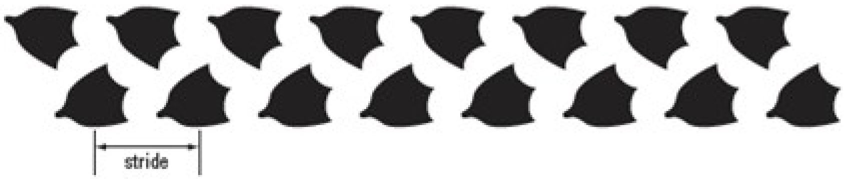
CCSS MODELING Write a glide reflection that can be used to predict the location of the next track for each set of animal tracks.

31. **turkey**

SOLUTION:
Assume that the initial position is (x, y).
The average stride length of a turkey is about 11 inches. Since the turkey has two legs, the average stride length between the two legs will be 11/2 or 5.5.
The glide reflection will be $(x + 5.5, y)$.

ANSWER:
$(x + 5.5, y)$ reflected in the line that separates the left prints from the right prints

32. **duck**



SOLUTION:

Assume that the initial position is (x, y).

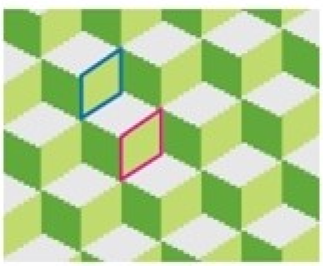
The average stride length of a duck is about 5 inches. Since the duck has two legs, the average stride length between the two legs will be 5/2 or 2.5.

The glide reflection will be $(x + 2.5, y)$.

ANSWER:

$(x + 2.5, y)$ reflected in the line that separates the left prints from the right prints

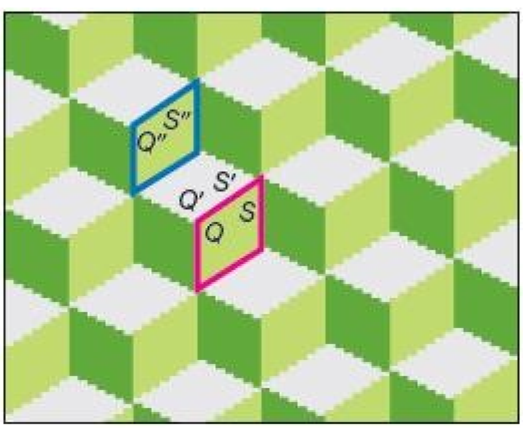
33. **KNITTING** Tonisha is knitting a scarf using the tumbling blocks pattern shown at the right. Describe the transformations combined to transform the red figure to the blue figure.



SOLUTION:

Step 1: Reflect figure over top edge. Q reflects to Q' and S to S'.

Step 2: Reflect figure over top edge. Q' reflects to Q'' and S' to S''.

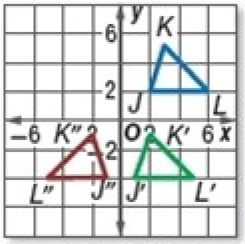


Thus, a double reflection describes the transformation from the pink to blue figure.

ANSWER:

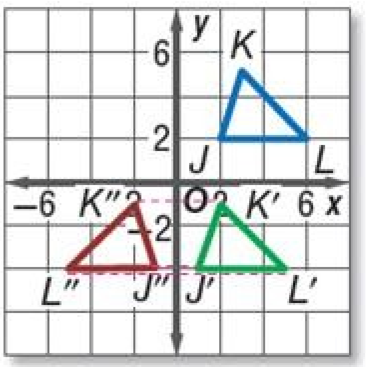
double reflection

Describe the transformations that combined to map each figure.

34. 

SOLUTION:

Step 1: Translate Δ*JKL*.



Describe the translations.

$J(2, 2) \Rightarrow J'(1, -4)$

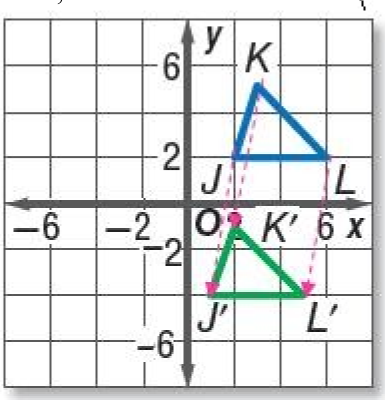
$K(3, 5) \Rightarrow K'(2, -1)$

$L(6, 2) \Rightarrow L'(5, -4)$

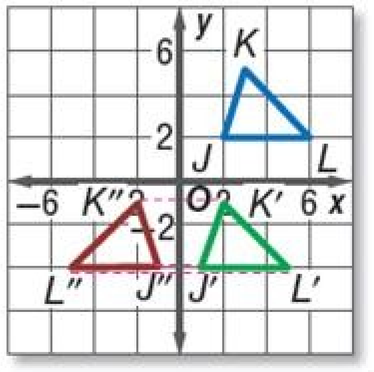
x translation: $5 - 6 = -1$
y translation: $-4 - 2 = -6$

$\langle x, y \rangle = \langle x - 1, y - 6 \rangle$

Thus, the translation vector is $\langle -1, -6 \rangle$.



Step 2: Reflect Δ*J'K'L* in the *y*-axis.



Describe the transformation..

$J'(1, -4) \Rightarrow J'(-1, -4)$

$K'(2, -1) \Rightarrow K'(-2, -1)$

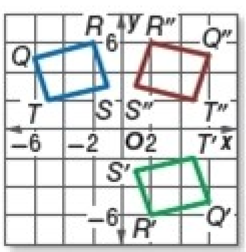
$L'(5, -4) \Rightarrow L'(-5, -4)$

The transformation can be described by $\langle x, y \rangle = \langle -x, y \rangle$.

The transformation is a reflection in the *y*-axis.

ANSWER:

translation along $\langle -1, -6 \rangle$ and reflection in the *y*-axis

35. 

SOLUTION:

Step 1: Rotate figure *QRST*.

Describe the transformation.

$Q(-6, 5) \Rightarrow Q'(6, -5)$

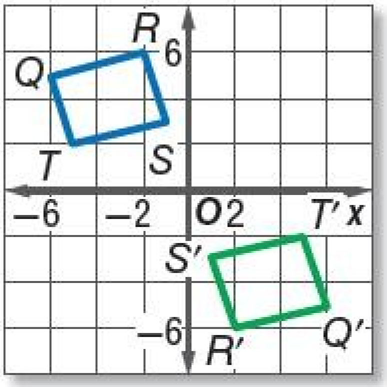
$R(-2, 6) \Rightarrow R'(2, -6)$

$S(-1, 3) \Rightarrow S'(1, -3)$

$T(-5, 2) \Rightarrow T'(5, -2)$.

