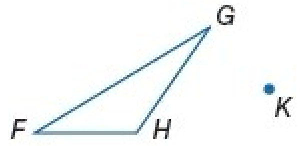
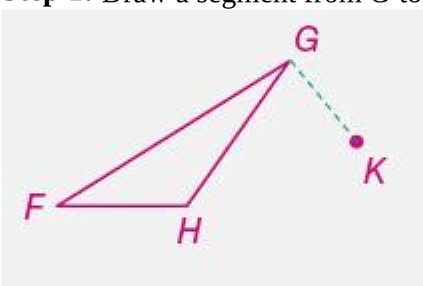
Copy each polygon and point *K*. Then use a protractor and ruler to draw the specified rotation of each figure about point *K*.

1. 45°

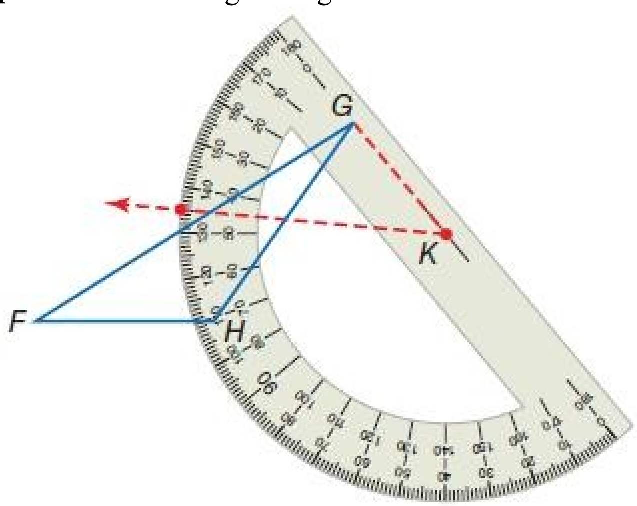


SOLUTION:

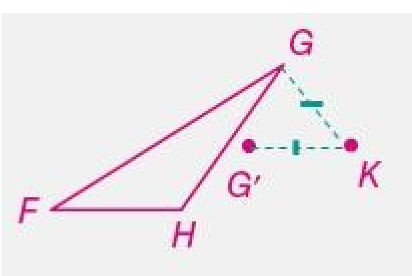
Step 1: Draw a segment from *G* to *K*.



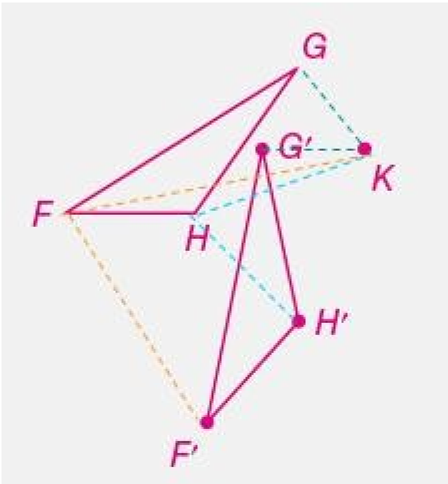
Step 2: Draw a 45° angle using $\overline{GK}$.

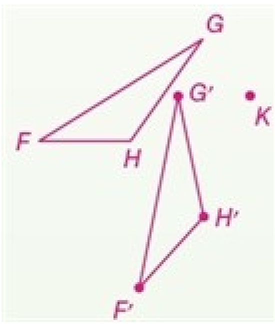


Step 3: Use a ruler to draw *G*' such that *G'K* = *GK*.

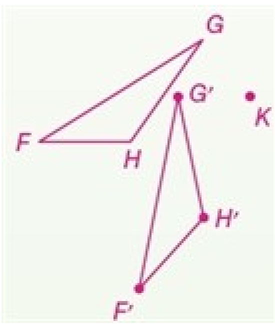


Step 4: Repeat Steps 1-3 for the vertices *F* and *H* and draw Δ*F'G'H'*.

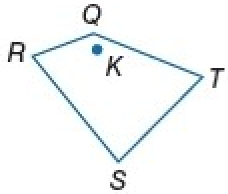




ANSWER:

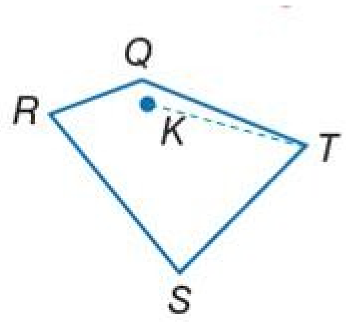


2. 120°

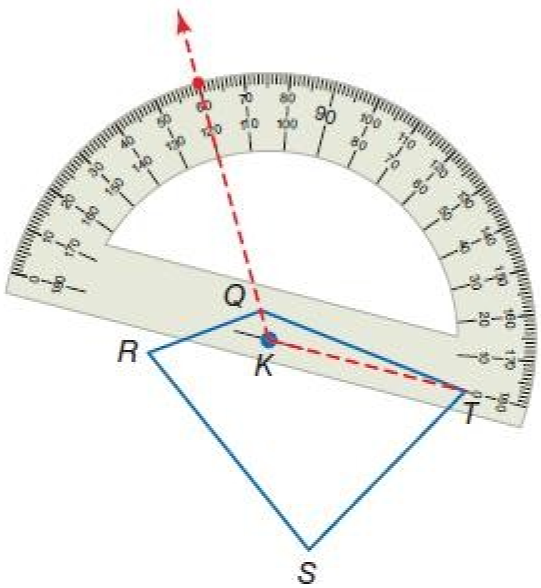


SOLUTION:

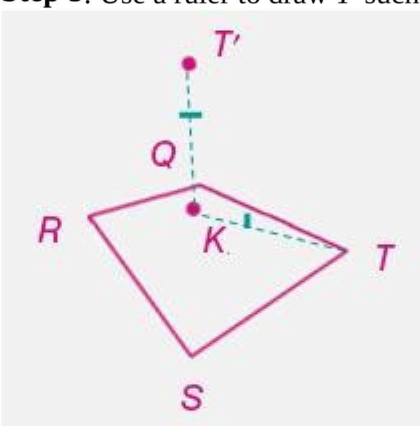
Step 1: Draw a segment from *T* to *K*.



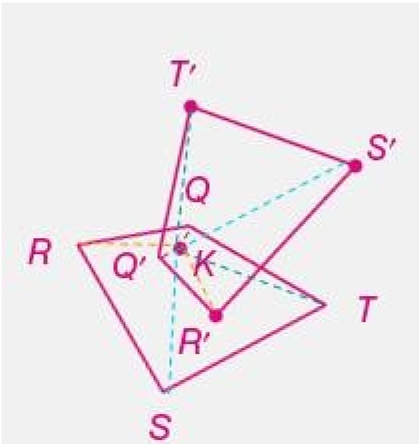
Step 2: Draw a 120°angle using $\overline{TK}$.



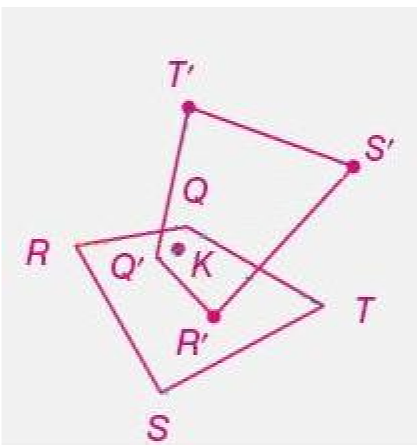
Step 3: Use a ruler to draw *T'* such that *T'K* = *TK*.



Step 4: Repeat Steps 1-3 for the vertices *Q*, *S*, and *R* and draw *Q'R'S'T'*.



ANSWER:



3. Triangle *DFG* has vertices *D*(–2, 6), *F*(2, 8), and *G*(2, 3). Graph ΔDFG and its image after a rotation 180° about the origin.

SOLUTION:

To rotate a point 180° counterclockwise about the origin, multiply the *x*- and *y*-coordinate of each vertex by –1.

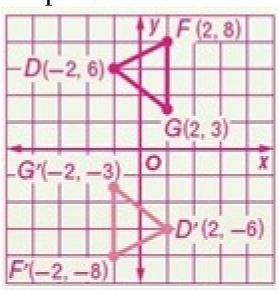
$(x, y) \rightarrow (-x, -y)$

$(-2, 6) \rightarrow (2, -6)$

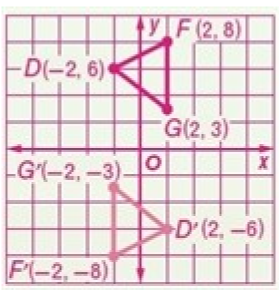
$(2, 8) \rightarrow (-2, -8)$

$(2, 3) \rightarrow (-2, -3)$

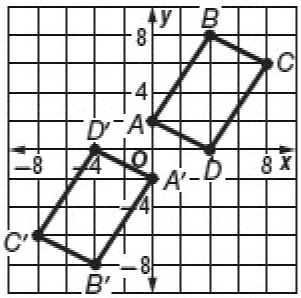
Graph ΔDFG and its image.



ANSWER:



4. **MULTIPLE CHOICE** For the transformation shown, what is the measure of the angle of rotation of *ABCD* about the origin?



A 90°
B 180°
C 270°
D 360°

SOLUTION:

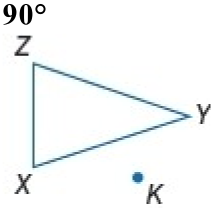
The coordinates of *A*, *B*, *C*, and *D* are (0, 2), (4, 8), (8, 6), and (0, 4) respectively. The coordinates of *A*', *B*', *C*', and *D*' are (0, –2), (–4, –8), (–8, –6), and (0, –4) respectively. The *x* and *y* coordinates of the rotated image are multiplied by –1. So, the image has been rotated 180° about the origin.
The correct choice is B.

