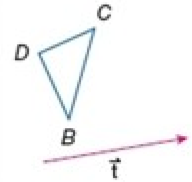
9-2 Translations

Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

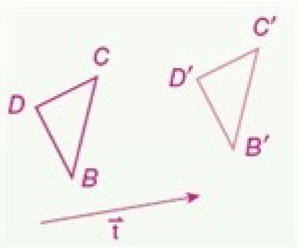
1. 

SOLUTION:

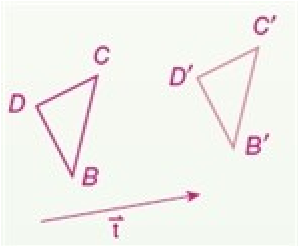
Step 1: Draw a line through each vertex parallel to vector $\vec{t}$.

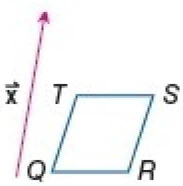
Step 2 : Measure the length of vector $\vec{t}$. Locate point *B*' by marking off this distance along the line through vertex *B*, starting at *B* and in the same direction as the vector.

Step 3: Repeat Step 2 to locate points *C*' and *D*'. Then connect vertices *B*', *C*', and *D*' to form the translated image.



ANSWER:





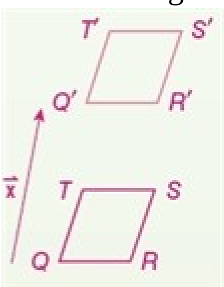
2.

SOLUTION:

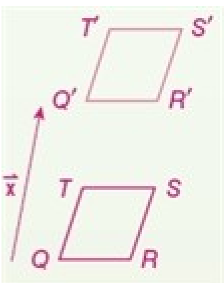
Step 1: Draw a line through each vertex parallel to vector $\vec{x}$.

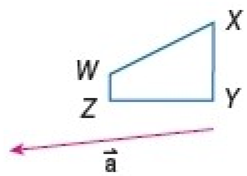
Step 2 : Measure the length of vector $\vec{x}$. Locate point *Q*' by marking off this distance along the line through vertex *Q*, starting at *Q* and in the same direction as the vector.

Step 3: Repeat Step 2 to locate points *R*', *S*', and *T*'. Then connect vertices *Q*', *R*', *S*', and *T*' to form the translated image.



ANSWER:



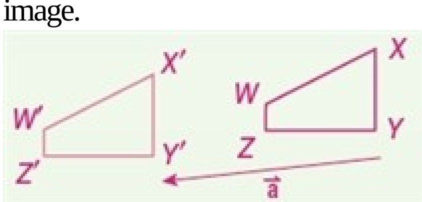
3. 

SOLUTION:

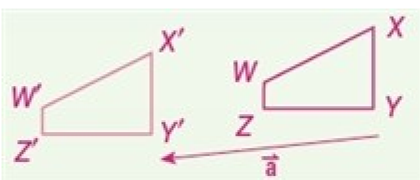
Step 1: Draw a line through each vertex parallel to vector $\vec{a}$.

Step 2 : Measure the length of vector $\vec{a}$. Locate point *W'* by marking off this distance along the line through vertex *W*, starting at *W* and in the same direction as the vector.

Step 3: Repeat Step 2 to locate points *X'*, *Y'* and *Z'*. Then connect vertices *W'*, *X'*, *Y'* and *Z'* to form the translated image.



ANSWER:

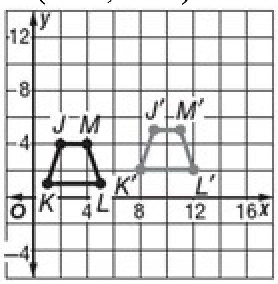


Graph each figure and its image along the given vector.

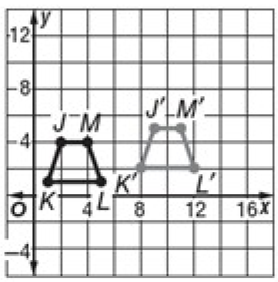
4. trapezoid *JKLM* with vertices *J*(2, 4), *K*(1, 1), *L*(5, 1) and *M*(4, 4); $\langle 7, 1 \rangle$

SOLUTION:

The coordinates of the vertices of the translated trapezoid are *J'*(2 + 7, 4 + 1), *K'*(1 + 7, 1 + 1), *L'*(5 + 7, 1 + 1) and *M'*(4 + 7, 4 + 1).



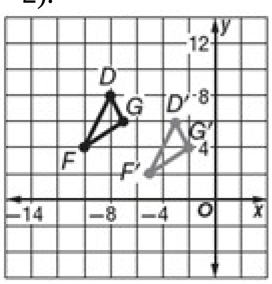
ANSWER:



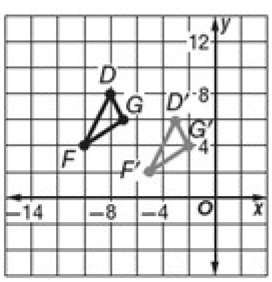
5. ΔDFG with vertices *D*(–8, 8), *F*(–10, 4), and *G*(–7, 6); $\langle 5, -2 \rangle$

SOLUTION:

The coordinates of the vertices of the translated triangle are *D'*(–8 + 5, 8 – 2), *F'*(–10 + 5, 4 – 2), and *G'*(–7 + 5, 6 – 2).



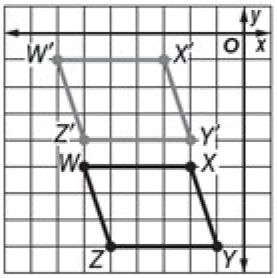
ANSWER:



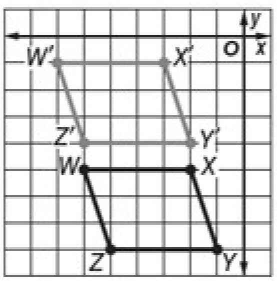
6. parallelogram *WXYZ* with vertices *W*(–6, –5), *X*(–2, –5), *Y*(–1, –8), and Z(–5, –8); $\langle -1, 4 \rangle$

SOLUTION:

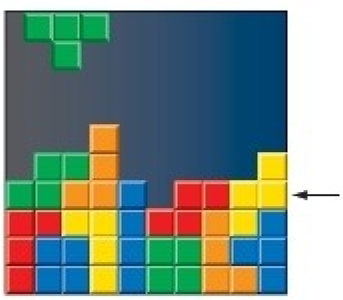
The coordinates of the vertices of the translated parallelogram are *W'*(–6 – 1, –5 + 4), *X'*(–2 – 1, –5 + 4), *Y'*(–1 – 1, –8 + 4), and Z'(–5 – 1, –8 + 4).



ANSWER:



7. **VIDEO GAMES** The object of the video game shown is to manipulate the colored tiles left or right as they fall from the top of the screen to completely fill each row without leaving empty spaces. If the starting position of the tile piece at the top of the screen is (x, y), use function notation to describe the translation that will fill the indicated row.



SOLUTION:

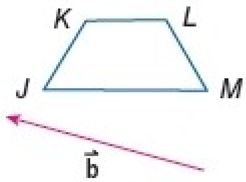
There is only one block space for the row to be complete. For the object to fit in the place, it should be moved 3 places right and 5 places down. So, the translation vector should be $\langle 3, -5 \rangle$.

Therefore, the translation is $(x, y) \rightarrow (x + 3, y - 5)$.

ANSWER:

$(x, y) \rightarrow (x + 3, y - 5)$.

CCSS TOOLS Copy the figure and the given translation vector. Then draw the translation of the figure along the translation vector.

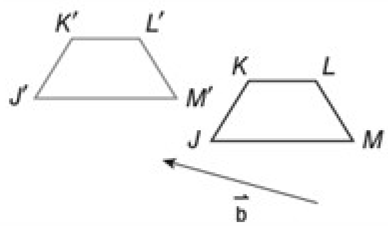
8. 