Notice that the signs of *x*- and *y*-coordinates change.
The rotation can be describe by $(x, y) \rightarrow (-x, -y)$.

Thus, the rotation is a $180°$ rotation about the origin.



Step 2: Reflection *Q'R'S'T'*.

Describe the transformation.

$Q'(6, -5) \Rightarrow Q'(6, 5)$

$R'(2, -6) \Rightarrow R'(2, 6)$

$S'(1, -3) \Rightarrow S''(1, 3)$

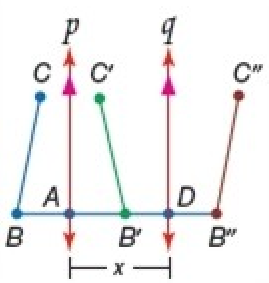
$T'(5, -2) \Rightarrow T''(5, 2)$

Notice that the signs of y-coordinates change.
Thus, the reflection can be describe by $(x, y) \rightarrow (x, -y)$.

Therefore, the reflection is in the *x*-axis.

ANSWER:
rotation 180° about the origin and reflection in the *x*-axis

36. **PROOF** Write a two-column proof of Theorem 9.2.
Given: A reflection in line p maps $\overline{BC}$ to $\overline{B'C'}$.
A reflection in line q maps $\overline{B'C'}$ to $\overline{B''C''}$.
$p \parallel q, AD = x$



Prove: a. $\overline{BB''} \perp p, \overline{BB''} \perp q$
b. $BB'' = 2x$

SOLUTION:
In the 1st row, state the given information about the reflections and the distance between p and q.

In the 2nd row, use the definition of perpendicular bisectors to show p is the perpendicular bisector of $\overline{BB'}$, and q is the perpendicular bisector of $\overline{B'B''}$.

In the 3rd row, use segment addition to show $BB' + B'B'' = BB''$.

In the 4th row, use the concept that a line perpendicular to a portion of a segment is perpendicular to the whole segment.

In the 5th row, apply the reflexive property.

In the 6th row, define the relationship between segment length using the definition of congruent segments.

In the 7th row, apply the segment addition postulate

In the 8th row, use substitution to substitute AB' for BA and $B'D$ for BD'.

In the 9th row, use the addition property to combine like terms.

In the 10th row, use the distributive property to factor out a common factor of 2.

In the 11th row, use the segment addition postulate.

In the 12th row, use substitution with row 10 and row 11.

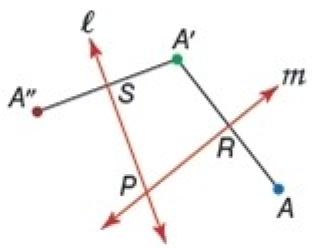
In the 13th row, substitute x for AD.

Proof:
Statements (Reasons):
1. A reflection in line p maps $\overline{BC}$ to $\overline{B'C'}$; a reflection in line q maps $\overline{B'C'}$ to $\overline{B''C''}$; $p \parallel q$; x is the distance between p and q. (Given)
2. p is the perpendicular bisector of $\overline{BB'}$, and q is the perpendicular bisector of $\overline{B'B''}$. (Def. of $\perp$ bisector)
3. $BB' + B'B'' = BB''$ (Seg. Add. Post.)
4. $\overline{BB''} \perp p$, $\overline{BB''} \perp q$ (A line perpendicular to a portion of a segment is perpendicular to the whole segment.)
5. $\overline{BA} \cong \overline{AB'}$; $\overline{B'D} \cong \overline{DB''}$ (Def. of refl.)
6. $BA = AB'$; $B'D = DB''$ (Def. of $\cong$)
7. $BA + AB' + B'D + DB'' = BB''$ (Seg. Add. Post.)
8. $AB' + AB' + B'D + B'D = BB''$ (Subs.)
9. $2AB' + 2B'D = BB''$ (Add. Prop.)
10. $2(AB' + B'D) = BB''$ (Dist. Prop.)
11. $AB' + B'D = AD$ (Seg. Add. Post.)
12. $2AD = BB''$ (Subs.)
13. $2x = BB''$ (Subs.)

ANSWER:
Proof:
Statements(Reasons):
1. A reflection in line p maps $\overline{BC}$ to $\overline{B'C'}$; a reflection in line q maps $\overline{B'C'}$ to $\overline{B''C''}$; $p \parallel q$; x is the distance between p and q. (Given)

2. p is the perpendicular bisector of $\overline{BB'}$, and q is the perpendicular bisector of $\overline{B'B''}$. (Def. of $\perp$ bisector)
3. $BB' + B'B'' = BB''$ (Seg. Add. Post.)
4. $\overline{BB''} \perp p$, $\overline{BB''} \perp q$ (A line perpendicular to a portion of a segment is perpendicular to the whole segment.)
5. $\overline{BA} \cong \overline{AB'}$; $\overline{B'D} \cong \overline{DB''}$ (Def. of refl.)
6. $BA = AB'$; $B'D = DB''$ (Def. of $\cong$)
7. $BA + AB' + B'D + DB'' = BB''$ (Seg. Add. Post.)
8. $AB' + AB' + B'D + B'D = BB''$ (Subs.)
9. $2AB' + 2B'D = BB''$ (Add. Prop.)
10. $2(AB' + B'D) = BB''$ (Dist. Prop.)
11. $AB' + B'D = AD$ (Seg. Add. Post.)
12. $2AD = BB''$ (Subs.)
13. $2x = BB''$ (Subs.)

37. **PROOF** Write a paragraph proof of Theorem 9.3.



Given: Lines l and m intersect at point P. A is any point not on l or m.
Prove: **a.** If you reflect point A in m, and then reflect its image A' in l, A'' is the image of A after a rotation about point P.
b. $m\angle APA'' = 2(m\angle SPR)$

SOLUTION:
Walk through the proof step by step. Review the givens and what needs to be proven. You are given intersecting lines and a point not on the lines. You need to prove that a reflected point over two lines is the image after a rotation. You also need to show the relationship between the angle between the line and the rotation. Use the properties that you have learned about reflection, rotations, and translations to walk through the proof.

Proof: We are given that ℓ and m intersect at point P and that A is not on ℓ or m. Reflect A over m to A' and reflect A' over ℓ to A''. By the definition of reflection, m is the perpendicular bisector of $\overline{AA'}$ at R, and l is the perpendicular bisector of $\overline{A'A''}$ at S. $\overline{AR} \cong \overline{A'R}$ by the definition of a perpendicular bisector.