ANSWER:

B

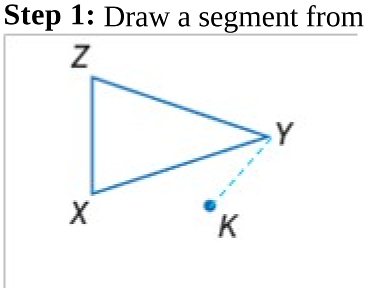
CCSS TOOLS Copy each polygon and point *K*. Then use a protractor and ruler to draw the specified rotation of each figure about point *K*.

5. **90°**

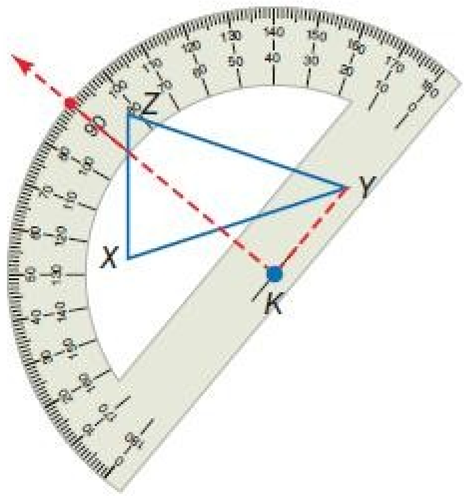


SOLUTION:

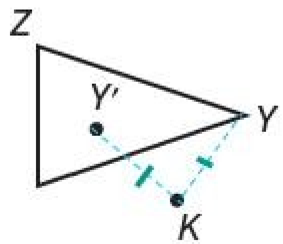
Step 1: Draw a segment from *Y* to *K*.



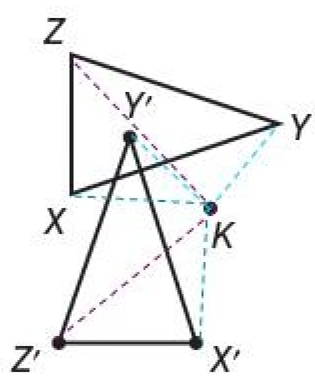
Step 2: Draw a 90° angle using $\overline{YK}$.



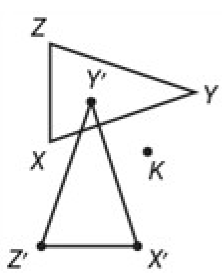
Step 3: Use a ruler to draw *Y*' such that *Y'K* = *YK*.



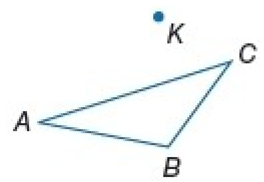
Step 4: Repeat Steps 1-3 for the vertices *X* and *Z* and draw Δ*X'Y'Z'*.



ANSWER:

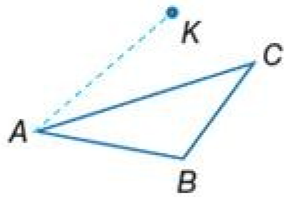


6. **15°**

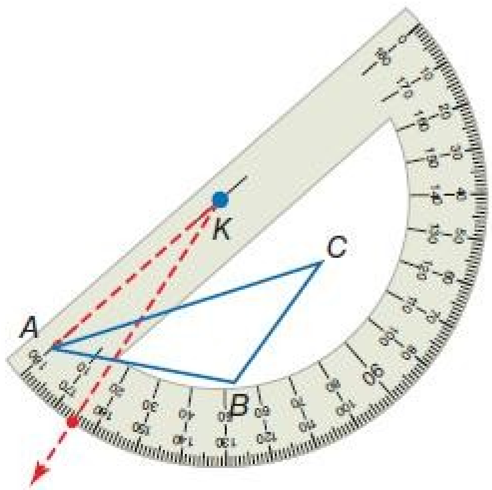


SOLUTION:

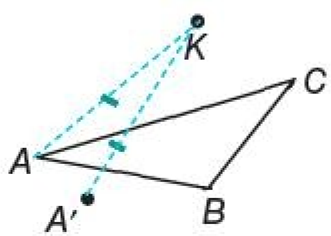
Step 1: Draw a segment from A to K.



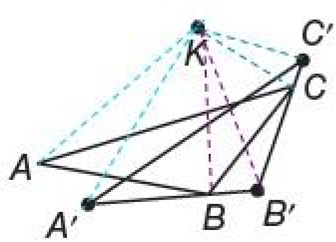
Step 2: Draw a 15° angle using $\overline{AK}$.



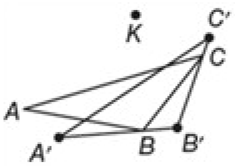
Step 3: Use a ruler to draw A' such that $A'K = AK$.



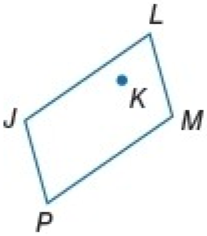
Step 4: Repeat Steps 1-3 for the vertices B and C and draw $\Delta A'B'C'$.



ANSWER:

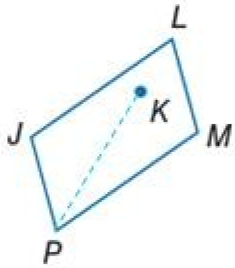


7. **145°**

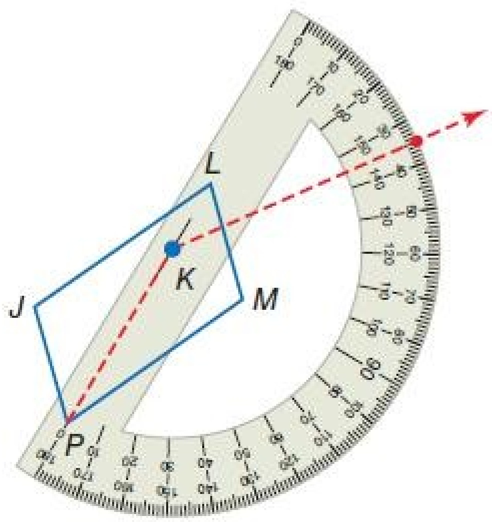


SOLUTION:

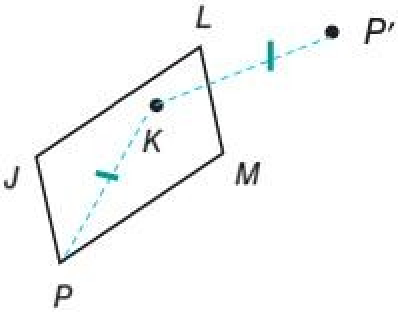
Step 1: Draw a segment from *P* to *K*.



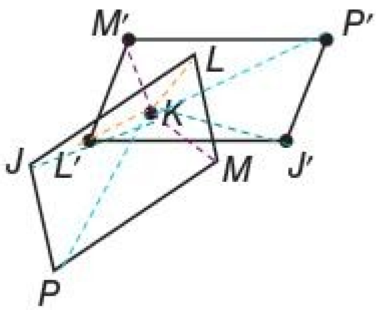
Step 2: Draw a 145° angle using $\overline{PK}$.



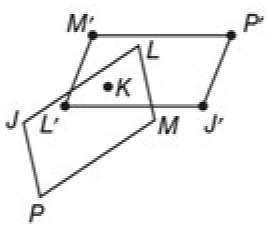
Step 3: Use a ruler to draw *P*' such that *P'K* = *PK*.



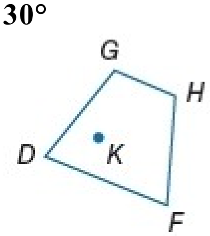
Step 4: Repeat Steps 1-3 for the vertices *K*, *L*, and M and draw *K'L'M'P'*.



ANSWER:

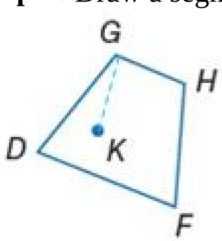


8. **30°**

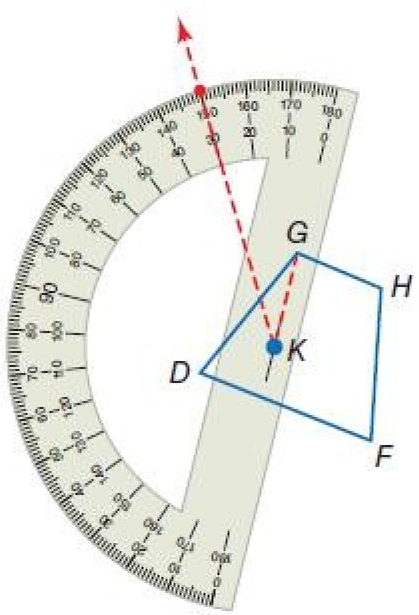


SOLUTION:

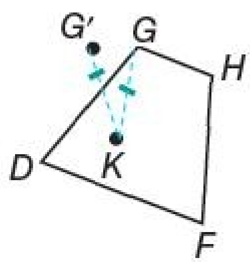
Step 1: Draw a segment from *G* to *K*.



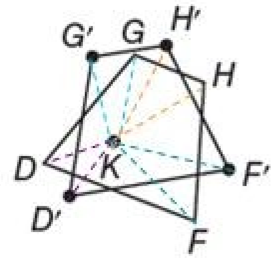
Step 2: Draw a 30° angle using $\overline{GK}$.