SOLUTION:

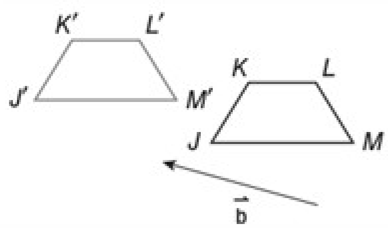
Step 1: Draw a line through each vertex parallel to vector $\vec{b}$.

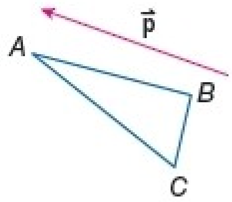
Step 2 : Measure the length of vector $\vec{b}$. Locate point *J*' by marking off this distance along the line through vertex *J*, starting at *J* and in the same direction as the vector.

Step 3: Repeat Step 2 to locate points *K*', *L*', and *M*'. Then connect vertices *J*', *K*', *L*', and *M*' to form the translated image.



ANSWER:



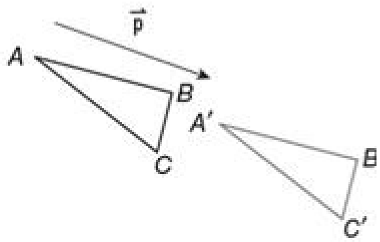
9. 

SOLUTION:

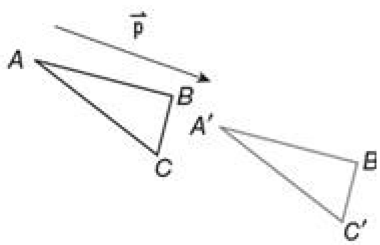
Step 1: Draw a line through each vertex parallel to vector $\vec{p}$.

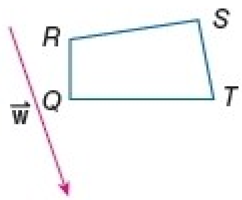
Step 2 : Measure the length of vector $\vec{p}$. Locate point A' by marking off this distance along the line through vertex A, starting at A and in the same direction as the vector.

Step 3: Repeat Step 2 to locate points B' and C'. Then connect vertices A', B', and C' to form the translated image.



ANSWER:



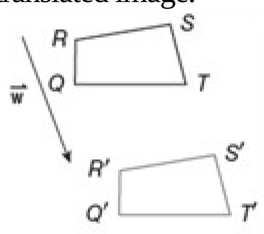
10. 

SOLUTION:

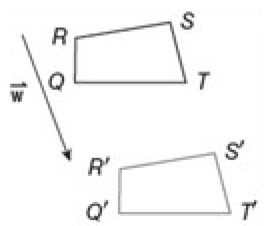
Step 1: Draw a line through each vertex parallel to vector $\vec{w}$.

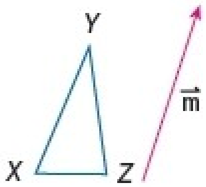
Step 2 : Measure the length of vector $\vec{w}$. Locate point *Q*' by marking off this distance along the line through vertex *Q*, starting at *Q* and in the same direction as the vector.

Step 3: Repeat Step 2 to locate points *R*', *S*', and *T*'. Then connect vertices *Q*', *R*', *S*', and *T*' to form the translated image.



ANSWER:



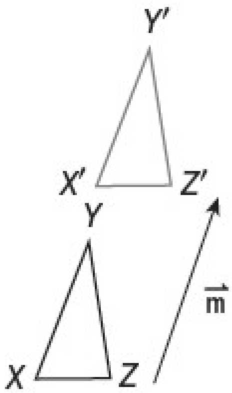
11. 

SOLUTION:

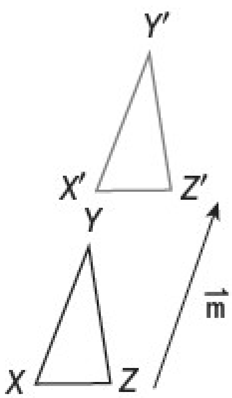
Step 1: Draw a line through each vertex parallel to vector $\vec{m}$.

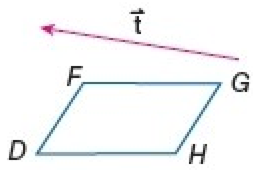
Step 2 : Measure the length of vector $\vec{m}$. Locate point X' by marking off this distance along the line through vertex X, starting at X and in the same direction as the vector.

Step 3: Repeat Step 2 to locate points Y' and Z'. Then connect vertices X', Y', and Z' to form the translated image.



ANSWER:



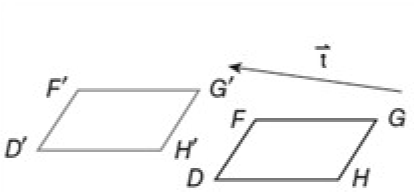
12. 

SOLUTION:

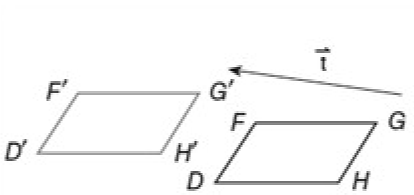
Step 1: Draw a line through each vertex parallel to vector $\vec{t}$.

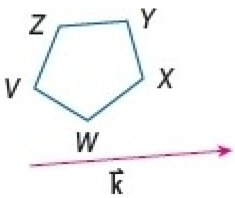
Step 2 : Measure the length of vector $\vec{t}$. Locate point *D*' by marking off this distance along the line through vertex *D*, starting at *D* and in the same direction as the vector.

Step 3: Repeat Step 2 to locate points *F*', *G*' and *H*'. Then connect vertices *D*', *F*', *G*' and *H'* to form the translated image.



ANSWER:



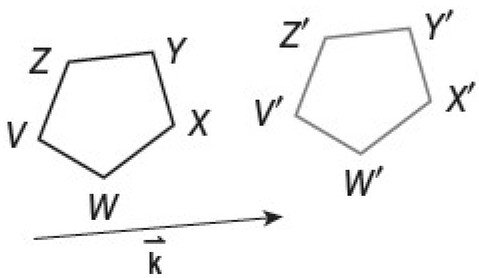
13. 

SOLUTION:

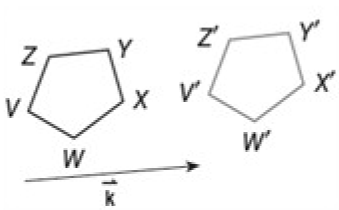
Step 1: Draw a line through each vertex parallel to vector $\vec{k}$.

Step 2 : Measure the length of vector $\vec{k}$. Locate point *V'* by marking off this distance along the line through vertex *V*, starting at *V* and in the same direction as the vector.

Step 3: Repeat Step 2 to locate points *W'*, *X'*, *Y'* and *Z'*. Then connect vertices *V'*, *W'*, *X'*, *Y'* and *Z'* to form the translated image.



ANSWER:

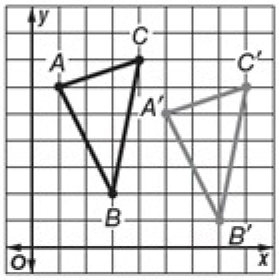


Graph each figure and its image along the given vector.

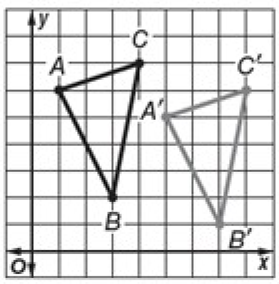
14. $\triangle ABC$ with vertices $A(1, 6)$, $B(3, 2)$, and $C(4, 7)$; $\langle 4, -1 \rangle$

SOLUTION:

The coordinates of the vertices of the translated triangle are $A'(1 + 4, 6 – 1)$, $B'(3 + 4, 2 – 1)$, and $C'(4 + 4, 7 – 1)$.