Through any two points there is exactly one line, so we can draw auxiliary segments, $\overline{AP}, \overline{A'P}$, and $\overline{A''P}$. Angle ARP, angle $A'RP$, angle $A'SP$ and angle $A''SP$ are right angles by the definition of perpendicular bisectors.

$\overline{RP} \cong \overline{RP}$ and $\overline{SP} \cong \overline{SP}$ by the Reflexive Property. $\Delta ARP \cong \Delta A'RP$ *and* $\Delta A'RP \cong \Delta A''SP$ by the SAS Congruence Postulate. Using CPCTC, $\overline{AP} \cong \overline{A'P}$, $\overline{A'P} \cong \overline{A''P}$, and $\overline{AP} \cong \overline{A''P}$ by the Transitive Property.

By the definition of a rotation, A'' is the image of A after a rotation about point P. Also using CPCTC, $\angle APR \cong \angle A'PR$ and $\angle A'PS \cong \angle A''PS$.

By the definition of congruence, $m\angle APR = m\angle A'PR$ *and* $m\angle A'PS =$

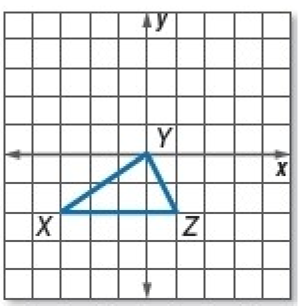
$m\angle A''PS$. $m\angle APR + m\angle A'PR + m\angle A'PS + m\angle A''PS = m\angle APA''$ and $m\angle A'PS + m\angle A'PR = m\angle SPR$ by the Angle Addition Postulate.

$m\angle A'PR + m\angle A'PR + m\angle A'PS + m\angle A'PS = m\angle APA''$ by Substitution, which simplifies to $2(m\angle A'PR + m\angle A'PS) = m\angle APA''$. By Substitution. $2(m\angle SPR) = m\angle APA'''$

ANSWER:

Proof: We are given that *l* and m intersect at point *P* and that *A* is not on *l* or *m*. Reflect *A* over m to A' and reflect A' over *l* to A''. By the definition of reflection, *m* is the perpendicular bisector of $\overline{AA'}$ at *R*, and *l* is the perpendicular bisector of $\overline{A'A''}$ at S. $\overline{AR} \cong \overline{A'R}$ by the definition of a perpendicular bisector. Through any two points there is exactly one line, so we can draw auxiliary segments, $\overline{AP}, \overline{A'P}$, and $\overline{A''P}$. Angle *ARP*, angle *A'RP*, angle *A'SP* and angle *A''SP* are right angles by the definition of perpendicular bisectors. $\overline{RP} \cong \overline{RP}$ *and* $\overline{SP} \cong \overline{SP}$ by the Reflexive Property. $\Delta ARP \cong \Delta A'RP$ *and* $\Delta A'RP \cong \Delta A''SP$ by the SAS Congruence Postulate. Using CPCTC, $\overline{AP} \cong \overline{A'P}$ and $\overline{A'P} \cong \overline{A''P}$ and $\overline{AP} \cong \overline{A''P}$ by the Transitive Property. By the definition of a rotation, A'' is the image of *A* after a rotation about point *P*. Also using CPCTC, $\angle APR \cong \angle A'PR$ and $\angle A'PS \cong \angle A''PS$. By the definition of congruence, $m\angle APR = m\angle A'PR$ *and* $m\angle A'PS = m\angle A''PS$. $m\angle APR + m\angle A'PR + m\angle A'PS + m\angle A''PS = m\angle APA''$ and $m\angle A'PS + m\angle A'PR = m\angle SPR$ by the Angle Addition Postulate. $m\angle A'PR + m\angle A'PR + m\angle A'PS + m\angle A'PS = m\angle APA''$ by Substitution, which simplifies to $2(m\angle A'PR + m\angle A'PS) = m\angle APA''$. By Substitution. $2(m\angle SPR) = m\angle APA'''$

38. **ERROR ANALYSIS** Daniel and Lolita are translating ΔXYZ along $\langle 2, 2\rangle$ and reflecting it in the line *y* = 2. Daniel says that the transformation is a glide reflection. Lolita disagrees and says that the transformation is a composition of transformations. Is either of them correct? Explain your reasoning.



SOLUTION:

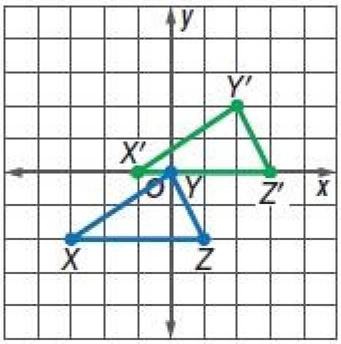
Step 1: Translate along $\langle 2, 2\rangle$.

$(x, y) \Rightarrow (x + 2, y + 2)$

$X(-3, -2) \Rightarrow X'(-1, 0)$

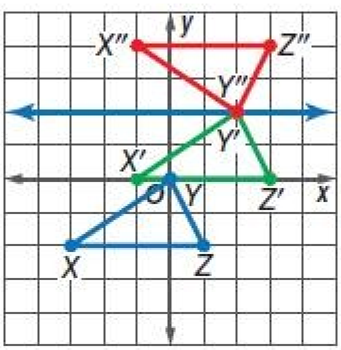
$Y(0, 0) \Rightarrow Y'(2, 2)$

$Z(1, -2) \Rightarrow Z'(3, 0)$



Step 2: Reflect over the $y = 2$ line.

$$2\text{ units from } y = 2: X'(-1, 0) \Rightarrow X''(-1, 4)$$
$$\text{on } y = 2: Y'(2, 2) \Rightarrow Y''(2, 2)$$
$$2\text{ units from } y = 2: Z'(3, 0) \Rightarrow Z''(3, 4)$$



The transformation is a composition of translation and a reflection, so it is a composition of transformations. Therefore, Lolita is correct. The line $y = 2$ is not parallel to the vector $\langle 2, 2 \rangle$, thus, the transformation cannot be a glide reflection.

ANSWER:

Lolita is correct; Sample answer: Since the line y = 2 is not parallel to the vector $\langle 2, 2 \rangle$, the transformation cannot be a glide reflection. It is a composition of translation and a reflection, so it is a composition of transformations.