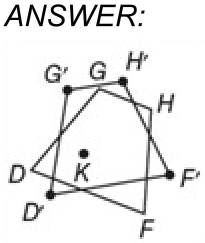
Step 3: Use a ruler to draw *G*' such that *G'K* = *GK*.



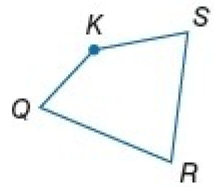
Step 4: Repeat Steps 1-3 for the vertices *D*, *F*, and *H* and draw figure *D'F'G'H'*.



ANSWER:

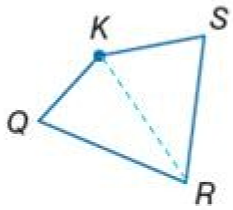


9. **260°**

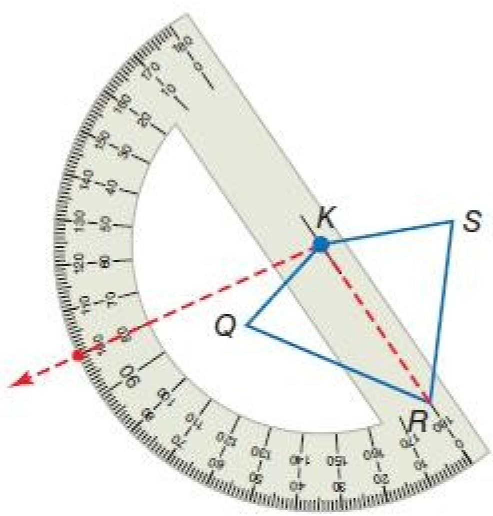


SOLUTION:

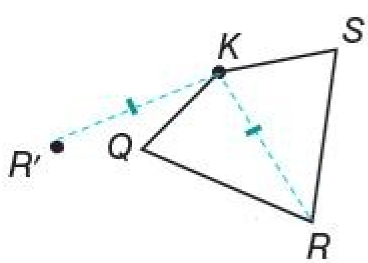
Step 1: Draw a segment from *R* to *K*.



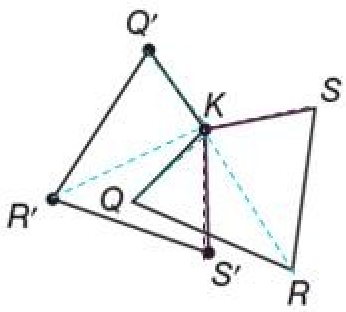
Step 2: Draw a 260° angle using $\overline{RK}$.



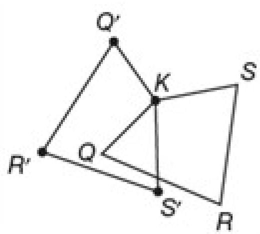
Step 3: Use a ruler to draw *R*' such that *R'K* = *RK*.



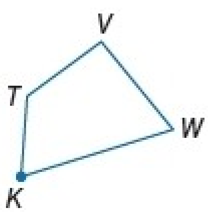
Step 4: Repeat Steps 1-3 for the vertices *Q*, *R*. and *S* and draw figure *K'Q'R'S'*.



ANSWER:

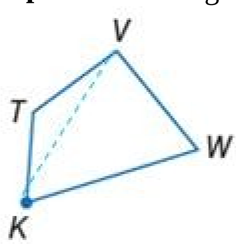


10. **50°**

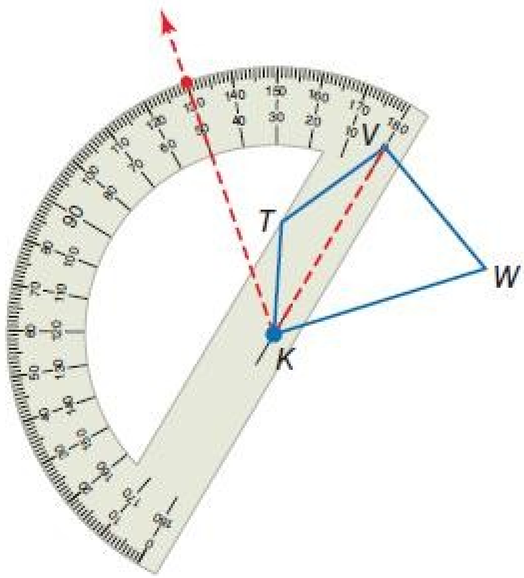


SOLUTION:

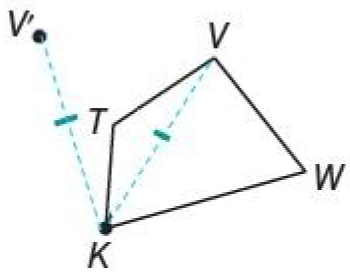
Step 1: Draw a segment from *V* to *K*.



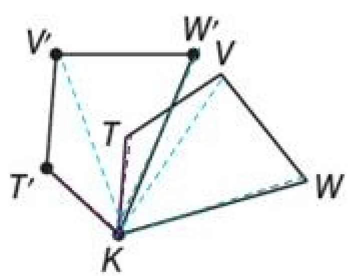
Step 2: Draw a 50° angle using $\overline{VK}$.



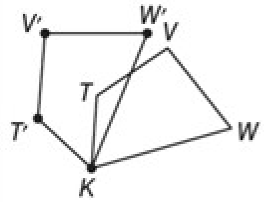
Step 3: Use a ruler to draw *V*' such that *V'K* = *VK*.



Step 4: Repeat Steps 1-3 for the vertices *T* and *W* and draw figure *K'T'V'W'*.



ANSWER:



PINWHEELS Find the angle of rotation to the nearest tenth of a degree that maps *P* onto *P'*. Explain your reasoning.

11. Refer to page 643.

SOLUTION:

The petals are equidistant from each other, so divide 360 degrees by the number of petals around the object to determine how many degrees there are between two petals.

120°; 360° ÷ 6 petals = 60° per petal. Two petal turns is 2 · 60° or 120°.

ANSWER:

120°; 360° ÷ 6 petals = 60° per petal. Two petal turns is 2 · 60° or 120°.

12. Refer to page 643.

SOLUTION:

The petals are equidistant from each other, so divide 360 degrees by the number of petals around the object to determine how many degrees there are between two petals.

90°; 360° ÷ 8 petals = 45° per petal. Two petal turns is 2 · 45° or 90°.

ANSWER:

90°; 360° ÷ 8 petals = 45° per petal. Two petal turns is 2 · 45° or 90°.

13. Refer to page 643.

SOLUTION:

The petals are equidistant from each other, so divide 360 degrees by the number of petals around the object to determine how many degrees there are between two petals.

154.2°; 360° ÷ 7 petals = 51.4° per petal. Three petal turns is 3 · 51.4° or 154.2°.

ANSWER:

154.2°; 360° ÷ 7 petals = 51.4° per petal. Three petal turns is 3 · 51.4° or 154.2°.

Graph each figure and its image after the specified rotation about the origin.

14. ΔJKL has vertices $J(2, 6)$, $K(5, 2)$, and $L(7, 5)$; 90°

SOLUTION:

To rotate a point 90° counterclockwise about the origin, multiply the y-coordinate of each vertex by –1 and interchange.

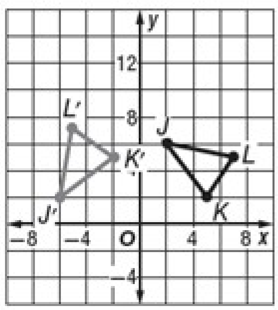
$(x, y) \rightarrow (-y, x)$

$(2,6) \rightarrow (-6,2)$

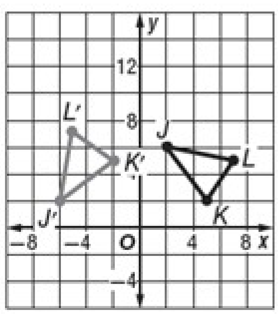
$(5,2) \rightarrow (-2,5)$

$(7,5) \rightarrow (-5,7)$

Graph ΔJKL and its image.



ANSWER:



15. rhombus *WXYZ* has vertices *W*(–3, 4), *X*(0, 7), *Y*(3, 4), and *Z*(0, 1); 90°

SOLUTION:

To rotate a point 90° counterclockwise about the origin, multiply the *y*-coordinate of each vertex by –1 and interchange.

$(x, y) \to (-y, x)$

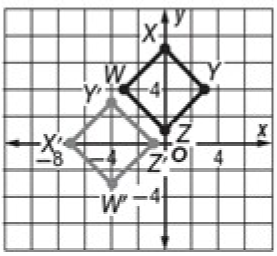
$(-3, 4) \to (-4, -3)$

$(0, 7) \to (-7, 0)$

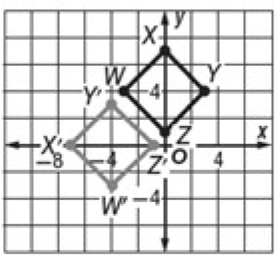
$(3, 4) \to (-4, 3)$

$(0, 1) \to (-1, 0)$

Graph rhombus *WXYZ* and its image.



ANSWER:



16. ΔFGH has vertices *F*(2, 4), *G*(5, 6), and *H*(7, 2); 180°

SOLUTION:

To rotate a point 180° counterclockwise about the origin, multiply the *x*- and *y*-coordinate of each vertex by –1.