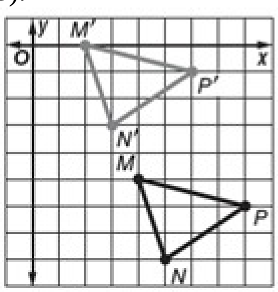
ANSWER:



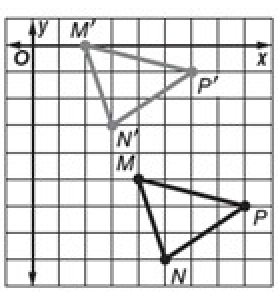
15. $\triangle MNP$ with vertices $M(4, –5)$, $N(5, –8)$, and $P(8, –6)$; $\langle -2, 5 \rangle$

SOLUTION:

The coordinates of the vertices of the translated triangle are $M'(4 – 2, –5 + 5)$, $N'(5 – 2, –8 + 5)$, and $P'(8 – , –6 + 5)$.



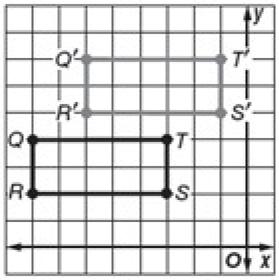
ANSWER:



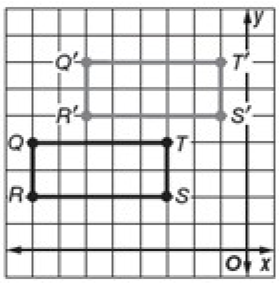
16. rectangle *QRST* with vertices *Q*(–8, 4), *R*(–8, 2), *S*(–3, 2), and *T*(–3, 4); $\langle 2, 3 \rangle$

SOLUTION:

The coordinates of the vertices of the translated rectangle are *Q'*(–8 + 2, 4 + 3), *R'*(–8 + 2, 2 + 3), *S'*(–3 + 2, 2 + 3), and *T'*(–3 + 2, 4 + 3).



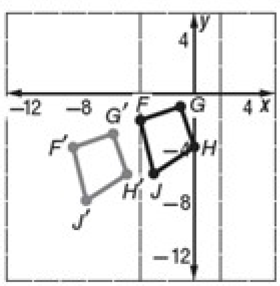
ANSWER:



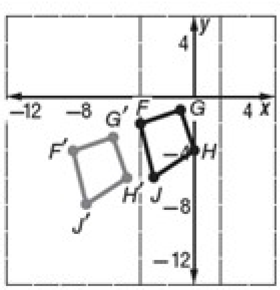
17. quadrilateral *FGHJ* with vertices *F*(–4, –2), *G*(–1, –1), *H*(0, –4), and *J*(–3, –6); $\langle -5, -2 \rangle$

SOLUTION:

The coordinates of the vertices of the translated quadrilateral are *F'*(–4 – 5, –2 – 2), *G'*(–1 – 5, –1 – 2), *H'*(0 – 5, –4 – 2), and *J'*(–3 – 5, –6 – 2).



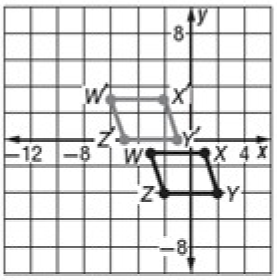
ANSWER:



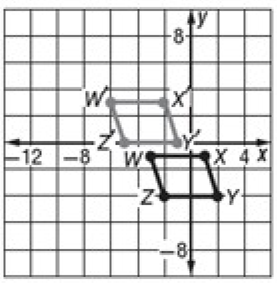
18. $\square WXYZ$ with vertices $W(-3, -1)$, $X(1, -1)$, $Y(2, -4)$, and $Z(-2, -4)$; $\langle -3, 4\rangle$

SOLUTION:

The coordinates of the vertices of the translated parallelogram are $W'(-3 - 3, -1 + 4)$, $X'(1 - 3, -1 + 4)$, $Y'(2 - 3, -4 + 4)$, and $Z'(-2 - 3, -4 + 4)$.



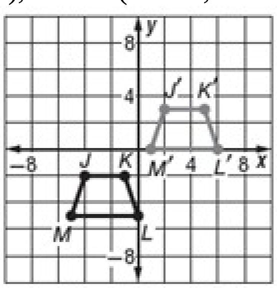
ANSWER:



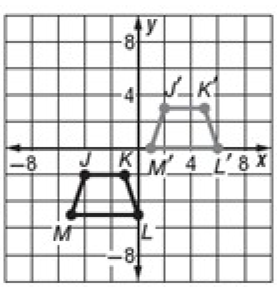
19. trapezoid $JKLM$ with vertices $J(-4, -2)$, $K(-1, -2)$, $L(0, -5)$, and $M(-5, -5)$; $\langle 6, 5\rangle$

SOLUTION:

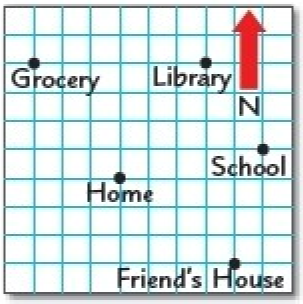
The coordinates of the vertices of the translated trapezoid are $J'(-4 + 6, -2 + 5)$, $K'(-1 + 6, -2 + 5)$, $L'(0 + 6, -5 + 5)$, and $M'(-5 + 6, -5 + 5)$.



ANSWER:



20. **CCSS MODELING** Brittany's neighborhood is shown on the grid at the right.



a. If she leaves home and travels 4 blocks north and 3 blocks east, what is her new location?

b. Use words to describe two possible translations that will take Brittany home from school.

SOLUTION:

a. Since the north side is marked upwards, travelling 4 blocks north and 3 blocks east means moving 4 blocks up and 3 blocks right. The point 4 blocks up and 3 blocks right to the home is the library. Therefore, her new location will be the library.

b. The resultant vector must be $\langle -5, 1\rangle$. Any combination of vectors that form this resultant vector will be satisfactory.

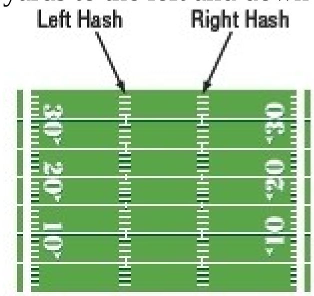
She can go west 5 blocks and then south 1 block, or she can go south 1 block and then west 5 blocks to reach the school.

ANSWER:

a. Library

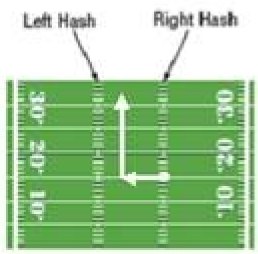
b. Sample answer: She can go west 5 blocks and then south 1 block, or she can go south 1 block and then west 5 blocks.

21. **FOOTBALL** A wide receiver starts from his 15-yard line on the right hash mark and runs a route that takes him 12 yards to the left and down field for a gain of 17 yards. Write a translation vector to describe the receiver's route.



SOLUTION:

The receiver starts on the right hash of the 15-yard line and moves 12 yards left and then 17 yards down field, or upwards as shown in the diagram.

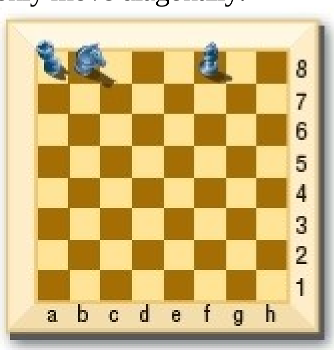


So, a translation vector that represents the receiver's route would be $\langle -12, 17 \rangle$.