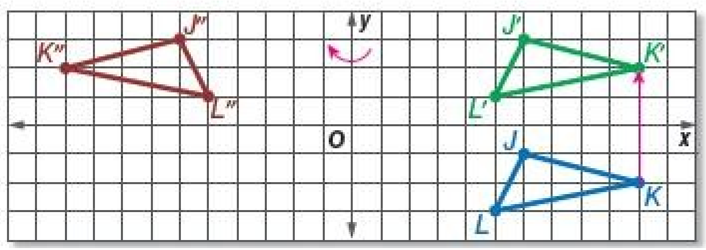
39. **WRITING IN MATH** Do any points remain invariant under glide reflections? Under compositions of transformations? Explain.

SOLUTION:

Invariant points map onto themselves.

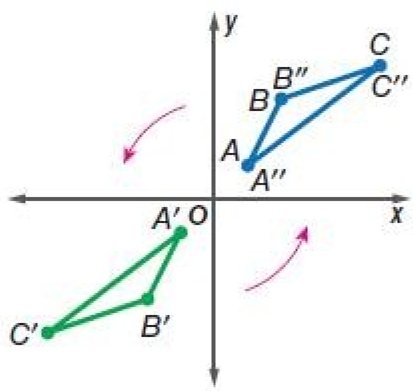
No points remain invariant under glide reflections. This is because all of the points are translated along a vector.

Consider the glide reflection shown below. There are no invariant points.

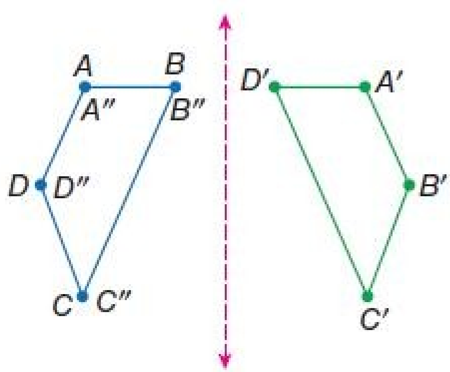


It is possible for points to remain invariant under a composition of transformations. There may be invariant points when a figure is rotated and reflected, rotated twice, or reflected twice.

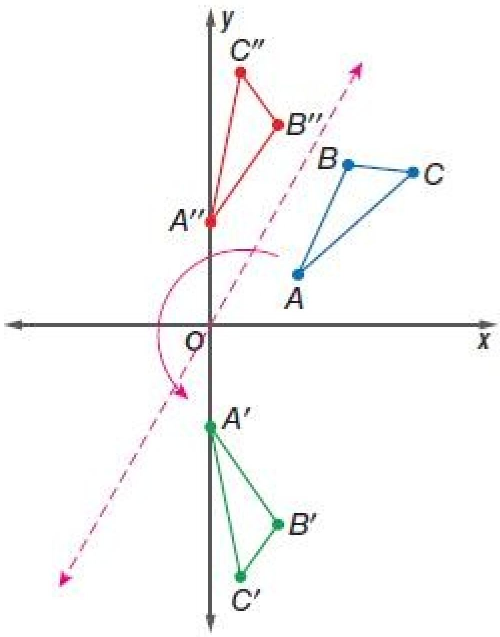
Two 180º rotations will bring the image back to the original image.



Two reflections about the same line will do the same.



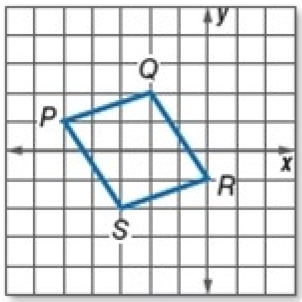
A rotation and reflection can also bring the image back to the original image. Triangle ABC is rotated 240 degrees about the origin and then reflected twice; in the *x*-axis and in the dashed line. The final image maps to the original image.



ANSWER:

Sample answer: No; there are no invariant points in a glide reflection because all of the points are translated along a vector. Perhaps for compositions of transformations, there may be invariant points when a figure is rotated and reflected, rotated twice, or reflected twice.

40. **CHALLENGE** If *PQRS* is translated along $\langle 3, -2\rangle$, reflected in $y = -1$, and rotated 90° about the origin, what are the coordinates of *P'''Q'''R'''S'''*?



SOLUTION:

The coordinates of *P*, *Q*, *R*, and S are (–5, 1), (–2, 2), (0, –1), and (–3, –2) respectively.

Step 1: Translation along $\langle 3, -2\rangle$.

$$(x, y) \Rightarrow (x + 3, y - 2)$$
$$P(-5, 1) \Rightarrow P'(-2, -1)$$
$$Q(-2, 2) \Rightarrow Q'(1, 0)$$
$$R(0, -1) \Rightarrow R'(3, -3)$$
$$S(-3, -2) \Rightarrow S'(0, -4)$$

Step 2: Reflection in $y = -1$.

$$P'(-2, -1) \Rightarrow P''(-2, -1)$$
$$Q'(1, 0) \Rightarrow Q''(1, -2)$$
$$R'(3, -3) \Rightarrow R''(3, 1)$$
$$S'(0, -4) \Rightarrow S''(0, 2)$$

Step 3: Rotation 90º about the origin.

$$(x, y) \Rightarrow (-y, x)$$
$$P''(-2, -1) \Rightarrow P'''(1, -2)$$
$$Q''(1, -2) \Rightarrow Q'''(2, 1)$$
$$R''(3, 1) \Rightarrow R'''(-1, 3)$$
$$S''(0, 2) \Rightarrow S'''(-2, 0)$$

The coordinates of *P'''Q'''R'''S'''* are *P'''*(1, –2), *Q'''*(2, 1), *R'''*(–1, 3), *S'''* (–2, 0).

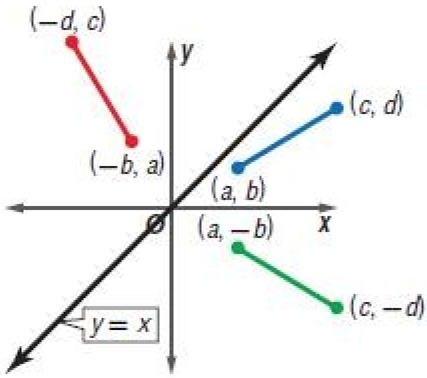
ANSWER:

$P'''(1, -2)$, Q''' (2, 1), R''' (–1, 3), S''' (–2, 0)

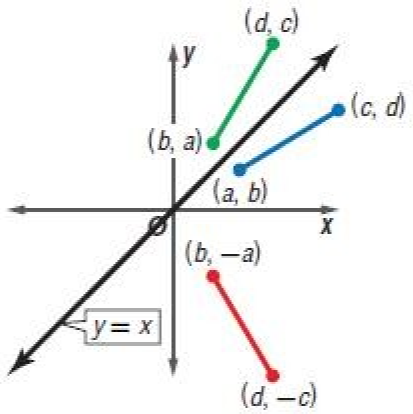
41. **CCSS ARGUMENTS** If an image is to be reflected in the line $y = x$ and the x-axis, does the order of the reflections affect the final image? Explain.