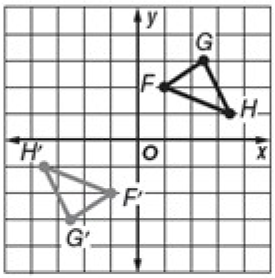
$(x, y) \rightarrow (-x, -y)$

$(2,4) \rightarrow (-2,-4)$

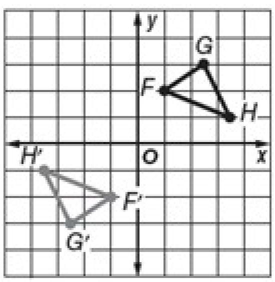
$(5,6) \rightarrow (-5,-6)$

$(7,2) \rightarrow (-7,-2)$

Graph ΔFGH and its image.



ANSWER:



17. trapezoid *ABCD* has vertices *A*(–7, –2), *B*(–6, –6), *C*(–1, –1), and *D*(–5, 0); 180°

SOLUTION:

To rotate a point 180° counterclockwise about the origin, multiply the *x*- and *y*-coordinate of each vertex by –1.

$(x, y) \rightarrow (-x, -y)$

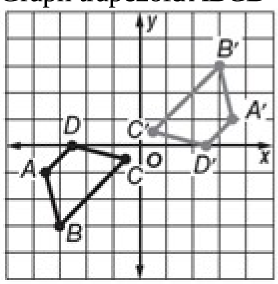
$(-7, -2) \rightarrow (7, 2)$

$(-6, -6) \rightarrow (6, 6)$

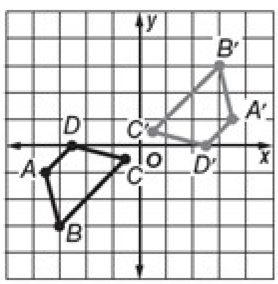
$(-1, -1) \rightarrow (1, 1)$

$(-5, 0) \rightarrow (5, 0)$

Graph trapezoid *ABCD* and its image.



ANSWER:



18. ΔRST has vertices $R(-6, -1)$, $S(-4, -5)$, and $T(-2, -1)$; 270°

SOLUTION:

To rotate a point 270° counterclockwise about the origin, multiply the x-coordinate of each vertex by –1 and interchange.

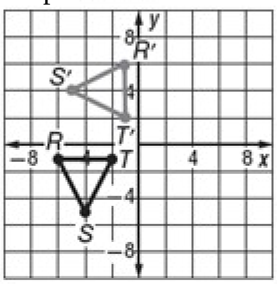
$(x, y) \rightarrow (y, -x)$

$(-6, -1) \rightarrow (-1, 6)$

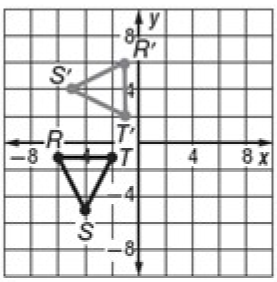
$(-4, -5) \rightarrow (-5, 4)$

$(-2, -1) \rightarrow (-1, 2)$

Graph ΔRST and its image.



ANSWER:



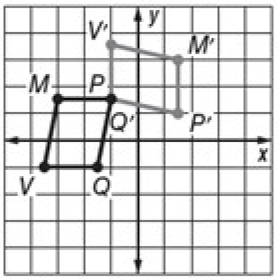
19. parallelogram $MPQV$ has vertices $M(-6, 3)$, $P(-2, 3)$, $Q(-3, -2)$, and $V(-7, -2)$; 270°

SOLUTION:

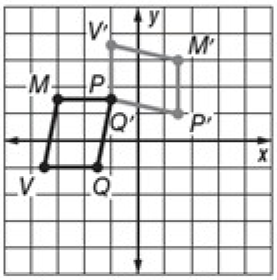
To rotate a point 270° counterclockwise about the origin, multiply the x-coordinate of each vertex by –1 and interchange.

$(x, y) \rightarrow (y, -x)$
$(-6, 3) \rightarrow (3, 6)$
$(-2, 3) \rightarrow (3, 2)$
$(-3, -2) \rightarrow (-2, 3)$
$(-7, -2) \rightarrow (-2, 7)$

Graph $MPQV$ and its image.

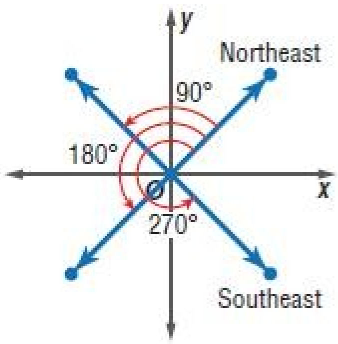


ANSWER:



20. **WEATHER** A weathervane is used to indicate the direction of the wind. If the vane is pointing northeast and rotates 270°, what is the new wind direction?

SOLUTION:



The weathervane would point southeast after a rotation of 270º.

ANSWER:

southeast

21. **CCSS MODELING** The photograph of the Grande Roue, or Big Wheel, appears blurred because of the camera's shutter speed—the length of time the camera's shutter was open.
Refer to the photo on page 644.
a. Estimate the angle of rotation in the photo. (Hint: Use points A and A'.)
b. If the Ferris wheel makes one revolution per minute, use your estimate from part **a** to estimate the camera's shutter speed.

SOLUTION:

a. Consider the light color rectangle between A and A'. The rectangle repeats at regular intervals around the wheel. You can use the number of rectangles to determine the angle of rotation. There are a total of 18 rectangles in the given big wheel.

$$\text{angle of rotation} = \frac{\text{circumference of the circular wheel}}{\text{number of rectangles}}$$

$$\text{angle of rotation} = \frac{2\pi\left(\frac{60}{2}\right)}{18}$$

$$\text{angle of rotation} \approx 10°$$

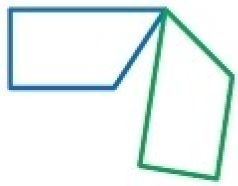
b. Since the wheel makes one revolution per minute, the wheel revolves 360 degrees in 60 seconds. Let x be the camera's shutter speed.

$$\frac{360}{60} = \frac{10}{x}$$
$$360x = 600$$
$$x \approx 1.7$$

ANSWER:

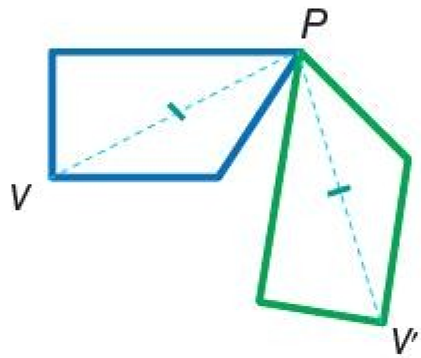
a. 10°
b. about 1.7 seconds

Each figure shows a preimage and its image after a rotation about point *P*. Copy each figure, locate point *P*, and find the angle of rotation.

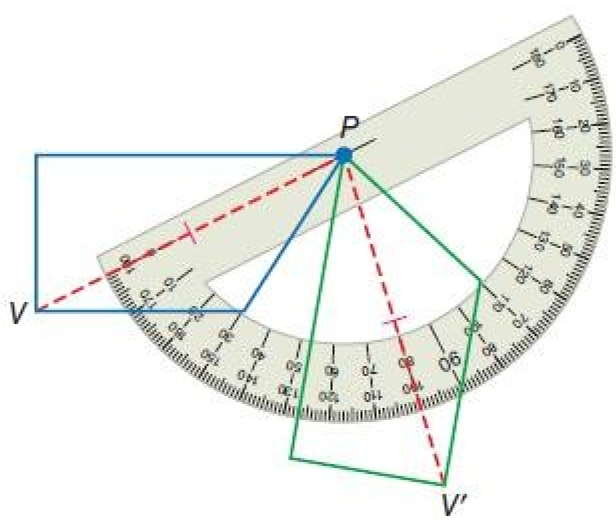
22. 

SOLUTION:

Use a protractor and ruler. Notice that $VP = V'P$. Then P is the center of rotation.

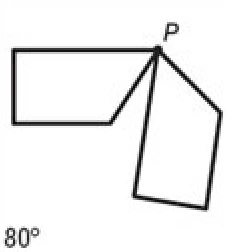


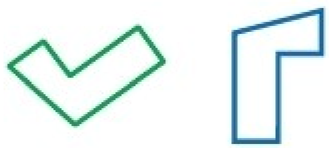
Use a protractor to determine the angle of rotation. Place the protractor on VP with the center point on P. Then determine the angle.



The angle of rotation is 80°.

ANSWER:

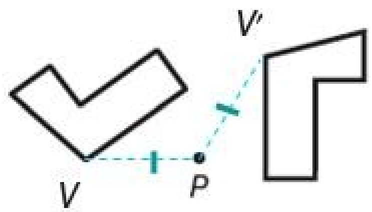




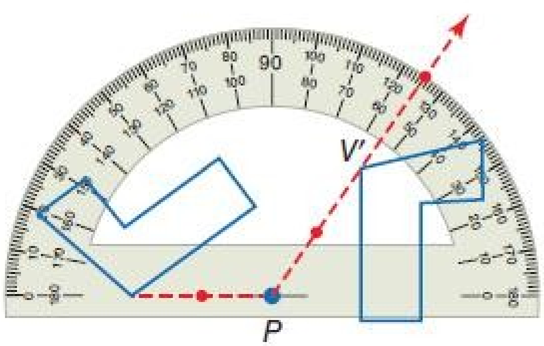
23.