ANSWER:

$\langle -12, 17 \rangle$

22. **CHESS** Each chess piece has a path that it can follow to move. The rook, which begins in square a8, can only move vertically or horizontally. The knight, which begins in square b8, can move two squares horizontally and then one square vertically, or two squares vertically and one square horizontally. The bishop, which begins in square f8, can only move diagonally.



a. The knight moves 2 squares vertically and 1 square horizontally on its first move, then two squares horizontally and 1 square vertically on its second move. What are the possible locations for the knight after two moves?

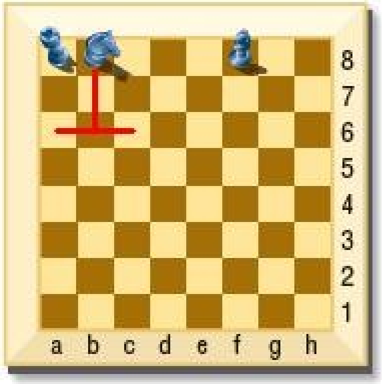
b. After two moves, the rook is in square d3. Describe a possible translation to describe the two moves.

c. Describe a translation that can take the bishop to square a1. What is the minimum number of moves that can be used to accomplish this translation?

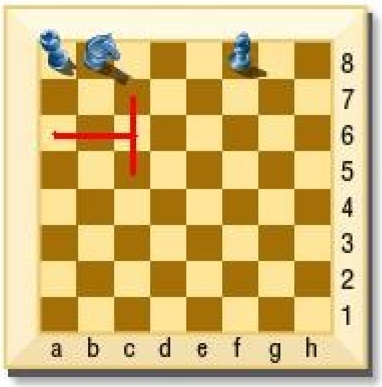
SOLUTION:

9-2 Translations

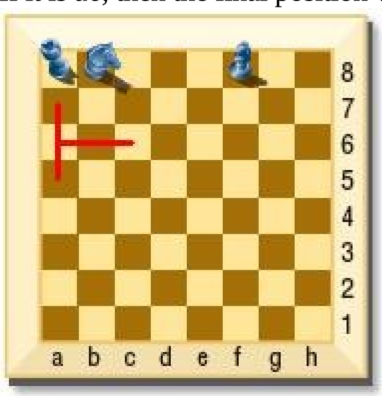
a. From b8 when the knight moves 2 squares vertically it reaches b6. There it has two choices for the horizontal movement a6 and c6.



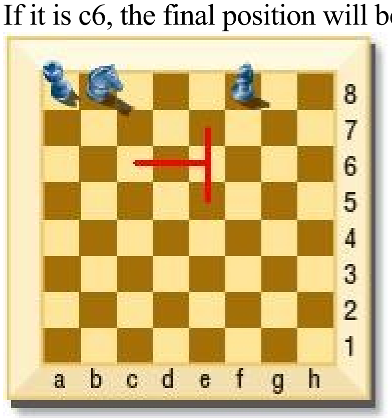
If the knight moves to a6, the second move takes it to either c5 or c7.



If the knight moves to c6, the two horizontal squares in the second move can be either to a6 or e6.
If it is a6, then the final position will be either a5 or a7.

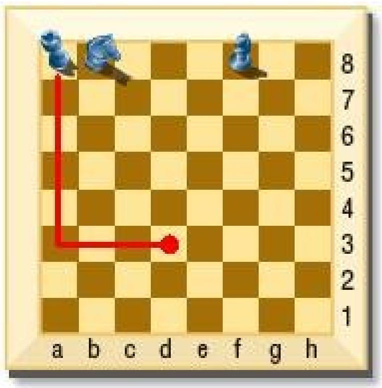


If it is c6, the final position will be either e5 or e7.

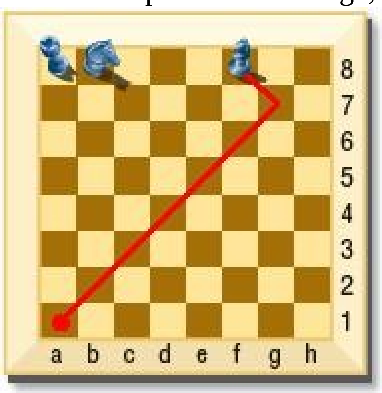


Therefore, the possible locations after the two moves are a7, a5, c7, c5, e7, or e5.

b. The rook could have moved vertically 5 squares down on the first move and then horizontally three squares right on the second turn.



c. The bishop could move to g7, then g7 to a1. The minimum number of moves is 2.



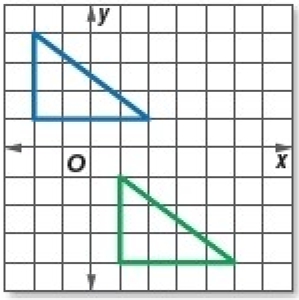
ANSWER:

a. a7, a5, c7, c5, e7, or e5

b. Sample answer: The rook could have moved vertically 5 squares down on the first move and then horizontally three squares right on the second turn.

c. Sample answer: The bishop could move to g7, then g7 to a1. The minimum number of moves is 2.

Write each translation vector.

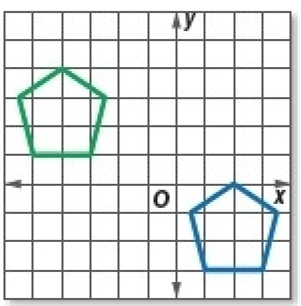
23. 

SOLUTION:
Use one point as a test point. The top vertex of the blue triangle is at (–2, 4). The top vertex of the green triangle is at (1, –1), Subtract the coordinates.

1 – (–2) = 3
–1 – 4 = –5

The triangle has been translated 3 units right and 5 units down. So, the translation vector is $\langle 3, -5 \rangle$.

ANSWER:
$\langle 3, -5 \rangle$.

24. 

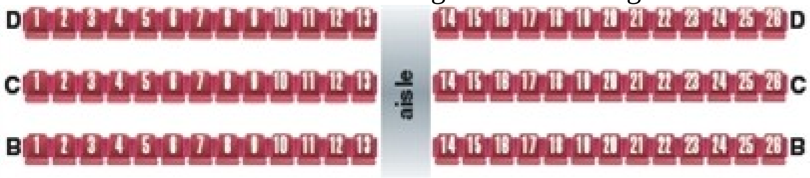
SOLUTION:
Use one point as a test point. The top vertex of the blue pentagon is at (2, 0). The top vertex of the green pentagon is at (–4, 4), Subtract the coordinates.

–4 – (2) = –6
4 – 0 = 4

The pentagon has been translated 6 units left and 4 units up. So, the translation vector is $\langle -6, 4 \rangle$.

ANSWER:
$\langle -6, 4 \rangle$.

25. **CONCERTS** Dexter's family buys tickets every year for a concert. Last year they were in seats C3, C4, C5, and C6. This year, they will be in seats D16, D17, D18, and D19. Write a translation in words and using vector notation that can be used to describe the change in their seating.



SOLUTION:

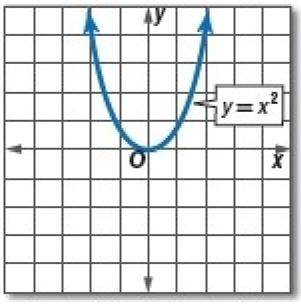
Pick a seat from the new row and compare it to the corresponding seat in the previous row. The seat D16 is 13 seats right and 1 row back to C3. Therefore, the translation vector can be represented as $\langle 13, -1 \rangle$.

ANSWER:

They move to the right 13 seats and back one row; $\langle 13, -1 \rangle$.

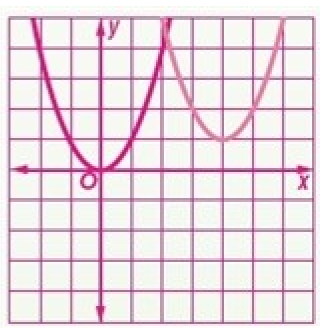
CCSS SENSE-MAKING Graph the translation of each function along the given vector. Then write the equation of the translated image.

26. $\langle 4, 1\rangle$



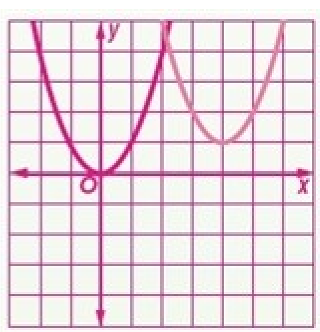
SOLUTION:

The translation of $\langle 4, 1\rangle$ translates each point of $y = x^2$ 4 units right and 1 unit up. Thus, the vertex of (0, 0) will translate to (4, 1). (1, 1) translates to (5, 2) and (–1, 1) translates to (3, 2), etc.



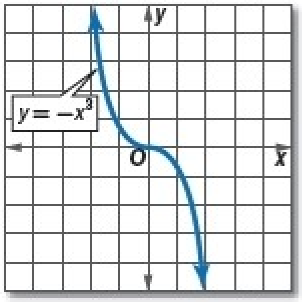
The equation for the translated function is $y = (x - 4)^2 + 1$.

ANSWER:



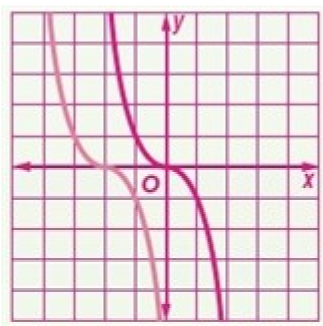
$y = (x - 4)^2 + 1$

27. $\langle -2, 0 \rangle$



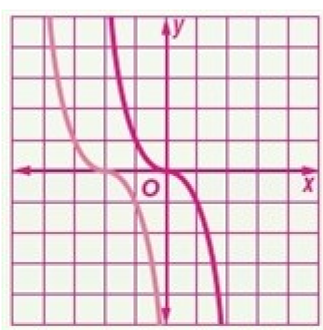
SOLUTION:

The translation of $\langle -2, 0 \rangle$ translates each point of $y = -x^3$ 2 units left. Thus, the intercept of (0, 0) will translate to (–2, 0). (1, –1) translates to (–1, –1) and (–1, 1) translates to (–3, 1), etc.



The equation of the translated function is $y = -(x + 2)^3$.

ANSWER:

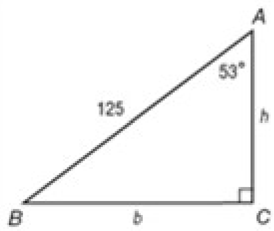


$y = -(x + 2)^3$

28. **ROLLER COASTERS** The length of the roller coaster track from the top of a hill to the bottom of the hill is 125 feet at a 53° angle with the vertical. If the position at the top of the hill is (x, y), use function notation to describe the translation to the bottom of the hill. Round to the nearest foot.

SOLUTION:

We have a triangle as shown.



Use trigonometric ratios to find the vertical and horizontal changes of the coordinates of points from A to B.

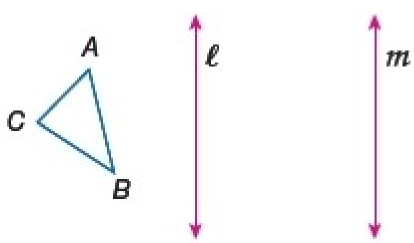
$$\begin{aligned} \sin 53° &= \frac{b}{125} \\ 125 \sin 53° &= b \\ 100 &\approx b \\ \cos 53° &= \frac{h}{125} \\ 125 \cos 53° &= h \\ 75 &\approx h \end{aligned}$$

The vertex B is 75 ft down and 100 ft left to A. Therefore, the translation is $(x, y) \rightarrow (x - 75, y - 100)$.

ANSWER:

$(x, y) \rightarrow (x + 75, y - 100)$

29. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate reflections over a pair of parallel lines.



a. GEOMETRIC On patty paper, draw ΔABC and a pair of vertical lines l and m. Reflect in line l by folding the patty paper. Then reflect, $\Delta A'B'C'$, in line m. Label the final image $\Delta A''B''C''$.

b. GEOMETRIC Repeat the process in part a for ΔDEF reflected in vertical lines n and p and ΔJKL reflected in vertical lines q and r.

c. TABULAR Copy and complete the table below.

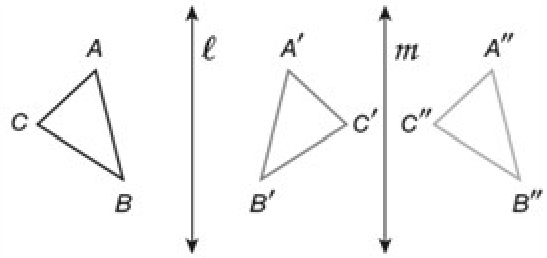
Distance Between Corresponding Points (cm)		Distance Between Vertical Lines (cm)	
A and *A″*, *B* and *B″*, *C* and *C″*		ℓ and m	
D and *D″*, *E* and *E″*, *F* and *F″*		n and p	
J and *J″*, *K* and *K″*, *L* and *L″*		q and r	

d. VERBAL Describe the result of two reflections in two vertical lines using one transformation.

SOLUTION:

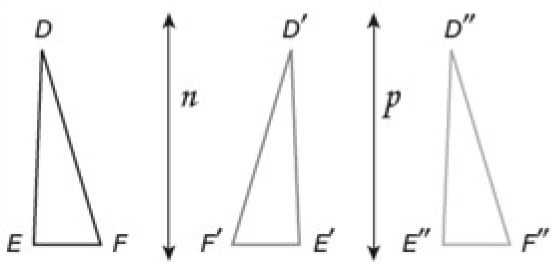
a.

Step 1: Draw *ABC* and line *l* and line *m*.
Step 2: Fold the paper on line *l*. Sketch the figure *A'B'C'*.
Step 3: Fold the paper on line *m*. Sketch the figure *A''B''C''*.

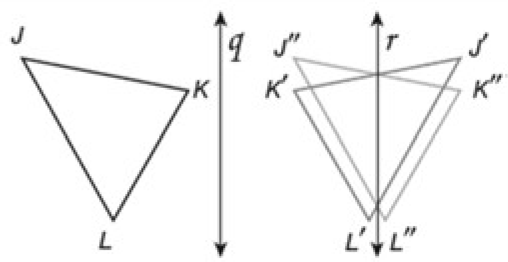


b.

Draw another triangle *DEF* and line *n* and line *p* . Reflect twice following the steps in part **a**.



Draw another triangle *JKL* and line *q* and line *r* . Reflect twice following the steps in part **a**.



c.

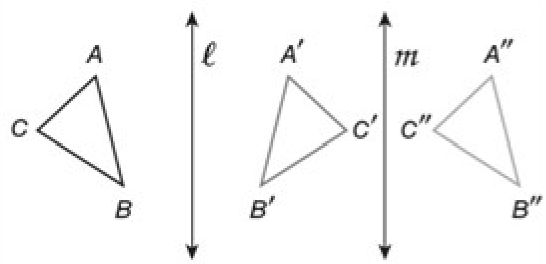
9-2 Translations

Distance Between Corresponding Points (cm)		Distance Between Vertical Lines (cm)	
A and A'', B and B'', C and C''	4.4	ℓ and m	2.2
D and D'', E and E'', F and F''	5.6	n and p	2.8
J and J'', K and K'', L and L''	2.8	q and r	1.4

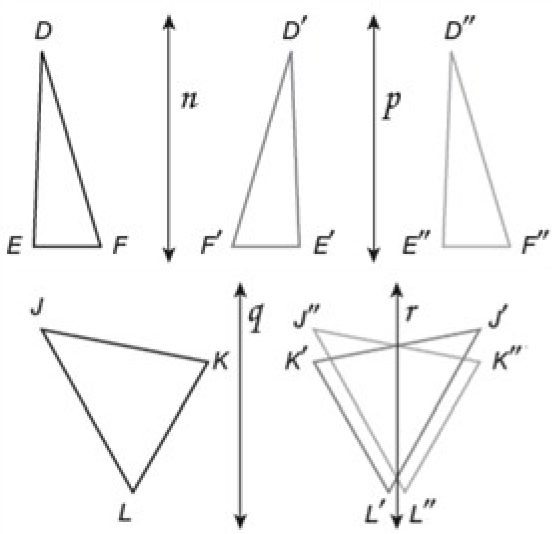
d. Sample answer: the composition of two reflections in vertical lines can be described by a horizontal translation that is twice the distance between the two vertical lines.

ANSWER:

a.



b.



Distance Between Corresponding Points (cm)		Distance Between Vertical Lines (cm)	
A and A'', B and B'', C and C''	4.4	ℓ and m	2.2
D and D'', E and E'', F and F''	5.6	n and p	2.8
J and J'', K and K'', L and L''	2.8	q and r	1.4

c.

d. Sample answer: the composition of two reflections in vertical lines can be described by a horizontal translation that is twice the distance between the two vertical lines.

9-2 Translations

30. **REASONING** Determine a rule to find the final image of a point that is translated along $\langle x+a, y+b\rangle$ and then $\langle x+c, y+d\rangle$.

SOLUTION:

Translating along $\langle a,b\rangle$ and then along $\langle c,d\rangle$ is same as translating along $\langle a+c,b+d\rangle$. Therefore, the translation vector is $\langle x+a+c, y+b+d\rangle$.

ANSWER:

$\langle x+a+c, y+b+d\rangle$.

31. **CHALLENGE** A line $y = mx + b$ is translated using the vector $\langle a, b \rangle$. Write the equation of the translated line. What is the value of the y-intercept?

SOLUTION:

The vector $\langle a, b \rangle$ maps the point $(x, y) \to (x + a, y + b)$.

Step 1: Identify two points on the line y = mx + b. When $x = 0$, y = b. When $y = 0$, $x = -\frac{b}{m}$. So two points on the line y = mx + b are (0, b) and $\left(-\frac{b}{m}, 0\right)$.

Step 2: Find the location of these points under the vector translation $\langle a, b \rangle$.
(0, b) → (0 + a, b + b) → (a, 2b)

$\left(-\frac{b}{m}, 0\right) \to \left(-\frac{b}{m} + a, 0 + b\right) \to \left(-\frac{b}{m} + a, b\right)$

Step 3: Use these points to determine the equation of the translated line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2b - b}{a - \left(-\frac{b}{m} + a\right)} = \frac{b}{a + \frac{b}{m} - a} = \frac{b}{\frac{b}{m}} = m$$

Since the line is translated with its orientation unchanged we would expect the slope of the line to remain unchanged. Substituting the point (a, 2b) into the equation $y = mx + b_n$ we will find b_n, the y-intercept of the translated line.

$y = mx + b_n$
$2b = ma + b_n$
$b_n = 2b - ma$

Note that if we had used the second point on the translated line, we would have arrived at the same value for the new y-intercept.

$y = mx + b_n$

$b = m\left(-\frac{b}{m} + a\right) + b_n$

$b = -b + ma + b_n$

$b_n = 2b - ma$

Substituting the value of the new y-intercept into the equation, we arrive at the equation for the translated line.

$y = mx + b_n$
$y = mx + 2b - ma$

Combine like terms.
$y = m(x - a) + 2b$

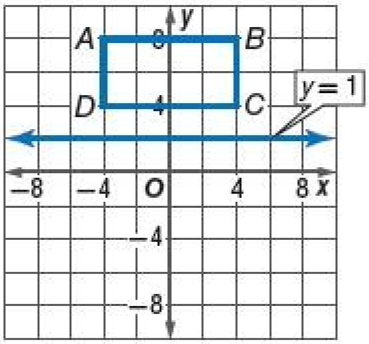
ANSWER:

$y = m(x - a) + 2b$; $2b - ma$

32. **OPEN ENDED** Draw a figure on the coordinate plane so that the figure has the same orientation after it is reflected in the line $y = 1$. Explain what must be true in order for this to occur.

SOLUTION:

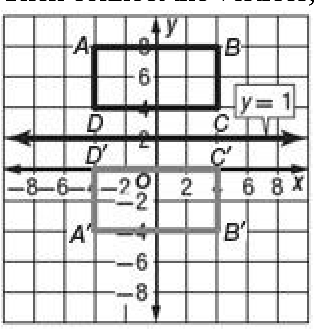
Draw figure $ABCD$ with vertices at A(–4, 8), B (4, 8), C (4, 4), and D(–, 4). Draw the line $y = 1$.



Find a corresponding point for each vertex so that a vertex and its image are equidistant from the line $y = 1$.

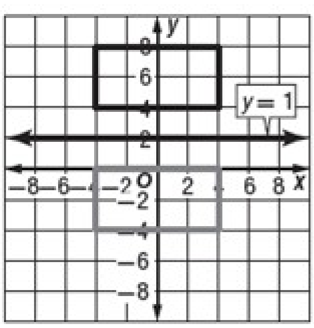
Vertex A is 3 units up from $y = 1$, locate the point 3 units down from $y = 1$. A' would be (–4, –4).
Vertex B is 3 units up from $y = 1$, locate the point 3 units down from $y = 1$. B' would be (4, –4).
Vertex C is 1 unit up from $y = 1$, locate the point 1 unit down from $y = 1$. C' would be (–4, 0).
Vertex D is 1 unit up from $y = 1$, locate the point 1 unit down from $y = 1$. D' would be (4, 0).

Then connect the vertices, A', B', C', and D' to form the reflected image.



In order for the figures $ABCD$ and $A'B'C'D'$ to be the same, they must match up when you fold the grid in half horizontally at the $y = 1$ line.

ANSWER:



Sample answer: in order for the figure to be the same, it must match up when you fold it in half horizontally at the $y = 1$ line.

33. **WRITING IN MATH** Compare and contrast function notation and vector notation for translations.

SOLUTION:

Sample answer: Both vector notation and function notation describe the distance a figure is translated in the horizontal and vertical directions. Vector notation only describes the movement. It does not determine the beginning and ending locations. Function notation does both. It will give the initial point, the distance of the horizontal and vertical translation, and the ending point. For example, the translation a units to the right and b units up from the point (x, y) would be written $\langle a, b \rangle$ in vector notation and $(x, y) \rightarrow (x + a, y + b)$ in function notation.

ANSWER:

Sample answer: Both vector notation and function notation describe the distance a figure is translated in the horizontal and vertical directions. Vector notation does not give a rule in terms of initial location, but function notation does. For example, the translation a units to the right and b units up from the point (x, y) would be written $\langle a, b \rangle$ in vector notation and $(x, y) \rightarrow (x + a, y + b)$ in function notation.

34. **WRITING IN MATH** Recall from Lesson 9-1 that an invariant point maps onto itself. Can invariant points occur with translations? Explain why or why not.

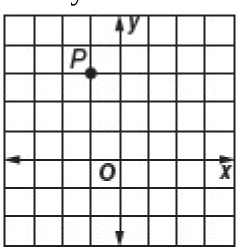
SOLUTION:

Sample answer: No; since an image must move in order for a translation to have taken place, and the orientation of the figure must remain the same, no point can be invariant in a translation. If any of the points remain invariant, the resulting figure is the original figure.

ANSWER:

Sample answer: No; since an image must move in order for a translation to have taken place, and the orientation of the figure must remain the same, no point can be invariant in a translation. If any of the points remain invariant, the resulting figure is the original figure.

35. Identify the location of point P under translation $(x + 3, y + 1)$.



A (0, 6)
B (0, 3)
C (2, –4)
D (2, 4)

SOLUTION:

The coordinates of the point P is (–1, 3). So, the translated point will be (–1 + 3, 3 + 1) = (2, 4). Therefore, the correct choice is D.

ANSWER:

D

36. **SHORT RESPONSE** Which vector best describes the translation of $A(3,-5)$ to $A'(-2,-8)$?

SOLUTION:

The translation is $A(3,-5) \rightarrow A'(-2,-8)$.
–2 – 3 = –5 and –8 – (–5) = –3
Therefore, the translation vector is $\langle -5,-3 \rangle$.

ANSWER:

$\langle -5,-3 \rangle$.

37. **ALGEBRA** Over the next four days, Amanda plans to drive 160 miles, 235 miles, 185 miles, and 220 miles. If her car gets an average of 32 miles per gallon of gas, how many gallons of gas should she expect to use in all?
F 25
G 30
H 35
J 40

SOLUTION:

First find the total distance she has to travel in the next 4 days.
160 + 235 + 185 + 220 = 800.
Divide the total distance by 32 to find the number of gallons of gas required.

$$\frac{800}{32} = 25$$

Therefore, the correct choice is F.

ANSWER:

F

38. **SAT/ACT** A bag contains 5 red marbles, 2 blue marbles, 4 white marbles, and 1 yellow marble. If two marbles are chosen in a row, without replacement, what is the probability of getting 2 white marbles?
A. $\frac{1}{66}$
B. $\frac{1}{11}$
C. $\frac{1}{9}$
D. $\frac{5}{33}$
E. $\frac{2}{5}$

SOLUTION:
Here, the choice of the first marble affects the choice of the second marble. So, the events are dependent events. The probability of two dependent events *A* and *B* is $P(A \text{ and } B) = P(A) \cdot P(B)$.

There is a total of 12 marbles of which 4 are white. So, the probability of choosing a white marble from 12 is $\frac{1}{3}$. After the first choice, the bag is left with 11 marbles of which 3 are white. The probability of choosing a white marble again is $\frac{3}{11}$. Then the probability of the events *A* and *B* is $\frac{1}{3} \cdot \frac{3}{11} = \frac{1}{11}$

Therefore, the correct choice is B.

ANSWER:
B

Graph each figure and its image under the given reflection.

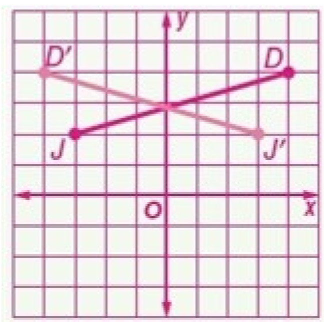
39. $\overline{DJ}$ with endpoints $D(4, 4)$, $J(–3, 2)$ in the y-axis

SOLUTION:

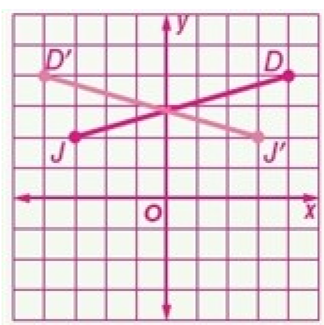
To reflect over the y-axis, multiply the x-coordinate of each vertex by –1.

$(x, y) \rightarrow (-x, y)$
$D(4, 4) \rightarrow D'(-4, 4)$
$J(-3, 2) \rightarrow J'(3, 2)$

Plot the points. Then connect the vertices, D' and J' to form the reflected image.



ANSWER:



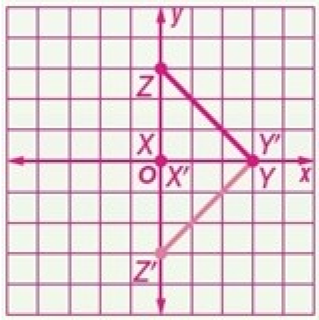
40. ΔXYZ with vertices $X(0, 0)$, $Y(3, 0)$, and $Z(0, 3)$ in the x-axis

SOLUTION:

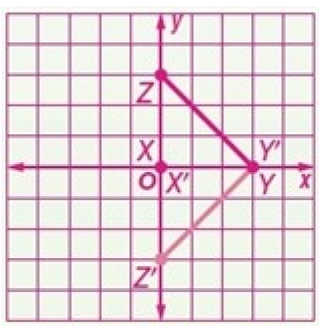
To reflect over the x-axis, multiply the y-coordinate of each vertex by –1.

$(x, y) \rightarrow (x, -y)$
$X(0, 0) \rightarrow X'(0, 0)$
$Y(3, 0), \rightarrow Y'(3, 0),$
$Z(0, 3) \rightarrow Z'(0, -3)$

Plot the points. Then connect the vertices, X', Y', and Z' to form the reflected image.



ANSWER:



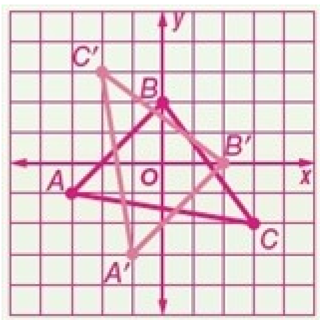
41. ΔABC with vertices $A(–3, –1)$, $B(0, 2)$, and $C(3, –2)$, in the line $y = x$

SOLUTION:

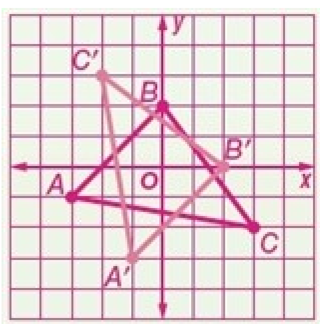
To reflect over the $y = x$ line, interchange the x- and y-coordinates of each vertex..

$(x, y) \rightarrow (y, x)$
$A(–3, –1) \rightarrow A'(–1, –3)$
$B(0, 2) \rightarrow B'(2, 0)$
$C(3, –2) \rightarrow C'(–2, 3)$

Plot the points. Then connect the vertices, A', B', and C' to form the reflected image.



ANSWER:



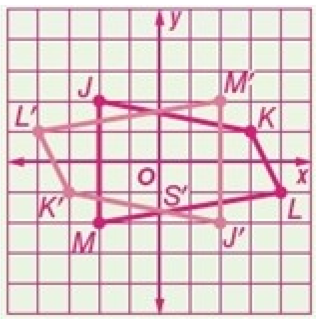
42. quadrilateral $JKLM$ with vertices $J(-2, 2)$, $K(3, 1)$, $L(4, -1)$, and $M(-2, -2)$ in the origin

SOLUTION:

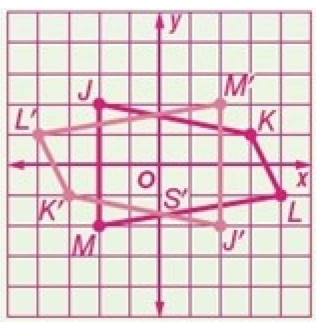
To reflect in the origin, multiply the x- and y-coordinate of each vertex by -1.

$(x, y) \rightarrow (-x, -y)$
$J(-2, 2) \rightarrow J(2, -2)$
$K(3, 1) \rightarrow K(-3, -1)$
$L(4, -1) \rightarrow L(-4, 1)$
$M(-2, -2) \rightarrow M(2, 2)$

Plot the points. Then connect the vertices, J', K', L', and M' to form the reflected image.



ANSWER:

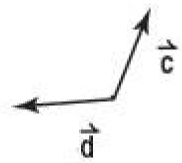


Copy the vectors to find each sum or difference.

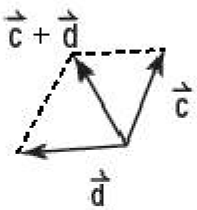
43. $\vec{c} + \vec{d}$

SOLUTION:

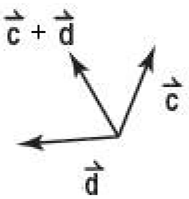
Using the parallelogram method, start by copying $\vec{c}$ and $\vec{d}$ so that the tail of $\vec{c}$ touches the tail of $\vec{d}$.



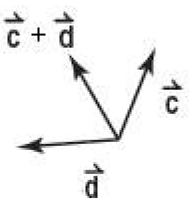
Then complete the parallelogram. Then draw the diagonal.



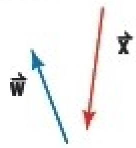
The diagonal is the resultant vector $\vec{c} + \vec{d}$.



ANSWER:

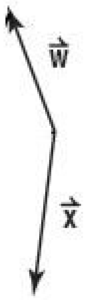


44. $\vec{w} + \vec{x}$

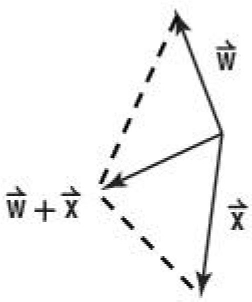


SOLUTION:

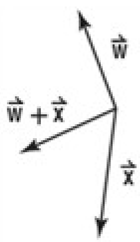
Using the parallelogram method, start by copying $\vec{w}$ and $\vec{x}$ so that the tail of $\vec{w}$ touches the tail of $\vec{x}$.



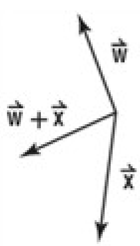
Then complete the parallelogram. Then draw the diagonal.



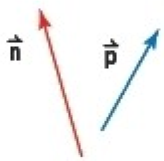
The diagonal is the resultant vector $\vec{w} + \vec{x}$.



ANSWER:

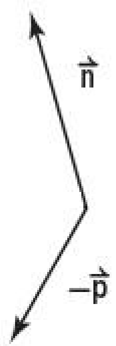


45. $\vec{n} - \vec{p}$

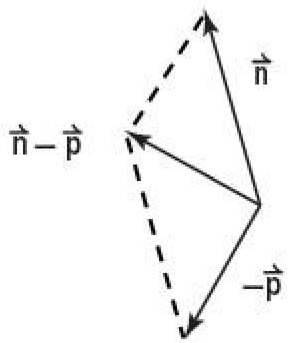


SOLUTION:

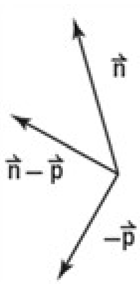
Using the parallelogram method, start by copying $\vec{n}$ and $-\vec{p}$ so that the tail of $\vec{n}$ touches the tail of $-\vec{p}$.



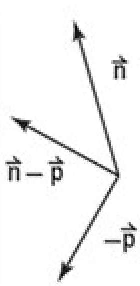
Then complete the parallelogram. Then draw the diagonal.



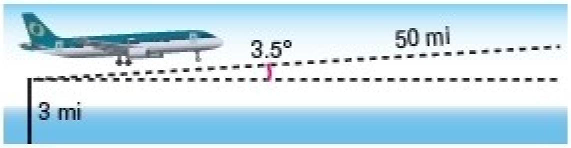
The diagonal is the resultant vector $\vec{n} - \vec{p}$.



ANSWER:



46. **NAVIGATION** An airplane is three miles above sea level when it begins to climb at a 3.5° angle. If this angle is constant, how far above sea level is the airplane after flying 50 miles?



SOLUTION:

Let x be the height that the airplane will be after flying 50 miles from the raising point.

$\sin 3.5° = \frac{x}{50}$

$50 \sin 3.5° = x$

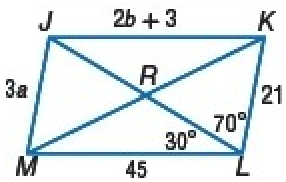
$3.1 \approx x$

The airplane was 3 miles above the sea level when it started rising. Therefore, after flying 50 miles, its will be about 3 + 3. 1 = 6.1 miles above the sea level.

ANSWER:

6.1 mi

Use $\square JKLM$ to find each measure.



47. $m\angle MJK$

SOLUTION:

The opposite angles of a parallelogram are congruent to each other. So, $\angle MJK \cong \angle MLK$.

m$\angle MLK$ = 30 + 70 = 100, so m$\angle MLK$ = 100.

ANSWER:

100

48. $m\angle JML$

SOLUTION:

The opposite angles of a parallelogram are congruent to each other. And the sum of the four angles in a parallelogram is 360.
It is given that $m\angle MLK$ = 70 + 30 or 100. Then $m\angle MJK$ = 100.

$360 = m\angle JML + 100 + m\angle LKJ + 100$	Sum angles of Quadrilateral
$360 = m\angle JML + 200 + m\angle LKJ$	Addition.
$160 = m\angle JML + m\angle LKJ$	-200 from each side.
$160 = 2m\angle JML$	Subsitutuion.
$80 = m\angle JML$	$\div$ each side by 2.

Thus, $m\angle JML$ is 80.

ANSWER:

80

49. $m\angle JKL$

SOLUTION:

The opposite angles of a parallelogram are congruent to each other. And the sum of the four angles in a **quadrilateral** is 360.
It is given that $m\angle MLK$ = 70 + 30 or 100. Then $m\angle MJK$ = 100.

$360 = m\angle JML + 100 + m\angle LKJ + 100$	Sum angles of Quadrilateral
$360 = m\angle JML + 200 + m\angle LKJ$	Addition.
$160 = m\angle JML + m\angle LKJ$	-200 from each side.
$160 = 2m\angle LKJ$	Subsitutuion.
$80 = m\angle LKJ$	$\div$ each side by 2.

Thus, $m\angle LKJ$ is 80.

ANSWER:

80

50. $m\angle KJL$

SOLUTION:

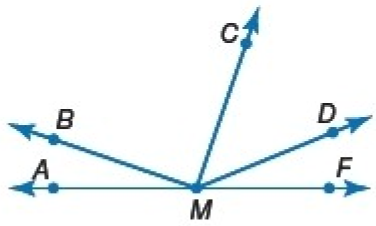
The angles $\angle JLM$ and $\angle KJL$ are corresponding angles. So, by the corresponding angle postulate, they are congruent. Therefore, $m\angle KJL = 30$.

ANSWER:

30

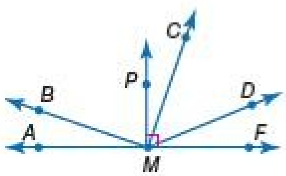
9-2 Translations

Copy the diagram shown, and extend each ray. Classify each angle as *right, acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



51. $\angle AMC$

SOLUTION:

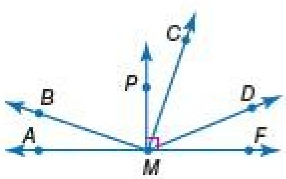


With the protractor, draw a vertical line at point M such that $\angle AMP$ is a right angle. Point M on $\angle AMC$ lies in the exterior of right angle $\angle AMP$, so $\angle AMC$ is an obtuse angle. Use the protractor to find that $m\angle AMC$ *is 110.*

ANSWER:
obtuse; 110

52. $\angle FMD$

SOLUTION:



With the protractor, draw a vertical line at point M such that $\angle AMP$ is a right angle. Point M on $\angle FMD$ lies in the interior of right angle $\angle FMP$, so $\angle FMD$ is an acute angle. Use the protractor to find that $m\angle FMD$ is 20.

ANSWER:
acute; 20

53. $\angle BMD$

SOLUTION:
$\angle BMD$ is an obtuse angle. Use the protractor to find that $m\angle BMD$ is 140.

ANSWER:
obtuse; 140

54. $\angle CMB$

SOLUTION:

$\angle CMB$ is a right angle. Use the protractor to find that $m\angle CMB$ is 90.

ANSWER:

right; 90