SOLUTION:

Yes; sample answer: If a segment with endpoints (a, b) and (c, d) is to be reflected about the x-axis, the coordinates of the endpoints of the reflected image are $(a, –b)$ and $(c, –d)$. If the segment is then reflected about the line $y = x$, the coordinates of the endpoints of the final image are $(–b, a)$ and $(–d, c)$.



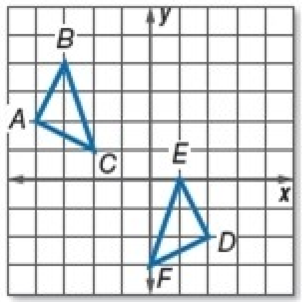
If the original image is first reflected about $y = x$, the coordinates of the endpoints of the reflected image are (b, a) and (d, c). If the segment is then reflected about the x-axis, the coordinates of the endpoints of the final image are $(b, –a)$ and $(d, –c)$.



ANSWER:

Yes; sample answer: If a segment with endpoints (a, b) and (c, d) is to be reflected about the x-axis, the coordinates of the endpoints of the reflected image are $(a, –b)$ and $(c, –d)$. If the segment is then reflected about the line $y = x$, the coordinates of the endpoints of the final image are $(–b, a)$ and $(–d, c)$. If the original image is first reflected about $y = x$, the coordinates of the endpoints of the reflected image are (b, a) and (d, c). If the segment is then reflected about the x-axis, the coordinates of the endpoints of the final image are $(b, –a)$ and $(d, –c)$.

42. **OPEN ENDED** Write a glide reflection or composition of transformations that can be used to transform ΔABC to ΔDEF .



SOLUTION:

Sample answer: first study triangles *ABC* and *DEF*. *BC* and *EF* have the same slope. Angles *A* and *D* are right angles. So, map point *A* to point *D*, point *B* to point *E*, and point *C* to point *F*.

The *x*-coordinates of points *C* and *F* are two units apart. The *y*-coordinates of *B* and *E* are 4 units apart. Translate *ABC* along the vector <0, –4> so each pair of corresponding points lie on the same horizontal line. Next, reflect triangle *ABC* in the line *x* = –1 to transform ΔABC to ΔDEF .

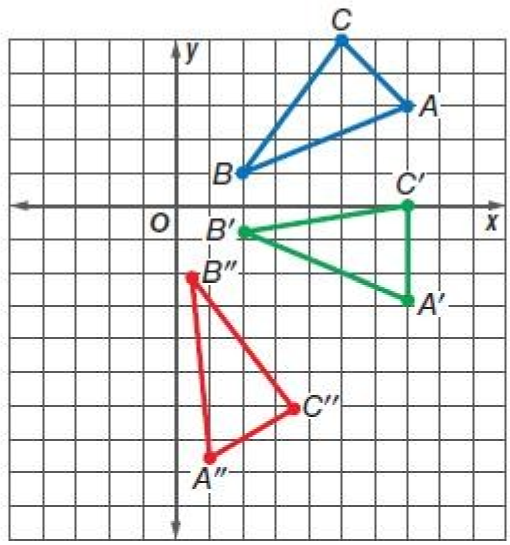
ANSWER:

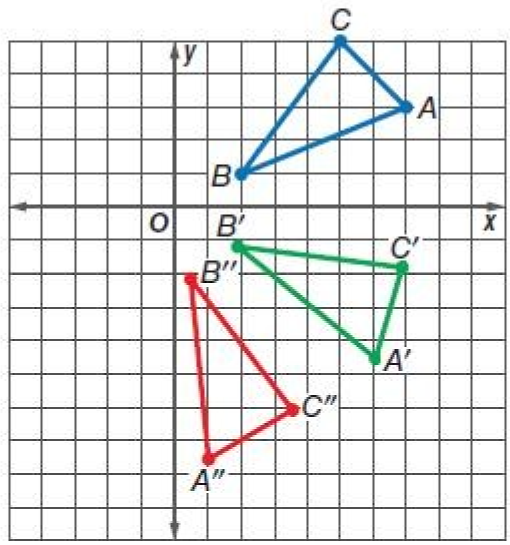
Sample answer: ΔABC can be translated along $\langle 0, -4 \rangle$ and reflected in *x* = −1 to form ΔDEF .

43. **REASONING** When two rotations are performed on a single image, does the order of the rotations *sometimes*, *always*, or *never* affect the location of the final image? Explain.

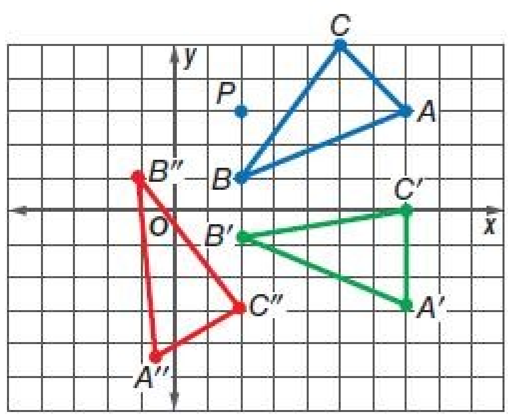
SOLUTION:

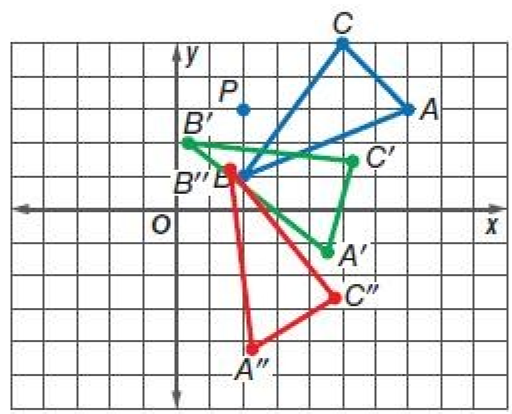
Sometimes; sample answer: When two rotations are performed on a single image, the order of the rotations does not affect the final image when the two rotations are centered at the same point.
For example, if $\triangle ABC$ is rotated 45° clockwise about the origin and then rotated 60° clockwise about the origin, $\triangle A''B''C''$ is the same as if the figure were first rotated 60° clockwise about the origin and then rotated 45° clockwise about the origin.