SOLUTION:

Use a protractor and ruler. Identify and label a corresponding vertex. Label a point *P* between *V* and *V'*. Use the ruler to measure the distance. Move point *P* until *VP* and *V'P* are equal. *P* is the center of rotation.

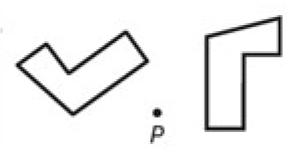


Use a protractor to determine the angle of rotation. Place the protractor on line *VP* with the center on *P*. Then determine the angle through point *V'*.



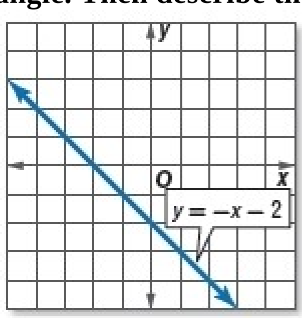
The angle of rotation is 125°.

ANSWER:



125°

ALGEBRA Give the equation of the line $y = -x - 2$ after a rotation about the origin through the given angle. Then describe the relationship between the equations of the image and preimage.



9-3 Rotations

24. 90°

SOLUTION:

To rotate a point 90° counterclockwise about the origin, multiply the y-coordinate by –1 and then interchange the x- and y-coordinates.

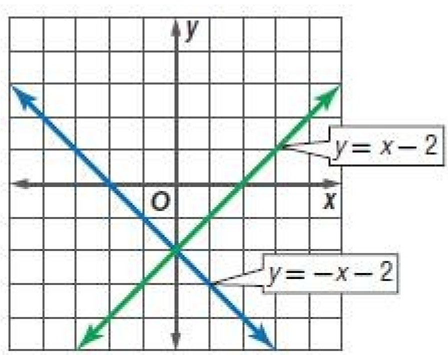
Choose two points on the line $y = -x - 2$ and find the coordinates after each is rotated 90°.

$(x, y) \rightarrow (-y, x)$
$(0, -2) \rightarrow (2, 0)$
$(-2, 0) \rightarrow (0, -2)$

Use the points to find the slope and equation of the line containing the rotated points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{0 - (-2)}{2 - 0}$$
$$m = \frac{2}{2}$$
$$m = 1$$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = (1)(x - 0)$$
$$y + 2 = x$$
$$y = x - 2$$



So, the equation of the line with equation $y = -x - 2$ after it is rotated 90° is $y = x - 2$. Since their slopes are negative reciprocals, the lines are perpendicular.

ANSWER:

$y = x - 2$; perpendicular

25. 180°

SOLUTION:

To rotate a point 180° counterclockwise about the origin, multiply the x- and y-coordinate by –1.

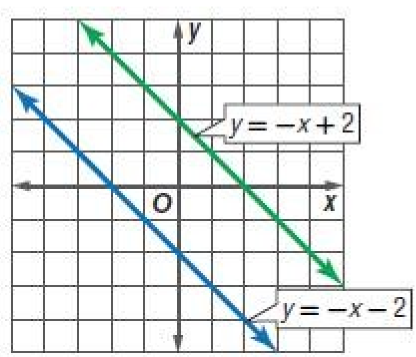
Choose two points on the line $y = -x - 2$ and find the coordinates after each is rotated 180°.

$(x, y) \rightarrow (-x, -y)$
$(0, -2) \rightarrow (0, 2)$
$(-2, 0) \rightarrow (2, 0)$

Use the points to find the slope and equation of the line containing the rotated points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{0 - (2)}{2 - 0}$$
$$m = -\frac{2}{2}$$
$$m = -1$$

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -1(x - 0)$$
$$y - 2 = -x$$
$$y = -x + 2$$



So, the equation of the line with equation $y = -x - 2$ after it is rotated 180° is $y = -x + 2$. Since their slopes are equal, the lines are parallel.

ANSWER:

$y = -x + 2$; parallel

26. 270°

SOLUTION:

To rotate a point 270° counterclockwise about the origin, multiply the *x*-coordinate by –1 and then interchange the *x*- and *y*-coordinates.

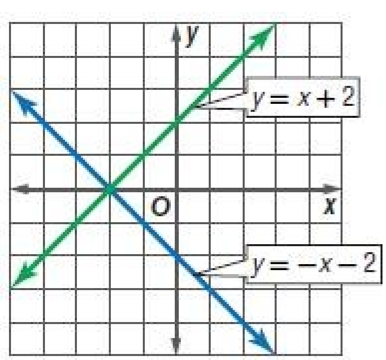
Choose two points on the line $y = -x - 2$ and find the coordinates after each is rotated 270°.

$(x, y) \rightarrow (y, -x)$
$(0, -2) \rightarrow (-2, 0)$
$(-2, 0) \rightarrow (0, 2)$

Use the points to find the slope and equation of the line containing the rotated points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{0 - 2}{-2 - 0}$$
$$m = \frac{-2}{-2}$$
$$m = 1$$

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 1(x - 0)$$
$$y - 2 = x$$
$$y = x + 2$$



So, the equation of the line with equation $y = -x - 2$ after it is rotated 270° is $y = x + 2$. Since their slopes are negative reciprocals, the lines are perpendicular.

ANSWER:

$y = x + 2$; perpendicular

27. 360°

SOLUTION:

The image under a 360° rotation is equal to the preimage. Thus, when rotating a point 360° counterclockwise about the origin, the x- and y-coordinates remain the same. Therefore, the equation of the line after a 360° rotation would still be $y = -x - 2$. Since they have the same equation, the lines are collinear.

ANSWER:

$y = -x - 2$; collinear

ALGEBRA Rotate the line the specified number of degrees about the *x*- and *y*-intercepts and find the equation of the resulting image.

28. $y = x - 5$; 90°

SOLUTION:

The slope of line $y = x - 5$ is $m = 1$ and the x-intercept is 5. The image of a line rotated 90° about its x-intercept would be a perpendicular line containing the same x-intercept. The slope of the rotated line is $m = -1$.

Use the slope and x-intercept point of (5, 0) to write an equation.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = (-1)(x - 5)$$
$$y = -x + 5$$

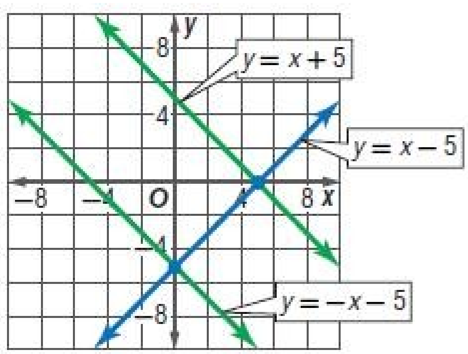
So, the equation of the image of the line $y = x - 5$ rotated 90° about the x-intercept is $y = -x + 5$.

The y-intercept of $y = x - 5$ is –5. The image of a line rotated 90° about its y-intercept would be a perpendicular line containing the same y-intercept. The slope of the rotated line is $m = -1$.

Use the slope and y-intercept point of (0, –5) to write an equation.

$$y - y_1 = m(x - x_1)$$
$$y - (-5) = (-1)(x - 0)$$
$$y + 5 = -x$$
$$y = -x - 5$$

So, the equation of the image of the line $y = x - 5$ rotated 90° about the y-intercept is $y = -x - 5$.



ANSWER:

x-intercept: $y = -x + 5$;
y-intercept: $y = -x - 5$

29. $y = 2x + 4$; 180°

SOLUTION:

The slope of line $y = 2x + 4$ is $m = 2$ and the x-intercept is –2. The image of a line rotated 180° about its x-intercept would be a line with the same slope and containing the same x-intercept. The slope of the rotated line is $m = 2$.

Use the slope and x-intercept point of (–2, 0) to write an equation.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x - (-2))$$
$$y = 2x + 4$$

So, the equation of the image of the line $y = 2x + 4$ rotated 180° about the x-intercept is $y = 2x + 4$.

The y-intercept of $y = 2x + 4$ is 4. The image of a line rotated 180° about its y-intercept would be a line with the same slope and containing the same y-intercept. The slope of the rotated line is $m = 2$.

Use the slope and y-intercept point of (0, 4) to write an equation.

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 2(x - 0)$$
$$y - 4 = 2x$$
$$y = 2x + 4$$

So, the equation of the image of the line $y = 2x + 4$ rotated 180° about the y-intercept is $y = 2x + 4$.

ANSWER:

x-intercept: $y = 2x + 4$;
y-intercept: $y = 2x + 4$

30. $y = 3x - 2$; 270°

SOLUTION:

The slope of line $y = 3x - 2$ is $m = 3$ and the x-intercept is $\frac{2}{3}$. The image of a line rotated 270° about its x-intercept would be a perpendicular line containing the same x-intercept. The slope of the rotated line is $m = -\frac{1}{3}$.

Use the slope and x-intercept point of $\left(\frac{2}{3}, 0\right)$ to write an equation.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \left(-\frac{1}{3}\right)\left(x - \frac{2}{3}\right)$$
$$y = -\frac{1}{3}x + \frac{2}{9}$$

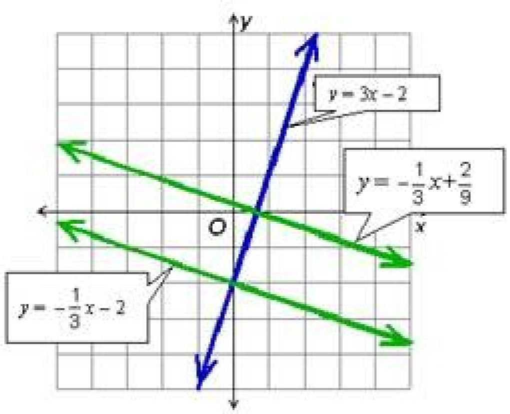
So, the equation of the image of the line $y = 3x - 2$ rotated 270° about the x-intercept is $y = -\frac{1}{3}x + \frac{2}{9}$.

The y-intercept of $y = 3x - 2$ is –2. The image of a line rotated 270° about its y-intercept would be a perpendicular line containing the same y-intercept. The slope of the rotated line is $m = -\frac{1}{3}$.

Use the slope and y-intercept point of (0, –2) to write an equation.

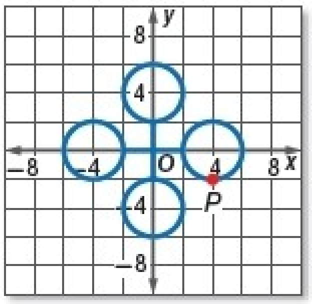
$$y - y_1 = m(x - x_1)$$
$$y - (-2) = \left(-\frac{1}{3}\right)(x - 0)$$
$$y + 2 = -\frac{1}{3}x$$
$$y = -\frac{1}{3}x - 2$$

So, the equation of the image of the line $y = 3x - 2$ rotated 270° about the y-intercept is $y = -\frac{1}{3}x - 2$.



ANSWER:
x-intercept: $y = -\frac{1}{3}x + \frac{2}{9}$;
y-intercept: $y = -\frac{1}{3}x - 2$

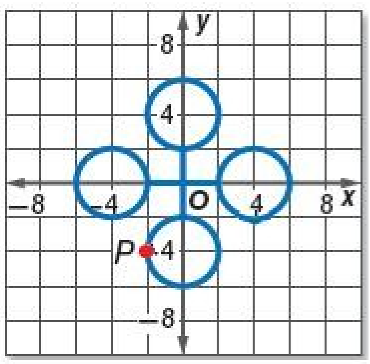
31. **RIDES** An amusement park ride consists of four circular cars. The ride rotates at a rate of 0.25 revolution per second. In addition, each car rotates 0.5 revolution per second. If Jane is positioned at point P when the ride begins, what coordinates describe her position after 31 seconds?



SOLUTION:

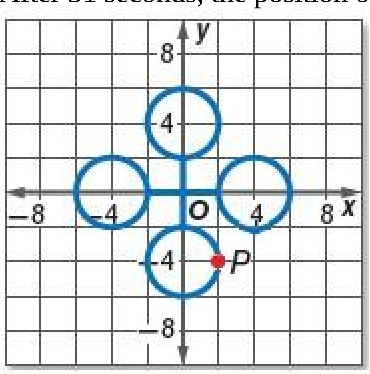
The entire ride completes 0.25 revolution every second, so the cars will be at their original locations every 4 seconds (after one full revolution). Therefore, after 4, 8, 12, 16, 20, 24, and 28 seconds, Jane's car will be at its original position. We want the position of the car after 31 seconds, which is $\frac{3}{4}$ of the way between 28 and 32 seconds, so the car will have made a 270° rotation at that time. To rotate a point 270° from its original location, multiply the *x*-coordinate by –1, then interchange the coordinates.

The original position *P* is at (4, –2), so the new one is at (–2, –4). This change reflects the rotation of the entire ride. We must also consider the rotation of the cars.



The cars complete 0.5 revolution every second, so they will be at their original position every 2 seconds. We want Jane's position at 31 seconds, which is halfway between 30 and 32 seconds, so her car will be halfway through one revolution. Therefore, the position *P* will be at the opposite end of the circle at 31 seconds. For Jane's car, the opposite end of (–2, –4) is (2, –4).

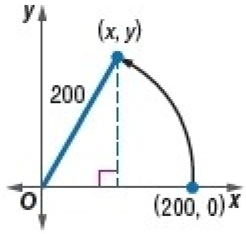
After 31 seconds, the position of *P* will be (2, –4).



ANSWER:

(2, –4)

32. **BICYCLE RACING** Brandon and Nestor are participating in a bicycle race on a circular track with a radius of 200 feet.



a. If the race starts at (200, 0) and both complete one rotation in 30 seconds, what are their coordinates after 5 seconds?
b. Suppose the length of race is 50 laps and Brandon continues the race at the same rate. If Nestor finishes in 26.2 minutes, who is the winner?

SOLUTION:

a. Let *x* represent the angle of rotation in 5 seconds.

$$\frac{1\text{rotation}}{\text{time}} = \frac{\text{current position}}{\text{time}}$$
$$\frac{360^\circ}{30} = \frac{x^\circ}{5}$$
$$30x = 1800$$
$$x = 60^\circ$$

To find the coordinates, use sine and cosine ratios.

$$\sin 60 = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 60 = \frac{y}{200}$$
$$y = 200\sin 60$$
$$y \approx 173.2$$

$$\cos 60 = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 60 = \frac{x}{200}$$
$$x = 200\cos 60$$
$$x = 100$$

The coordinates are (100, 173.2).

b. One lap means one revolution around the circle. So, the race consists of 50 revolutions. For one revolution, it takes 30 seconds. So for 50 laps, it takes 50(30) or 1500 seconds.

Convert the number of seconds into minutes.

$$\frac{1500}{60} = 25$$

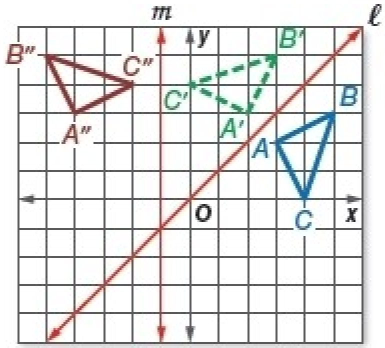
25 min. < 26.2 min., so Brandon is the winner.

ANSWER:

a. (100, 173.2)
b. Brandon; 25 min. < 26.2 min.

33. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate reflections over a pair of intersecting lines.

a. GEOMETRIC On a coordinate *y* plane, draw a triangle and a pair of intersecting lines. Label the triangle *ABC* and the lines *l* and *m*. Reflect ΔABC in the line *l*. Then reflect $\Delta A'B'C'$ in the line *m*. Label the final image $A''B''C''$.



b. GEOMETRIC Repeat the process in part a two more times in two different quadrants. Label the second triangle *DEF* and reflect it in intersecting lines n and p. Label the third triangle *MNP* and reflect it in intersecting lines *q* and *r*.

c. TABULAR Measure the angle of rotation of each triangle about the point of intersection of the two lines. Copy and complete the table below.

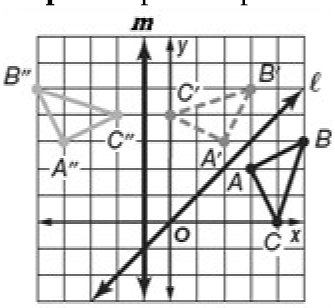
Angle of Rotation Between Figures		Angle Between Intersecting Lines	
△*ABC* and △*A″B″C″*		ℓ and *m*	
△*DEF* and △*D″E″F*		*n* and *p*	
△*MNP* and △*M″N″P*		*q* and *r*	

d. VERBAL Make a conjecture about the angle of rotation of a figure about the intersection of two lines after the figure is reflected in both lines.

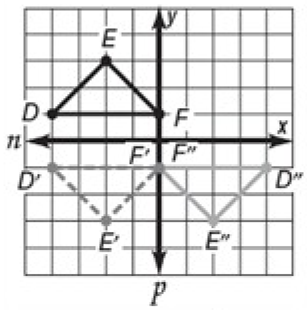
SOLUTION:

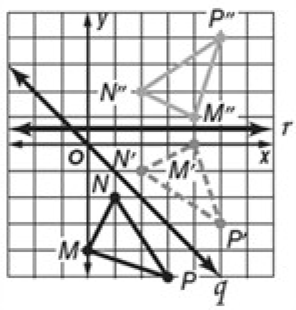
a.

Step 1: Draw a line through each vertex that is perpendicular to line *l*.
Step 2: Measure the distance from point *C* to the line *a*. Then locate *C*' the same distance from line *l* on the opposite side.
Step 3: Repeat Step 2 to locate points *A*' and *E*'. Then connect the vertices, *A*', *B*', and *C*' to form the reflected image.
Step 4: Repeat Step 1-3 to reflect *A'B'C'* over line *m*.



b. Use the steps from part a to reflect the figures.





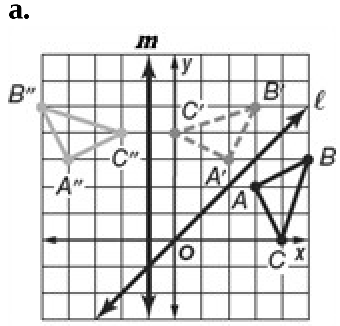
c.

Angle of Rotation Between Figures		Angle Between Intersecting Lines	
△*ABC* and △*A″B″C″*	90°	ℓ and *m*	45°
△*DEF* and △*D″E″F*	180°	*n* and *p*	90°
△*MNP* and △*M″N″P*	90°	*q* and *r*	45°

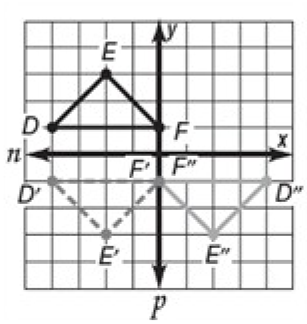
d. Sample answer: The measure of the angle of rotation about the point where the lines intersect is twice the measure of the angle between the two intersecting lines.