If $\triangle ABC$ is rotated 45° clockwise about the origin and then rotated 60° clockwise about *P*(2, 3), $\triangle A''B''C''$ is different than if the figure were first rotated 60° clockwise about *P*(2, 3) and then rotated 45° clockwise about the origin.





So, the order of the rotations sometimes affects the location of the final image.

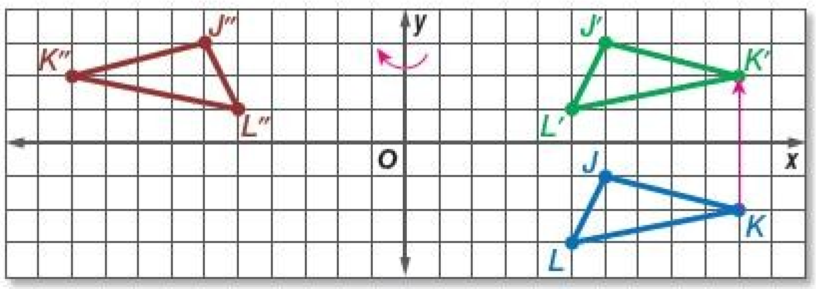
ANSWER:

Sometimes; sample answer: When two rotations are performed on a single image, the order of the rotations does not affect the final image when the two rotations are centered at the same point.

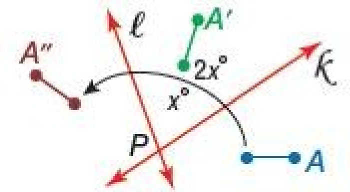
44. **WRITING IN MATH** Compare and contrast glide reflections and compositions of transformations.

SOLUTION:

Sample answer: Glide reflections are compositions of transformations. But not all compositions of transformations are glide reflections. Triangle *JKL* is translated and then reflected in the y-axis.This is a glide reflection.



Rotations can be included in compositions of transformations but not glide reflections. Segment A is reflected in line K and then reflected in line ℓ. This composition of transformations can be described by a rotation.

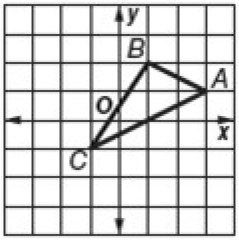


Translations and reflections can both be used in compositions of transformations, but only make up a glide reflection when a figure is translated along a vector and then reflected in a line parallel to that vector.

ANSWER:

Sample answer: Glide reflections are compositions of transformations. But not all compositions of transformations are glide reflections. Rotations can be included in compositions of transformations but not glide reflections. Translations and reflections can both be used in compositions of transformations, but only make up a glide reflection when a figure is translated along a vector and then reflected in a line parallel to that vector.

45. ΔABC is translated along the vector $\langle -2, 3\rangle$ and then reflected in the *x*-axis. What are the coordinates of A' after the transformation?



A (1, –4)
B (1, 4)
C (–1, 4)
D (–1, –4)

SOLUTION:

The coordinates of point *A* are (3, 1).

Step 1: Translate along $\langle -2, 3\rangle$.
$(x, y) \rightarrow (x - 2, y + 3)$
$A(3, 1) \rightarrow A'(1, 4)$

Step 2: Reflection in the *x*-axis.
$(x, y) \Rrightarrow (x, -y)$
$A(1, 4) \Rrightarrow A''(1, -4)$

So, the correct choice is A.

ANSWER:

A

46. **SHORT RESPONSE** What are the coordinates of D'' if $\overline{CD}$ with vertices *C*(2, 4) and *D*(8, 7) is translated along $\langle -6, 2\rangle$ and then reflected over the *y*-axis?

SOLUTION:

The coordinates of point *D* are (8, 7).

Step 1: Translate along $\langle -6, 2\rangle$.

$(x, y) \rightarrow (x - 6, y + 2)$
$D(8, 7) \rightarrow D'(2, 9)$

Step 2: Reflect over the *y*-axis.
$(x, y) \Rrightarrow (-x, y)$
$D'(2, 9) \Rrightarrow D''(-2, 9)$

The coordinates of *D*" is (–2, 9)

ANSWER:

(–2, 9)

47. **ALGEBRA** Write $\frac{18x^2 - 2}{3x^2 - 5x - 2}$ in simplest terms.

F $\frac{18}{3x+1}$
H $\frac{2(3x-1)}{x-2}$
G $\frac{2(3x+1)}{x-2}$
J $2(3x-1)$

SOLUTION:

$$\frac{18x^2-2}{3x^2-5x-2} = \frac{2(9x^2-1)}{3x^2-6x+x-2}$$
$$= \frac{2(3x-1)(3x+1)}{(3x+1)(x-2)}$$
$$= \frac{2(3x-1)\cancel{(3x+1)}}{\cancel{(3x+1)}(x-2)}$$
$$= \frac{2(3x-1)}{x-2}$$

So, the correct choice is H.

ANSWER:
H

48. **SAT/ACT** If $f(x) = x^3 - x^2 - x$, what is the value of $f(-3)$?

A –39
B –33
C –21
D –15
E –12

SOLUTION:

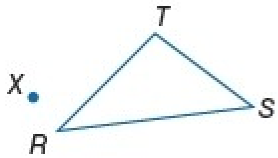
$$f(x) = x^3 - x^2 - x$$
$$f(-3) = (-3)^3 - (-3)^2 - (-3)$$
$$= -27 - 9 + 3$$
$$= -33$$

So, the correct choice is B.

ANSWER:
B

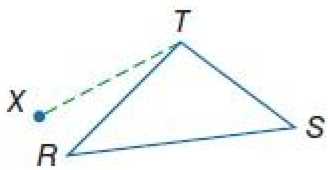
Copy each polygon and point *X*. Then use a protractor and ruler to draw the specified rotation of each figure about point *X*.

49. 60°

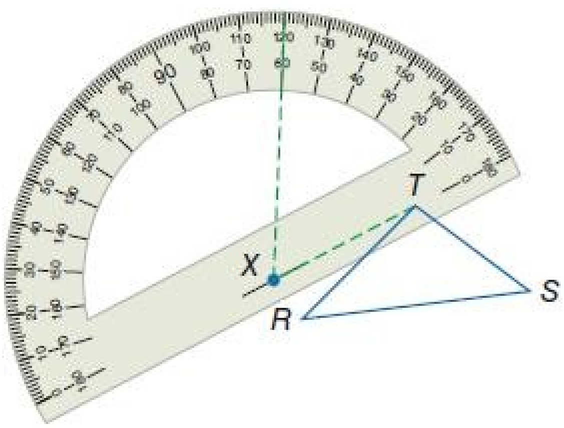


SOLUTION:

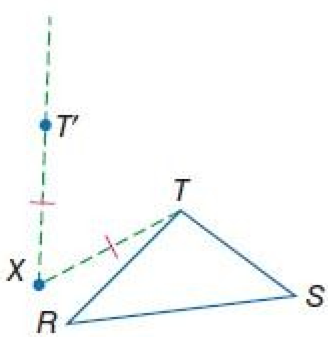
Step 1: Draw segment from *T* to *X*.



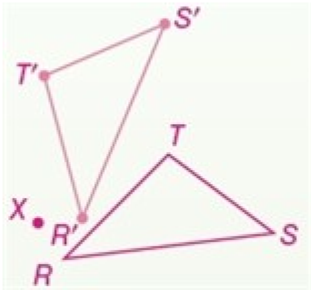
Step 2: Draw a 60 degree angle using *XT*.



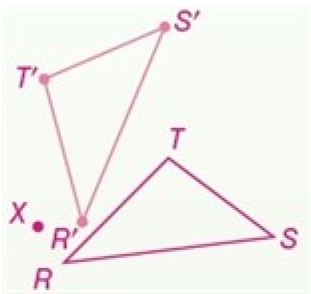
Step 3: Use a ruler to draw *T*' such that $XT = XT'$.



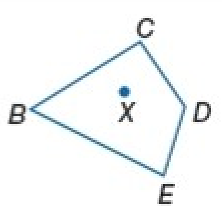
Step 4: Repeat Steps 1-3 for vertices *R* and *S* to complete the triangle.



ANSWER:

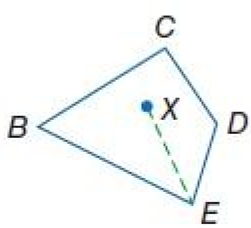


50. 120°

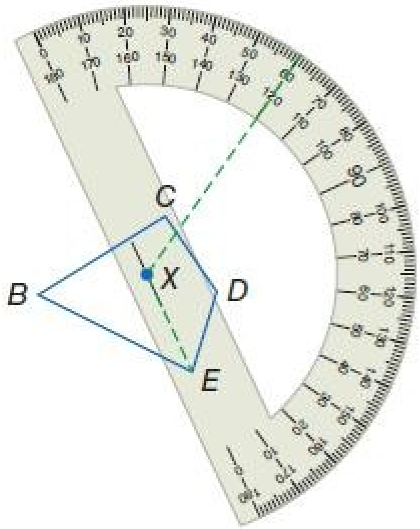


SOLUTION:

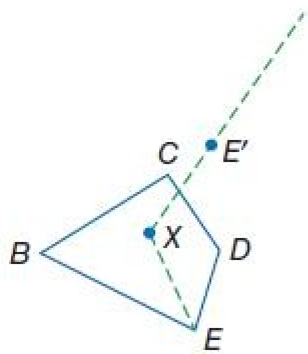
Step 1: Connect *X* to *E*.



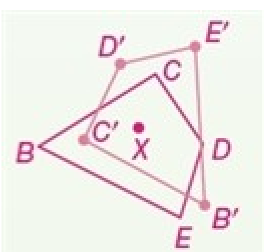
Step 2: Draw a 120° angle with XE.



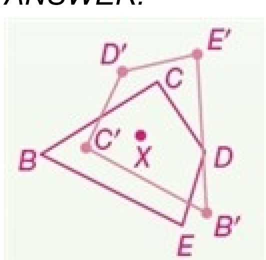
Step 3: Use a ruler to find E' such that $XE = XE'$.



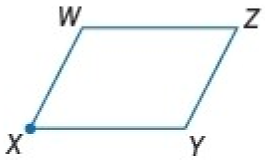
Step 4: Repeat steps 1-3 for vertices B, C, and D to complete the quadrilateral.



ANSWER:

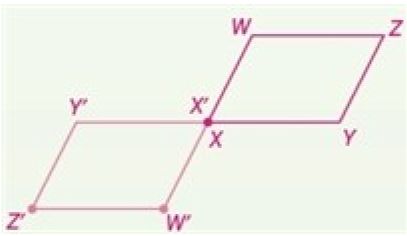


51. 180°

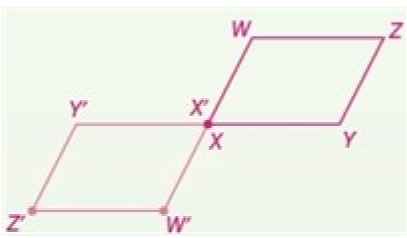


SOLUTION:

Make a 180° angle with XW. W' is on the same line as XW and $XW' = XW$. Repeat this process for the other three vertices.



ANSWER:

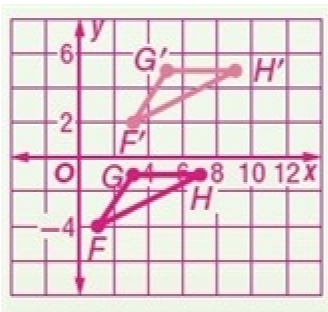


Graph each figure and its image along the given vector.

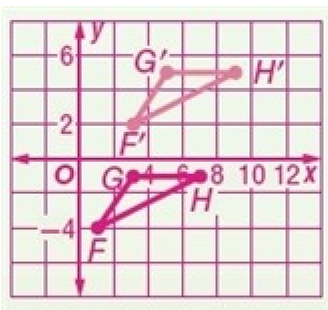
52. ΔFGH with vertices $F(1, –4)$, $G(3, –1)$, and $H(7, –1)$; $\langle 2, 6 \rangle$

SOLUTION:

The coordinates of the vertices of the translated triangle are $F'(1 + 2, –4 + 6)$, $G'(3 + 2, –1 + 6)$, and $H'(7 + 2, –1 + 6)$.