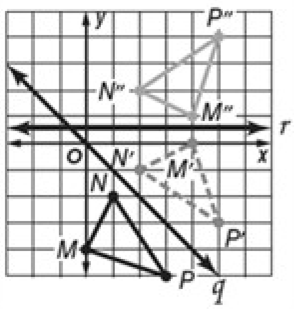
ANSWER:

a.



b.





c.

Angle of Rotation Between Figures		Angle Between Intersecting Lines	
△*ABC* and △*A''B''C''*	90°	ℓ and *m*	45°
△*DEF* and △*D''E''F*	180°	*n* and *p*	90°
△*MNP* and △*M''N''P*	90°	*q* and *r*	45°

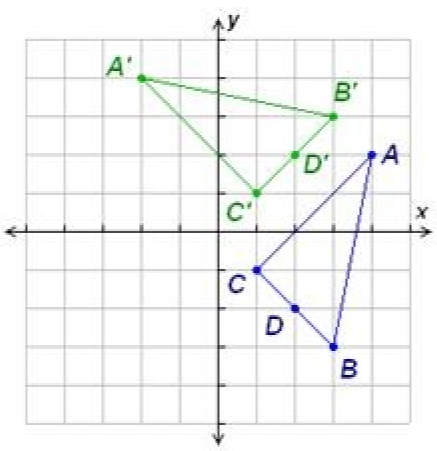
d. Sample answer: The measure of the angle of rotation about the point where the lines intersect is twice the measure of the angle between the two intersecting lines.

34. **WRITING IN MATH** Are collinearity and betweenness of points maintained under rotation? Explain.

SOLUTION:

Yes; sample answer: A rotation is a transformation which may change the orientation of the points but maintains congruence of the original figure and its image. So, the preimage can be mapped onto the image and corresponding segments will be congruent. Therefore, collinearity and betweenness of points are maintained in rotation.

In the diagram below, Δ*ABC* has vertices *A*(4, 2), *B*(3, –3), *C*(1, –1) and point *D*(2, –2) on $\overline{BC}$. After a 90° rotation, Δ*A'B'C'* has vertices *A*'(–2, 4), *B*'(3, 3), *C*'(1, 1) and point *D*'(2, 2) that is on $\overline{B'C'}$.



Points *B*, *C*, and *D* are collinear on Δ*ABC* and points *B*', *C*', and *D*' are collinear on Δ*A*'*B*'*C*'. Point D is between B and C on Δ*ABC* and point *D*' is between *B'* and *C*' on Δ*A*'*B*'*C*'. Therefore, collinearity and betweenness of points is maintained under this rotation.

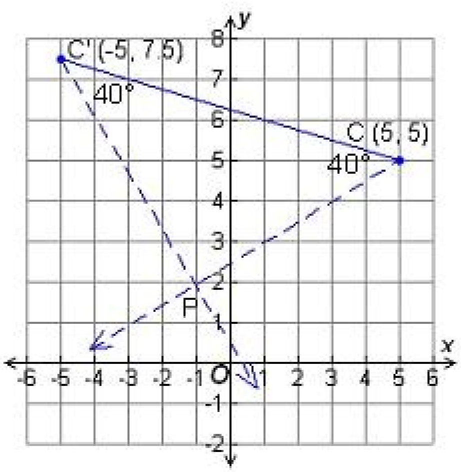
ANSWER:

Yes; sample answer: A rotation is a transformation which may change the orientation of the points but maintains congruence of the original figure and its image. So, the preimage can be mapped onto the image and corresponding segments will be congruent. Therefore, collinearity and betweenness of points are maintained in rotation.

35. **CHALLENGE** Point *C* has coordinates *C*(5, 5). The image of this point after a rotation of 100° about a certain point is C' (–5, 7.5). Use construction to estimate the coordinates of the center of this rotation. Explain.

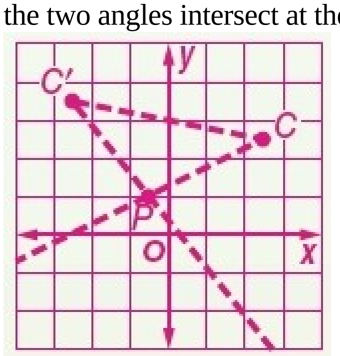
SOLUTION:

Sample answer: Plot points *C*(5, 5) and *C* '(-5, 7.5) on a coordinate plane. Draw $\overline{CC'}$. The image and preimage are the same distance from the point of rotation. Thus, if point *P* is the point of rotation, $\Delta CC'P$ is isosceles and the vertex angle of the triangle is the angle of rotation. So, $m\angle CPC'$ = 100° and both $m\angle PCC'$ and $m\angle PC'C$ are 40° because the base angles of an isosceles triangles are congruent. Using a protractor and straightedge construct a 40° angle with a vertex at *C* and a 40° angle with a vertex at C'. The intersection of the rays will be point *P*, which appears to have coordinates (1, -2). Thus, the center of rotation appears to be (–1, 2).



ANSWER:

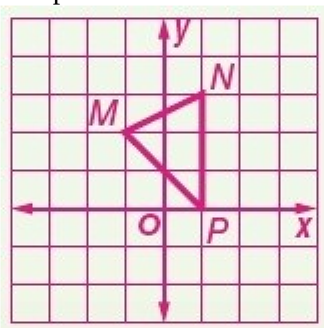
Sample answer: (–1, 2); Since $\Delta CC'P$ is isosceles and the vertex angle of the triangle is formed by the angle of rotation, both $m\angle PCC'$ and $m\angle PC'C$ are 40° because the base angles of isosceles triangles are congruent. When you construct a 40° angle with a vertex at C and a 40° angle with a vertex at C', the intersection of the rays forming the two angles intersect at the point of rotation, or (–1, 2).



36. **OPEN ENDED** Draw a figure on the coordinate plane. Describe a nonzero rotation that maps the image onto the preimage with no change in orientation.

SOLUTION:

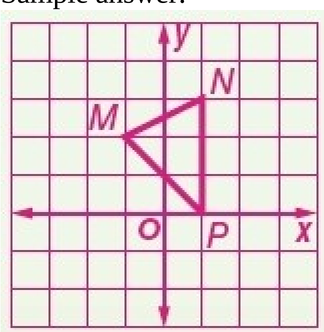
Sample answer:



A 360° rotation will map the image on the preimage. A 360° rotation is a sum of two 180° rotations. If one of the original points is N at (1, 3), the first 180° rotation moves N to (–1, –3). The second 180° rotation will move N back to (1, 3).

ANSWER:

Sample answer:



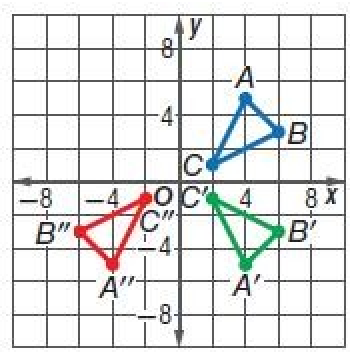
a 360° rotation

37. **CCSS ARGUMENTS** Is the reflection of a figure in the *x*-axis equivalent to the rotation of that same figure 180° about the origin? Explain.

SOLUTION:

A reflection of a figure in the *x*-axis is not equivalent to the rotation of that same figure 180° about the origin. When a figure is reflected about the *x*-axis, the *x*-coordinates of the transformed figure remain the same, and the *y*-coordinates are negated. When a figure is rotated 180° about the origin, both the *x*- and *y*-coordinates are negated. Therefore, the transformations are not equivalent.

In the following graph, Δ*ABC* is reflected over the x-axis to Δ*A'B'C'*. Δ*ABC* is rotated 180° about the origin to Δ*A''B''C''*.



ANSWER:

No; sample answer: When a figure is reflected about the *x*-axis, the *x*-coordinates of the transformed figure remain the same, and the *y*-coordinates are negated. When a figure is rotated 180° about the origin, both the *x*- and *y*-coordinates are negated. Therefore, the transformations are not equivalent.

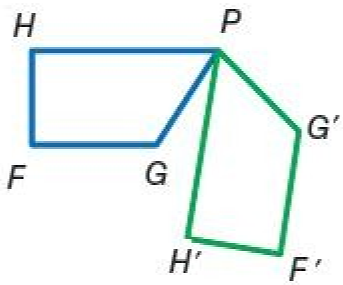
38. **WRITING IN MATH** Do invariant points *sometimes*, *always*, or *never* occur in a rotation? Explain your reasoning.

SOLUTION:

Invariant points are points that map onto themselves.

Sometimes, invariant points occur in a rotation. When a figure is rotated about a point on the figure, then the point of rotation is invariant. If a figure is rotated about a point not on the figure, then there are no invariant points in the rotation.

In the rotation below, point *P* is an invariant point.



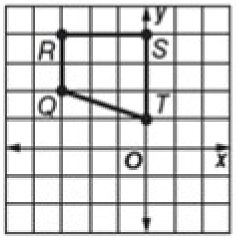
In the rotation below, there are no invariant points.

ANSWER:

Sometimes; sample answer: When a figure is rotated about a point on the figure, then the point of rotation is invariant. If a figure is rotated about a point not on the figure, then there are no invariant points in the rotation.

9-3 Rotations

39. What rotation of trapezoid *QRST* creates an image with point R' at (4, 3)?



A 270° counterclockwise about point *T*
B 185° counterclockwise about point *T*
C 180° clockwise about the origin
D 90° clockwise about the origin

SOLUTION:
Point *R* has coordinates(–3, 4). Consider each rotation.

A A 270° counterclockwise rotation about *T* will rotate (–3, 4) to (3, 4).
B A 185° counterclockwise rotation about *T* will rotate (–3, 4) somewhere between (3, –2) and (4, –1).
C A 180° clockwise rotation about the origin is equal to a a 180° counterclockwise rotation. Thus, $(x, y) \to (-x, -y)$ or (–3, 4) → (3, –4).
D A 90° clockwise rotation about the origin is equivalent to a 270° counterclockwise rotation: $(x, y) \to (y, -x)$. Thus, (–3, 4) → (4, 3).

Thus, the correct answer is D.

ANSWER:
D

40. **SHORT RESPONSE** ΔXYZ has vertices *X*(1, 7), *Y*(0, 2), and *Z*(–5, –2). What are the coordinates of X' after a rotation 270° counterclockwise about the origin?

SOLUTION:
To rotate a point 270° counterclockwise about the origin, multiply the *x*-coordinate of each vertex by –1 and interchange.
$(x, y) \to (y, -x)$
$(1, 7) \to (7, -1)$

ANSWER:
(7, –1)

41. **ALGEBRA** The population of the United States in July of 2007 was estimated to have surpassed 301,000,000. At the same time the world population was estimated to be over 6,602,000,000. What percent of the world population, to the nearest tenth, lived in the United States at this time?
F 3.1%
H 4.2%
G 3.5%
J 4.6%

SOLUTION:

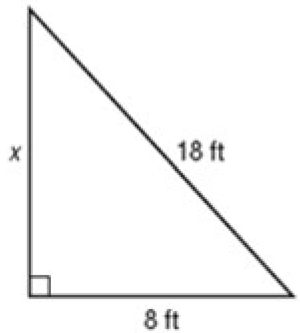
$$\frac{\text{United Sates population}}{\text{worldpopulation}} = \frac{301{,}000{,}000}{6{,}602{,}000{,}000} \times 100$$
$$\approx 4.6$$

So, the correct choice is J.

ANSWER:
J

42. **SAT/ACT** An 18-foot ladder is placed against the side of a house. The base of the ladder is positioned 8 feet from the house. How high up on the side of the house, to the nearest tenth of a foot, does the ladder reach?
A 10.0 ft
B 16.1 ft
C 19.7 ft
D 22.5 ft
E 26.0 ft

SOLUTION:
Assume that the angle formed by the side of a house and the ground is a right angle.
Make a sketch for the given situation.



Use the Pythagorean Theorem.

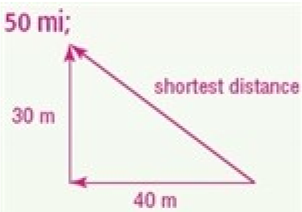
$$x^2 + 8^2 = 18^2$$
$$x^2 + 64 = 324$$
$$x^2 = 260$$
$$x \approx 16.1$$

So, the correct choice is B.

ANSWER:
B

43. **VOLCANOES** A cloud of dense gas and dust from a volcano blows 40 miles west and then 30 miles north. Make a sketch to show the translation of the dust particles. Then find the distance of the shortest path that would take the particles to the same position.
Refer to page 646.

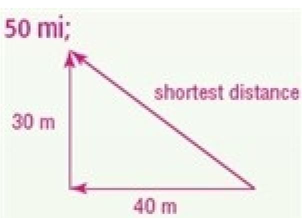
SOLUTION:



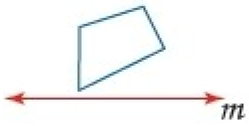
This distance can be found using the Pythagorean theorem.

$$c^2 = a^2 + b^2$$
$$c^2 = 30^2 + 40^2$$
$$c^2 = 900 + 1600$$
$$c^2 = 2500$$
$$c = \sqrt{2500}$$
$$c = 50$$

ANSWER:



Copy the figure and the given line of reflection. Then draw the reflected image in this line using a ruler.

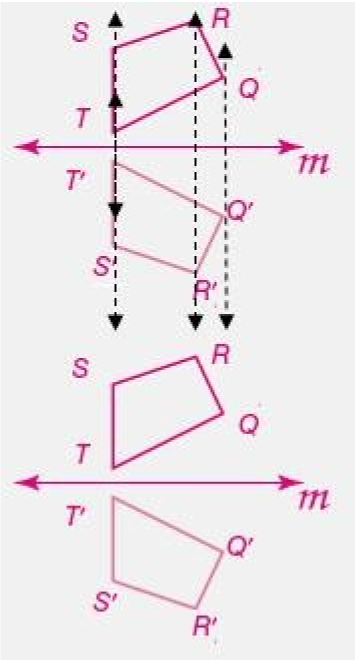
44. 

SOLUTION:

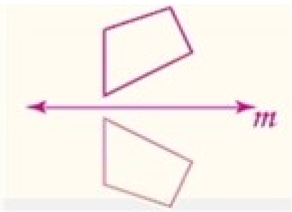
Step 1: Draw a line through each vertex that is perpendicular to line *m*.

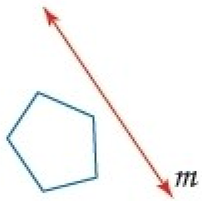
Step 2: Measure the distance from one vertex to the line *m*. Then locate the same distance from line *m* on the opposite side.

Step 3: Repeat Step 2 to locate the other three vertices. Then connect the vertices to form the reflected image.



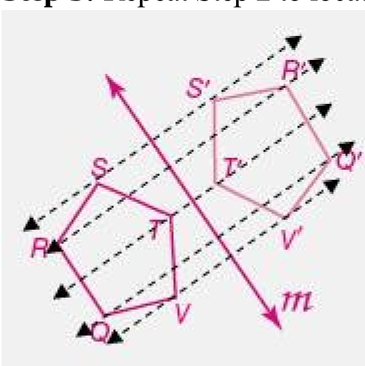
ANSWER:

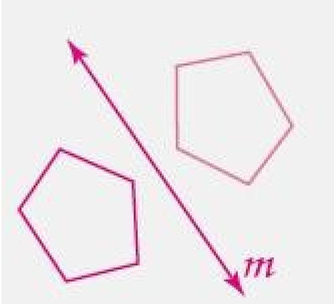


45. 

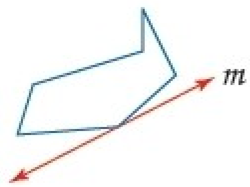
SOLUTION:

Step 1: Draw a line through each vertex that is perpendicular to line *m*.
Step 2: Measure the distance from one vertex to the line *m*. Then locate the same distance from line *m* on the opposite side.
Step 3: Repeat Step 2 to locate the other four vertices. Then connect the vertices to form the reflected image.





ANSWER:

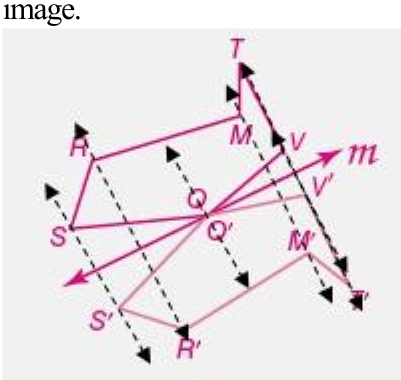
46. 

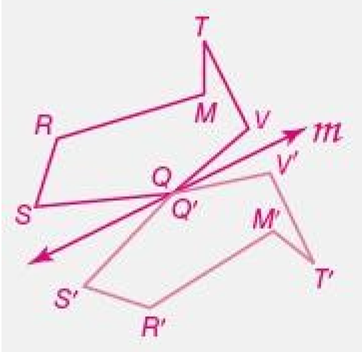
SOLUTION:

Step 1: Draw a line through each vertex that is perpendicular to line *m*.

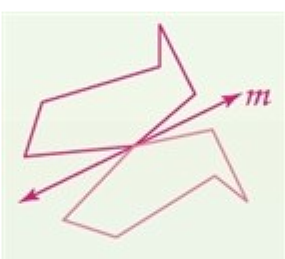
Step 2: Measure the distance from one vertex to the line *m*. Then locate the same distance from line *m* on the opposite side.

Step 3: Repeat Step 2 to locate points the other five vertices. Then connect the vertices to form the reflected image.

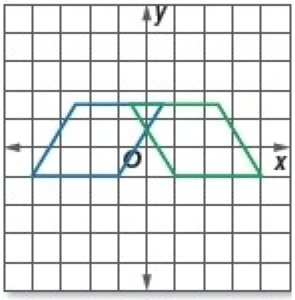




ANSWER:

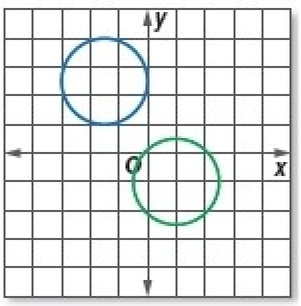


Identify the type of congruence transformation shown as a *reflection*, *translation*, or *rotation*.

47. 

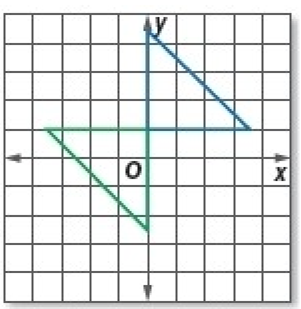
SOLUTION:
reflection (across the y-axis)

ANSWER:
reflection

48. 

SOLUTION:
translation (2.5 units to the right and 3.5 units down)

ANSWER:
translation

49. 

SOLUTION:
rotation (180° about the point (0, 1)) or reflection (across the line $y = -x + 1$)

ANSWER:
rotation or reflection