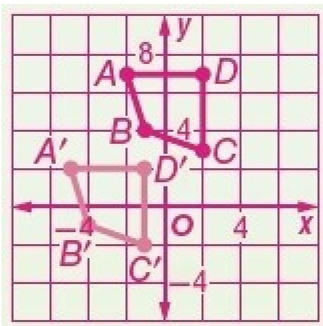
ANSWER:



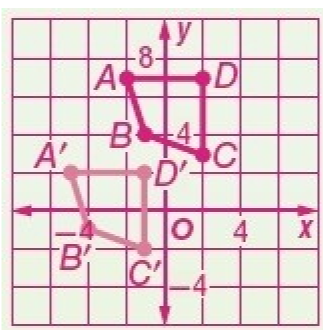
53. quadrilateral *ABCD* with vertices *A*(–2, 7), *B*(–1, 4), *C*(2, 3), and *D*(2, 7) $\langle -3, -5 \rangle$

SOLUTION:

The coordinates of the vertices of the translated quadrilateral are *A′*(–2 – 3, 7 – 5), *B′*(–1 – 3, 4 – 5), *C′*(2 – 3, 3 – 5), and *D′*(2 – 3, 7 – 5).



ANSWER:

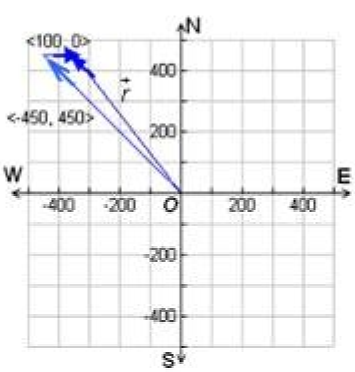


54. **AVIATION** A jet is flying northwest, and its velocity is represented by $\langle -450, 450 \rangle$ miles per hour. The wind is from the west, and its velocity is represented by $\langle 100, 0 \rangle$ miles per hour.

a. Find the resultant vector for the jet in component form.
b. Find the magnitude of the resultant.
c. Find the direction of the resultant.

SOLUTION:

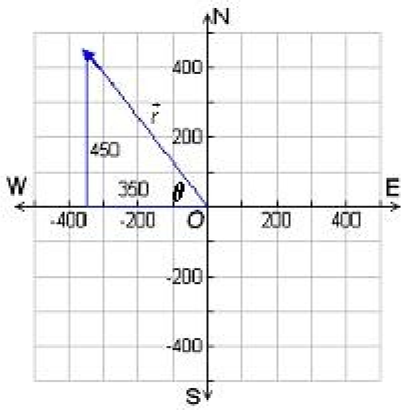
a. Draw a diagram. Let $\vec{r}$ represent the resultant vector.



Find the component form of the resultant vector by finding the sum of the vectors representing the plane and the wind.

$$\begin{aligned}\vec{r} &= \langle -450, 450\rangle + \langle 100, 0\rangle \\ &= \langle -450 + 100, 450 + 0\rangle \\ &= \langle -350, 450\rangle \text{ mph}\end{aligned}$$

b. Use the right triangle formed by the resultant vector and the negative *x*-axis to find the magnitude.



$$\begin{aligned}\text{magnitude} &= \sqrt{(-350)^2 + (450)^2} \\ &= \sqrt{122500 + 202500} \\ &= \sqrt{325000} \\ &\approx 570.1\end{aligned}$$

So, the magnitude of the resultant is about 570 mph.

c. Use the tangent function to find the measure of the angle θ.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{450}{350}$$

$$\theta = \tan^{-1}\left(\frac{9}{7}\right)$$

$$\theta \approx 52.1°$$

So, the resultant vector is at a direction of 52.1° above the negative *x*-axis or 52.1° north of west.

ANSWER:

a. $\langle -350, 450\rangle$ mph

b. 570.1

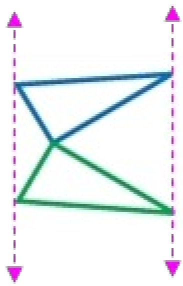
c. 52.1° north of west

Each figure shows a preimage and its reflected image in some line. Copy each figure and draw the line of reflection.

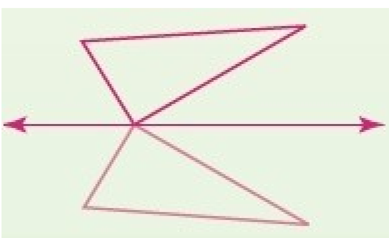
55. 

SOLUTION:

Draw a line through the corresponding points.

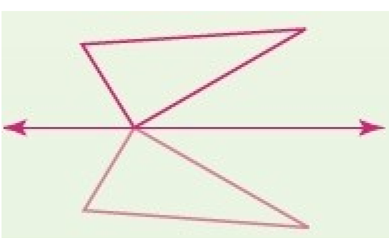


Draw a line perpendicular to the parallel lines, passing through the common point of both triangles.



This line is the line of reflection.

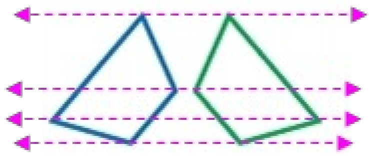
ANSWER:



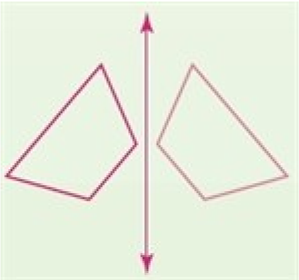
56. 

SOLUTION:

Draw a line through corresponding vertices.

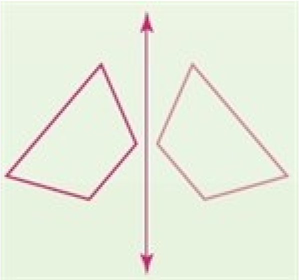


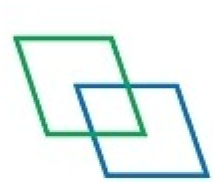
Then draw a line perpendicular to these lines that passes between the two trapezoids.



This line is the line of reflection.

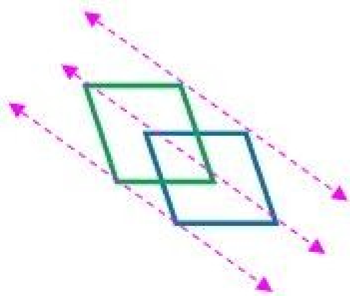
ANSWER:



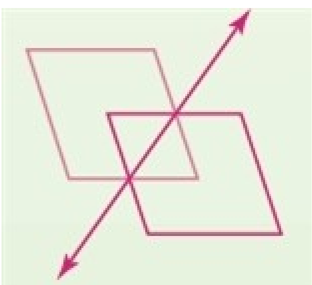
57. 

SOLUTION:

Draw a line through each pair of corresponding vertices.



Draw a line perpendicular to the lines that passes through the middle overlapping rhombus.



This line is the line of reflection.

ANSWER: