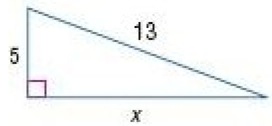
8-2 The Pythagorean Theorem and Its Converse

Find *x*.

1. 

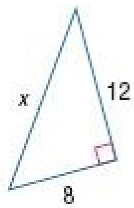
SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is 13 and the lengths of the legs are 5 and *x*.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 + 5^2 &= 13^2 \\ x^2 + 25 &= 169 \\ x^2 + 25 - 25 &= 169 - 25 \\ x^2 &= 144 \\ \sqrt{x^2} &= \sqrt{144} \\ x &= \sqrt{144} \\ x &= 12 \end{aligned}$$

ANSWER:

12

2. 

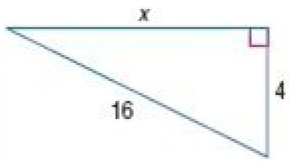
SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.
The length of the hypotenuse is *x* and the lengths of the legs are 8 and 12.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 8^2 + 12^2 &= x^2 \\ 64 + 144 &= x^2 \\ 208 &= x^2 \\ \sqrt{208} &= \sqrt{x^2} \\ \sqrt{16 \cdot 13} &= x \\ 4\sqrt{13} &= x \\ 14.4 &\approx x \end{aligned}$$

ANSWER:

$4\sqrt{13} \approx 14.4$



3.

SOLUTION:

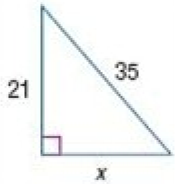
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is 16 and the lengths of the legs are 4 and x.

$$a^2 + b^2 = c^2$$
$$x^2 + 4^2 = 16^2$$
$$x^2 + 16 = 256$$
$$x^2 = 256 - 16$$
$$x = \sqrt{240}$$
$$x = \sqrt{16 \cdot 15}$$
$$x = 4\sqrt{15}$$
$$x \approx 15.5$$

ANSWER:

$$4\sqrt{15} \approx 15.5$$

4. Use a Pythagorean triple to find x. Explain your reasoning.



SOLUTION:

35 is the hypotenuse, so it is the greatest value in the Pythagorean Triple.

Find the common factors of 35 and 21.

$35 = 7 \cdot 5$

$21 = 7 \cdot 3$

The GCF of 35 and 21 is 7. Divide this out.

$35 \div 7 = 5$

$21 \div 7 = 3$

Check to see if 5 and 3 are part of a Pythagorean triple with 5 as the largest value.

$5^2 - 3^2 = 25 - 9 = 16 = 4^2$

We have one Pythagorean triple 3-4-5. The multiples of this triple also will be Pythagorean triple. So, $x = 7(4) = 28$. Use the Pythagorean Theorem to check it.

$35^2 \stackrel{?}{=} 21^2 + 28^2$

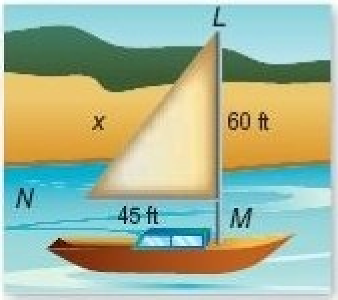
$1225 \stackrel{?}{=} 441 + 784$

$1225 = 1225 \checkmark$

ANSWER:

28; Since $35 = 7 \cdot 5$ and $21 = 7 \cdot 3$ and 3-4-5 is a Pythagorean triple, $x = 7 \cdot 4$ or 28.

5. **MULTIPLE CHOICE** The mainsail of a boat is shown. What is the length, in feet, of $\overline{LN}$?



A 52.5
B 65
C 72.5
D 75

SOLUTION:

The mainsail is in the form of a right triangle. The length of the hypotenuse is *x* and the lengths of the legs are 45 and 60.

$$\begin{aligned} a^2+b^2&=c^2 \\ 45^2+60^2&=x^2 \\ 2025+3600&=x^2 \\ 5625&=x^2 \\ \sqrt{5625}&=\sqrt{x^2} \\ 75&=x \end{aligned}$$

Therefore, the correct choice is D.

ANSWER:

D

Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *obtuse*, or *right*. Justify your answer.

6. 15, 36, 39

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$15+36>39\checkmark$

$36+39>15\checkmark$

$15+39>36\checkmark$

Therefore, the set of numbers can be measures of a triangle.

Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$39^2 \stackrel{?}{=} 15^2+36^2$

$1521 \stackrel{?}{=} 225+1296$

$1521=1521\checkmark$

Therefore, by the converse of Pythagorean Theorem, a triangle with the given measures will be a right triangle.

ANSWER:

Yes; right

$39^2 \stackrel{?}{=} 15^2+36^2$

$1521=225+1296$

7. 16, 18, 26

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$16+18>26$ ✓

$18+26>16$ ✓

$16+26>18$ ✓

Therefore, the set of numbers can be measures of a triangle.

Now, classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$26^2 \stackrel{?}{=} 16^2+18^2$$
$$676 \stackrel{?}{=} 256+324$$
$$676>580$$

Therefore, by Pythagorean Inequality Theorem, a triangle with the given measures will be an obtuse triangle.

ANSWER:

Yes; obtuse

$$26^2 \stackrel{?}{=} 16^2+18^2$$
$$676>256+324$$

8. 15, 20, 24

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$15+20>24$ ✓

$20+24>15$ ✓

$15+24>20$ ✓

Therefore, the set of numbers can be measures of a triangle.

Now, classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$24^2 \stackrel{?}{=} 15^2+20^2$$
$$576 \stackrel{?}{=} 225+400$$
$$576<625$$

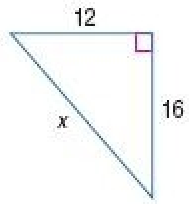
Therefore, by Pythagorean Inequality Theorem, a triangle with the given measures will be an acute triangle.

ANSWER:

Yes; acute

$$24^2 \stackrel{?}{=} 15^2+20^2$$
$$576<225+400$$

Find *x*.

9. 

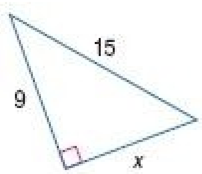
SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is *x* and the lengths of the legs are 12 and 16.

$$\begin{aligned} a^2+b^2&=c^2\\ 12^2+16^2&=x^2\\ 144+256&=x^2\\ 400&=x^2\\ \sqrt{400}&=\sqrt{x^2}\\ 20&=x \end{aligned}$$

ANSWER:

20

10. 

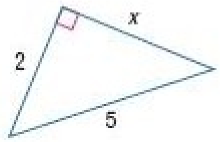
SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is 15 and the lengths of the legs are 9 and *x*.

$$\begin{aligned} a^2+b^2&=c^2\\ x^2+9^2&=15^2\\ x^2+81&=255\\ x^2+81-81&=255-81\\ x^2&=144\\ \sqrt{x^2}&=\sqrt{144}\\ x&=12 \end{aligned}$$

ANSWER:

12

11. 

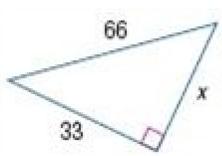
SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is 5 and the lengths of the legs are 2 and *x*.

$$\begin{aligned} a^2+b^2&=c^2 \\ x^2+2^2&=5^2 \\ x^2+4&=25 \\ x^2+4-4&=25-4 \\ x^2&=21 \\ \sqrt{x^2}&=\sqrt{21} \\ x&\approx 4.6 \end{aligned}$$

ANSWER:

$\sqrt{21} \approx 4.6$

12. 

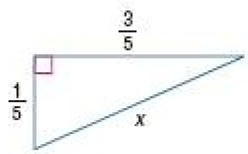
SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is 66 and the lengths of the legs are 33 and *x*.

$$\begin{aligned} a^2+b^2&=c^2 \\ x^2+33^2&=66^2 \\ x^2+1089&=4356 \\ x^2+1089-1089&=4356-1089 \\ x^2&=3267 \\ \sqrt{x^2}&=\sqrt{3267} \\ x&=33\sqrt{3} \\ x&\approx 57.2 \end{aligned}$$

ANSWER:

$33\sqrt{3} \approx 57.2$

13. 

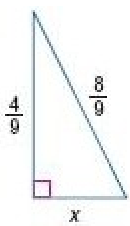
SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is *x* and the lengths of the legs are $\frac{1}{5}$ and $\frac{3}{5}$.

$$a^2+b^2=c^2$$
$$x^2+\left(\frac{1}{5}\right)^2=\left(\frac{3}{5}\right)^2$$
$$x^2+\frac{1}{25}=\frac{9}{25}$$
$$x^2+\frac{1}{25}-\frac{1}{25}=\frac{9}{25}-\frac{1}{25}$$
$$=\sqrt{\frac{8}{25}}$$
$$=\frac{2\sqrt{2}}{5}$$
$$\approx 0.6$$

ANSWER:

$\frac{\sqrt{10}}{5}\approx 0.6$

14. 

SOLUTION:

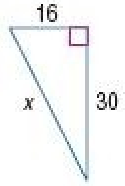
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is $\frac{8}{9}$ and the lengths of the legs are x and $\frac{4}{9}$.

$$\begin{aligned} a^2+b^2&=c^2 \\ x^2+\left(\frac{4}{9}\right)^2&=\left(\frac{8}{9}\right)^2 \\ x^2+\frac{16}{81}&=\frac{64}{81} \\ x^2+\frac{16}{81}-\frac{16}{81}&=\frac{64}{81}-\frac{16}{81} \\ x^2&=\frac{48}{81} \\ x&=\sqrt{\frac{48}{81}} \\ x&=\frac{4\sqrt{3}}{9} \\ x&\approx 0.8 \end{aligned}$$

ANSWER:

$\frac{4\sqrt{3}}{9}\approx 0.8$

CCSS PERSEVERANCE Use a Pythagorean Triple to find *x*.

15. 

SOLUTION:

Find the greatest common factors of 16 and 30.

$16 = 2 \cdot 2 \cdot 2 \cdot 2 = 2 \cdot 8$

$30 = 2 \cdot 3 \cdot 5 = 2 \cdot 15$

The GCF of 16 and 30 is 2. Divide this value out.

$16 \div 2 = 8$

$30 \div 2 = 15$

Check to see if 8 and 15 are part of a Pythagorean triple.

$8^2 + 15^2 = 64 + 225 = 289 = 17^2$

We have one Pythagorean triple 8-15-17. The multiples of this triple also will be Pythagorean triple. So, x = 2(17) = 34.

Use the Pythagorean Theorem to check it.

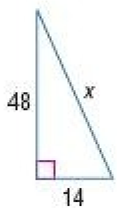
$34^2 \stackrel{?}{=} 16^2 + 30^2$

$1156 \stackrel{?}{=} 256 + 900$

$1156 = 1156 \checkmark$

ANSWER:

34

16. 

SOLUTION:

Find the greatest common factor of 14 and 48.

$14 = 2 \cdot 7$
$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 2 \cdot 24$

The GCF of 14 and 48 is 2. Divide this value out.
$14 \div 2 = 7$
$48 \div 2 = 24$

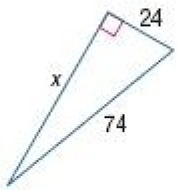
Check to see if 7 and 24 are part of a Pythagorean triple.
$7^2 + 24^2 = 49 + 576 = 625 = 25^2$

We have one Pythagorean triple 7-24-25. The multiples of this triple also will be Pythagorean triple. So, x = 2(25) = 50.
Use the Pythagorean Theorem to check it.

$50^2 \stackrel{?}{=} 14^2 + 48^2$
$2500 \stackrel{?}{=} 196 + 2304$
$2500 = 2500 \checkmark$

ANSWER:

50

17. 

SOLUTION:

74 is the hypotenuse, so it is the greatest value in the Pythagorean triple.

Find the greatest common factor of 74 and 24.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3 = 2 \cdot 12$$
$$74 = 2 \cdot 37$$

The GCF of 24 and 74 is 2. Divide this value out.

$$24 \div 2 = 12$$
$$74 \div 2 = 37$$

Check to see if 12 and 37 are part of a Pythagorean triple with 37 as the largest value.

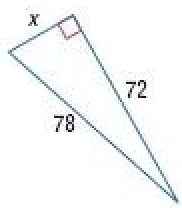
$$37^2 - 12^2 = 1369 - 144 = 1225 = 35^2$$

We have one Pythagorean triple 12-35-37. The multiples of this triple also will be Pythagorean triple. So, x = 2(35) = 70.
Use the Pythagorean Theorem to check it.

$$37^2 \stackrel{?}{=} 35^2 + 12^2$$
$$1369 \stackrel{?}{=} 1225 + 144$$
$$1369 = 1369$$

ANSWER:

70

18. 

SOLUTION:
78 is the hypotenuse, so it is the greatest value in the Pythagorean triple.

Find the greatest common factor of 78 and 72.

$78 = 2 \times 3 \times 13$
$72 = 2 \times 2 \times 2 \times 3 \times 3$

The GCF of 78 and 72 is 6. Divide this value out.

$78 \div 6 = 13$
$72 \div 6 = 12$

Check to see if 13 and 12 are part of a Pythagorean triple with 13 as the largest value.

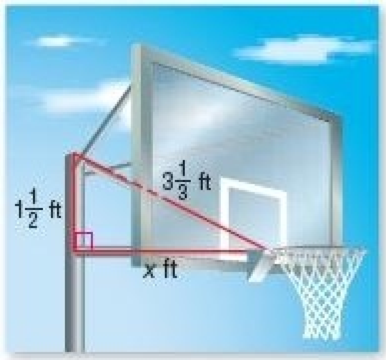
$13^2 - 12^2 = 169 - 144 = 25 = 5^2$

We have one Pythagorean triple 5-12-13. The multiples of this triple also will be Pythagorean triple. So, $x = 6(5) = 30$.
Use the Pythagorean Theorem to check it.

$$78^2 \stackrel{?}{=} 72^2 + 30^2$$
$$6084 \stackrel{?}{=} 5184 + 900$$
$$6084 = 6084 \checkmark$$

ANSWER:
30

19. **BASKETBALL** The support for a basketball goal forms a right triangle as shown. What is the length *x* of the horizontal portion of the support?



SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is $3\frac{1}{3}$ ft and the lengths of the legs are x ft and $1\frac{1}{2}$ ft.

$$a^2 + b^2 = c^2$$
$$x^2 + \left(1\frac{1}{2}\right)^2 = \left(3\frac{1}{3}\right)^2$$
$$x^2 + \left(\frac{3}{2}\right)^2 = \left(\frac{10}{3}\right)^2$$
$$x^2 + \frac{9}{4} = \frac{100}{9}$$
$$x^2 + \frac{9}{4} - \frac{9}{4} = \frac{100}{9} - \frac{9}{4}$$
$$x^2 = \frac{100}{9} \cdot \frac{4}{4} - \frac{9}{4} \cdot \frac{9}{9}$$
$$\sqrt{x^2} = \sqrt{\frac{400}{36} - \frac{81}{36}}$$
$$x = \sqrt{\frac{319}{36}}$$
$$x \approx 3$$

Therefore, the horizontal position of the support is about 3 ft.

ANSWER:

about 3 ft

20. **DRIVING** The street that Khaliah usually uses to get to school is under construction. She has been taking the detour shown. If the construction starts at the point where Khaliah leaves her normal route and ends at the point where she re-enters her normal route, about how long is the stretch of road under construction?

SOLUTION:

Let *x* be the length of the road that is under construction. The road under construction and the detour form a right triangle.

The length of the hypotenuse is *x* and the lengths of the legs are 0.8 and 1.8.

$$a^2 + b^2 = c^2$$
$$0.8^2 + 1.8^2 = x^2$$
$$0.64 + 3.24 = x^2$$
$$3.88 = x^2$$
$$\sqrt{3.88} = \sqrt{x^2}$$
$$2 \approx x$$

Therefore, a stretch of about 2 miles is under construction.

ANSWER:

about 2 mi

Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute, obtuse,* or *right*. Justify your answer.

21. 7, 15, 21

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$7+15>21$ ✓

$15+21>7$ ✓

$7+21>15$ ✓

Therefore, the set of numbers can be measures of a triangle.

Now, classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$21^2 \stackrel{?}{=} 7^2+15^2$$
$$441 \stackrel{?}{=} 49+225$$
$$441>274$$

Therefore, by Pythagorean Inequality Theorem, a triangle with the given measures will be an obtuse triangle.

ANSWER:

Yes; obtuse

$$21^2 \stackrel{?}{=} 7^2+15^2$$
$$441>49+225$$

22. 10, 12, 23

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$10+12>23$ ✗

$12+23>10$ ✓

$10+23>12$ ✓

Since the sum of the lengths of two sides is less than that of the third side, the set of numbers cannot be measures of a triangle.

ANSWER:

No; 23 > 10 + 12

23. 4.5, 20, 20.5

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$4.5 + 20 > 20.5 \checkmark$

$20 + 20.5 > 4.5 \checkmark$

$4.5 + 20.5 > 20 \checkmark$

Therefore, the set of numbers can be measures of a triangle.

Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$20.5^2 \stackrel{?}{=} 4.5^2 + 20^2$$

$$420.25 \stackrel{?}{=} 20.25 + 400$$

$$420.25 = 420.25 \checkmark$$

Therefore, by the converse of Pythagorean Theorem, a triangle with the given measures will be a right triangle.

ANSWER:

Yes; right

$$20.5^2 \stackrel{?}{=} 4.5^2 + 20^2$$

$$420.25 = 20.25 + 400$$

24. 44, 46, 91

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$44 + 46 > 91 \times$

$46 + 91 > 44 \checkmark$

$44 + 91 > 46 \checkmark$

Since the sum of the lengths of two sides is less than that of the third side, the set of numbers cannot be measures of a triangle.

ANSWER:

No; 91 > 44 + 46

25. 4.2, 6.4, 7.6

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$7.6 + 4.2 > 6.4$ ✓

$4.2 + 6.4 > 7.6$ ✓

$7.6 + 6.4 > 4.2$ ✓

Therefore, the set of numbers can be measures of a triangle.

Now, classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$7.6^2 \stackrel{?}{=} 4.2^2 + 6.4^2$

$57.76 \stackrel{?}{=} 17.64 + 40.96$

$57.76 < 58.6$

Therefore, by Pythagorean Inequality Theorem, a triangle with the given measures will be an acute triangle.

ANSWER:

Yes; acute

$7.6^2 \stackrel{?}{=} 4.2^2 + 6.4^2$

$57.76 < 17.64 + 40.96$

26. 4 , 12, 14

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$14 + 4 > 12$ ✓

$4 + 12 > 14$ ✓

$14 + 12 > 4$ ✓

Therefore, the set of numbers can be measures of a triangle.

Now, classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$14^2 \stackrel{?}{=} 4^2 + 12^2$

$196 \stackrel{?}{=} 16 + 144$

$196 > 160$

Therefore, by Pythagorean Inequality Theorem, a triangle with the given measures will be an obtuse triangle.

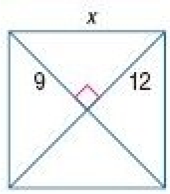
ANSWER:

Yes; obtuse

$14^2 \stackrel{?}{=} 4^2 + 12^2$

$196 > 16 + 144$

Find *x*.

27. 

SOLUTION:

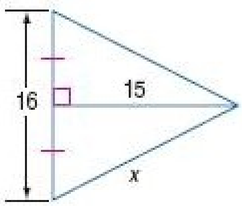
The triangle with the side lengths 9, 12, and *x* form a right triangle.

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is *x* and the lengths of the legs are 9 and 12.

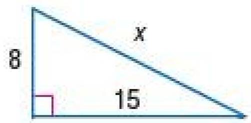
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 9^2 + 12^2 &= x^2 \\ 81 + 144 &= x^2 \\ 225 &= x^2 \\ \sqrt{225} &= \sqrt{x^2} \\ 15 &= x \end{aligned}$$

ANSWER:

15

28. 

SOLUTION:



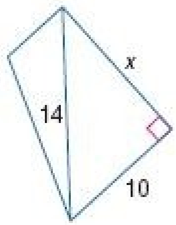
The segment of length 16 units is divided to two congruent segments. So, the length of each segment will be 8 units. Then we have a right triangle with the sides 15, 8, and x.

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is x and the lengths of the legs are 8 and 15.

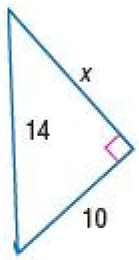
$$\begin{aligned} a^2+b^2&=c^2\\ 8^2+15^2&=x^2\\ 64+225&=x^2\\ 289&=x^2\\ \sqrt{289}&=\sqrt{x^2}\\ 17&=x \end{aligned}$$

ANSWER:

17

29. 

SOLUTION:



We have a right triangle with the sides 14, 10, and x.
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is 14 and the lengths of the legs are 10 and x.

$$\begin{aligned} a^2+b^2&=c^2\\ x^2+10^2&=14^2\\ x^2+100&=196\\ x^2+100-100&=196-100\\ x^2&=96\\ \sqrt{x^2}&=\sqrt{96}\\ x&=4\sqrt{6}\\ &\approx 9.8 \end{aligned}$$

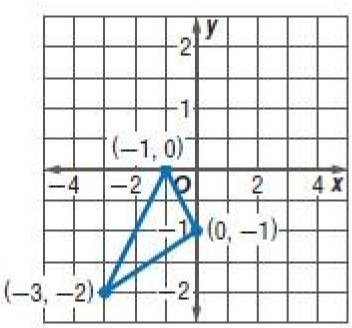
ANSWER:

$4\sqrt{6}\approx 9.8$

COORDINATE GEOMETRY Determine whether ΔXYZ is an *acute, right,* or *obtuse* triangle for the given vertices. Explain.

30. $X(-3, -2)$, $Y(-1, 0)$, $Z(0, -1)$

SOLUTION:



Use the distance formula to find the length of each side of the triangle.

$$XY = \sqrt{(-1-(-3))^2 + (0-(-2))^2} = \sqrt{2^2 + 2^2} = \sqrt{4+5} = \sqrt{8}$$

$$YZ = \sqrt{(0-(-1))^2 + (-1-0)^2} = \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

$$XZ = \sqrt{(0-(-3))^2 + (-1-(-2))^2} = \sqrt{3^2 + 1^2} = = \sqrt{9+1} = \sqrt{10}$$

Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$\left(\sqrt{10}\right)^2 \stackrel{?}{=} \left(\sqrt{8}\right)^2 + \left(\sqrt{2}\right)^2$$

$$10 \stackrel{?}{=} 8 + 2$$

$$10 = 10 \checkmark$$

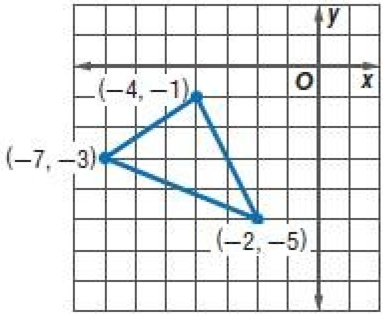
Therefore, by the converse of Pythagorean Theorem, a triangle with the given measures will be a right triangle.

ANSWER:

right; $XY = \sqrt{8}$, $YZ = \sqrt{2}$, $XZ = \sqrt{10}$

31. $X(-7, -3)$, $Y(-2, -5)$, $Z(-4, -1)$

SOLUTION:



Use the distance formula to find the length of each side of the triangle.

$$XY = \sqrt{(-2-(-7))^2 + (-5-(-3))^2} = \sqrt{5^2 + 2^2} = \sqrt{25+4} = \sqrt{29}$$
$$YZ = \sqrt{(-4-(-2))^2 + (-1-(-5))^2} = \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20}$$
$$XZ = \sqrt{(-4-(-7))^2 + (-1-(-3))^2} = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$

Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$\left(\sqrt{29}\right)^2 \stackrel{?}{=} \left(\sqrt{20}\right)^2 + \left(\sqrt{13}\right)^2$$
$$29 \stackrel{?}{=} 20 + 13$$
$$29 < 33$$

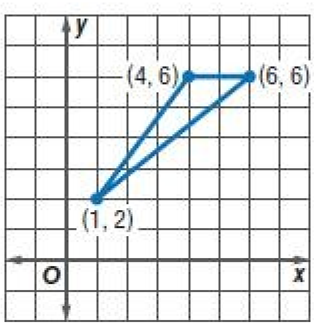
Therefore, by the Pythagorean Inequality Theorem, a triangle with the given measures will be an acute triangle.

ANSWER:

acute; $XY = \sqrt{29}$, $YZ = \sqrt{20}$, $XZ = \sqrt{13}$; $\left(\sqrt{29}\right)^2 < \left(\sqrt{20}\right)^2 + \left(\sqrt{13}\right)^2$

32. $X(1, 2)$, $Y(4, 6)$, $Z(6, 6)$

SOLUTION:



Use the distance formula to find the length of each side of the triangle.

$$XY = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$
$$YZ = \sqrt{(6-4)^2 + (6-6)^2} = \sqrt{2^2 + 0^2} = \sqrt{4} = 2$$
$$XZ = \sqrt{(6-1)^2 + (6-2)^2} = \sqrt{5^2 + 4^2} = \sqrt{25+16} = \sqrt{41}$$

Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$\left(\sqrt{41}\right)^2 \stackrel{?}{=} (5)^2 + (2)^2$$
$$41 \stackrel{?}{=} 25 + 4$$
$$41 > 29$$

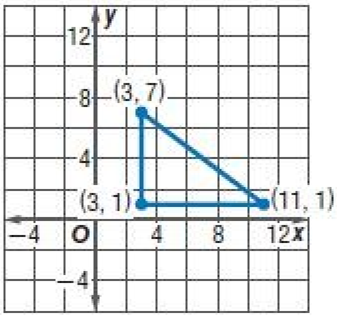
Therefore, by the Pythagorean Inequality Theorem, a triangle with the given measures will be an obtuse triangle.

ANSWER:

obtuse; $XY = 5$, $YZ = 2$, $XZ = \sqrt{41}$; $\left(\sqrt{41}\right)^2 > 5^2 + 2^2$

33. $X(3, 1)$, $Y(3, 7)$, $Z(11, 1)$

SOLUTION:



Use the distance formula to find the length of each side of the triangle.

$$XY = \sqrt{(3-3)^2 + (7-1)^2} = \sqrt{0^2 + 6^2} = \sqrt{36} = 6$$
$$YZ = \sqrt{(11-3)^2 + (1-7)^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10$$
$$XZ = \sqrt{(11-3)^2 + (1-1)^2} = \sqrt{8^2 + 0^2} = \sqrt{64} = 8$$

Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$$(10)^2 \stackrel{?}{=} (8)^2 + (6)^2$$
$$100 \stackrel{?}{=} 64 + 36$$
$$100 = 100 \checkmark$$

Therefore, by the converse of Pythagorean Theorem, a triangle with the given measures will be a right triangle.

ANSWER:

right; $XY = 6$, $YZ = 10$, $XZ = 8$; $6^2 + 8^2 = 10^2$

34. **JOGGING** Brett jogs in the park three times a week. Usually, he takes a $\frac{3}{4}$-mile path that cuts through the park. Today, the path is closed, so he is taking the orange route shown. How much farther will he jog on his alternate route than he would have if he had followed his normal path?

SOLUTION:

The normal jogging route and the detour form a right triangle. One leg of the right triangle is 0.45 mi. and let *x* be the other leg. The hypotenuse of the right triangle is $\frac{3}{4}$ mi. In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$
$$x^2 + 0.45^2 = \left(\frac{3}{4}\right)^2$$
$$x^2 + 0.2025 = 0.5625$$
$$x^2 + 0.2025 - 0.2025 = 0.5625 - 0.2025$$
$$x^2 = 0.36$$
$$\sqrt{x^2} = \sqrt{0.36}$$
$$x = 0.6$$

So, the total distance that he runs in the alternate route is 0.45 + 0. 6 = 1.05 mi. instead of his normal distance 0.75 mi.

Therefore, he will be jogging an extra distance of 0.3 miles in his alternate route.

ANSWER:

0.3 mi

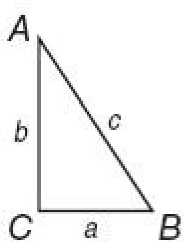
8-2 The Pythagorean Theorem and Its Converse

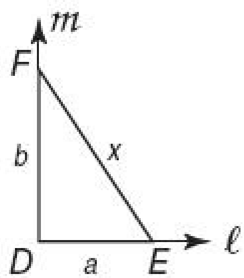
35. **PROOF** Write a paragraph proof of Theorem 8.5.

SOLUTION:

Theorem 8.5 states that if the sum of the squares of the lengths of the shortest sides of a triangle are equal to the square of the length of the longest side, then the triangle is a right triangle. Use the hypothesis to create a given statement, namely that $c^2 = a^2 + b^2$
for triangle ABC. You can accomplish this proof by constructing another triangle (Triangle DEF) that is congruent to triangle ABC, using SSS triangle congruence theorem. Show that triangle DEF is a right triangle, using the Pythagorean theorem. therefore, any triangle congruent to a right triangle but also be a right triangle.

Given: ΔABC with sides of measure a, b, and c, where $c^2 = a^2 + b^2$
Prove: ΔABC is a right triangle.

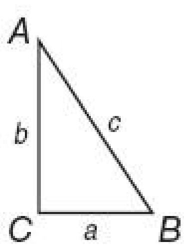


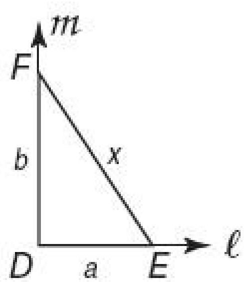


Proof: Draw $\overline{DE}$ on line ℓ with measure equal to a. At D, draw line $m \perp \overline{DE}$. Locate point F on m so that $DF = b$. Draw $\overline{FE}$ and call its measure x. Because ΔFED is a right triangle, $a^2 + b^2 = x^2$. But $a^2 + b^2 = c^2$, so $x^2 = c^2$ or $x =$ c. Thus, $\Delta ABC \cong \Delta FED$ by SSS. This means $\angle C \cong \angle D$. Therefore, $\angle C$ must be a right angle, making ΔABC a right triangle.

ANSWER:

Given: ΔABC with sides of measure a, b, and c, where $c^2 = a^2 + b^2$
Prove: ΔABC is a right triangle.





Proof: Draw $\overline{DE}$ on line ℓ with measure equal to a. At D, draw line $m \perp \overline{DE}$. Locate point F on m so that $DF = b$. Draw $\overline{FE}$ and call its measure x. Because ΔFED is a right triangle, $a^2 + b^2 = x^2$. But $a^2 + b^2 = c^2$, so $x^2 = c^2$ or $x =$ c. Thus, $\Delta ABC \cong \Delta FED$ by SSS. This means $\angle C \cong \angle D$. Therefore, $\angle C$ must be a right angle, making ΔABC a right triangle.

PROOF Write a two-column proof for each theorem.

36. Theorem 8.6

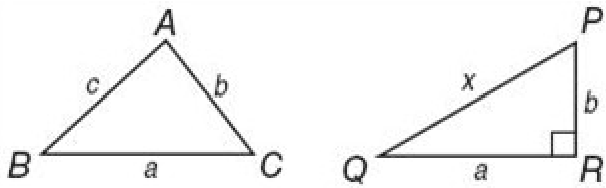
8-2 The Pythagorean Theorem and Its Converse

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two triangles, a right angle, relationship between sides. Use the properties that you have learned about right angles, acute angles, Pythagorean Theorem,angle relationships and equivalent expressions in algebra to walk through the proof.

Given: In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side. In ΔPQR , $\angle R$ is a right angle.

Prove: ΔABC is an acute triangle.



Proof:

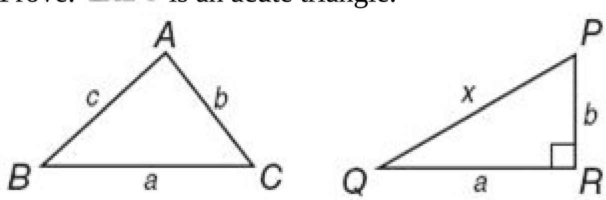
Statements (Reasons)

1. In $\triangle ABC$, $c^2 < a^2 + b^2$ where c is the length of the longest side. In ΔPQR , $\angle R$ is a right angle. (Given)
2. $a^2 + b^2 = x^2$ (Pythagorean Theorem)
3. $c^2 < x^2$ (Substitution Property)
4. $c < x$ (A property of square roots)
5. $m\angle R = 90°$ (Definition of a right angle)
6. $m\angle C < m\angle R$ (Converse of the Hinge Theorem)
7. $m\angle C < 90°$ (Substitution Property)
8. $\angle C$ is an acute angle. (Definition of an acute angle)
9. $\triangle ABC$ is an acute triangle. (Definition of an acute triangle)

ANSWER:

Given: In ΔABC , $c^2 < a^2 + b^2$ where c is the length of the longest side. In ΔPQR , $\angle R$ is a right angle.

Prove: ΔABC is an acute triangle.



Proof:

Statements (Reasons)

1. In ΔABC , $c^2 < a^2 + b^2$ where c is the length of the longest side. In ΔPQR , $\angle R$ is a right angle. (Given)
2. $a^2 + b^2 = x^2$ (Pythagorean Theorem)
3. $c^2 < x^2$ (Substitution Property)
4. $c < x$ (A property of square roots)

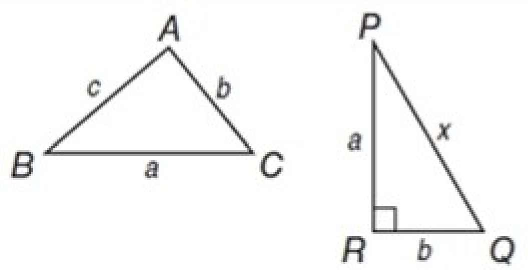
5. $m\angle R = 90°$ (Definition of a right angle)
6. $m\angle C < m\angle R$ (Converse of the Hinge Theorem)
7. $m\angle C < 90°$ (Substitution Property)
8. $\angle C$ is an acute angle. (Definition of an acute angle)
9. ΔABC is an acute triangle. (Definition of an acute triangle)

37. Theorem 8.7

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given two triangles, relationship between angles.Use the properties that you have learned about triangles, angles and equivalent expressions in algebra to walk through the proof.

Given: In ΔABC, $c^2 > a^2 + b^2$, where c is the length of the longest side.
Prove: ΔABC is an obtuse triangle.

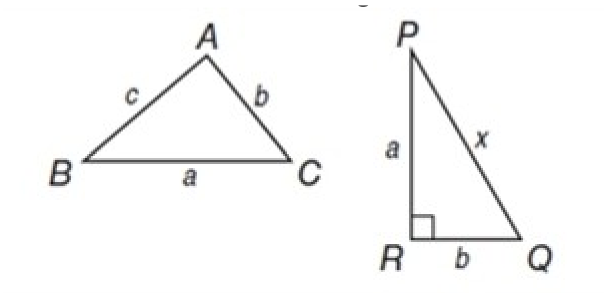


Statements (Reasons)

1. In ΔABC, $c^2 > a^2 + b^2$, where c is the length of the longest side. In ΔPQR, $\angle R$ is a right angle. (Given)
2. $a^2 + b^2 = x^2$ (Pythagorean Theorem)
3. $c^2 > x^2$ (Substitution Property)
4. $c > x$ (A property of square roots)
5. $m\angle R = 90°$ (Definition of a right angle)
6. $m\angle C > m\angle R$ (Converse of the Hinge Theorem)
7. $m\angle C > 90°$ (Substitution Property of Equality)
8. $\angle C$ is an obtuse angle. (Definition of an obtuse angle)
9. ΔABC is an obtuse triangle. (Definition of an obtuse triangle)

ANSWER:

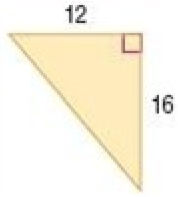
Given: In ΔABC, $c^2 > a^2 + b^2$, where c is the length of the longest side.
Prove: ΔABC is an obtuse triangle.



Statements (Reasons)

1. In ΔABC, $c^2 > a^2 + b^2$, where c is the length of the longest side. In ΔPQR, $\angle R$ is a right angle. (Given)
2. $a^2 + b^2 = x^2$ (Pythagorean Theorem)
3. $c^2 > x^2$ (Substitution Property)
4. $c > x$ (A property of square roots)
5. $m\angle R = 90°$ (Definition of a right angle)
6. $m\angle C > m\angle R$ (Converse of the Hinge Theorem)
7. $m\angle C > 90°$ (Substitution Property of Equality)
8. $\angle C$ is an obtuse angle. (Definition of an obtuse angle)
9. ΔABC is an obtuse triangle. (Definition of an obtuse triangle)

CCSS SENSE-MAKING Find the perimeter and area of each figure.

38. 

SOLUTION:

The area of a triangle is given by the formula

$A = \frac{1}{2}bh$ where *b* is the base and *h* is the height of the triangle. Since the triangle is a right triangle the base and the height are the legs of the triangle. So,

$A = \frac{1}{2}(12)(16) = 96$ sq. unit.

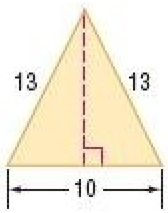
The perimeter is the sum of the lengths of the three sides. Use the Pythagorean Theorem to find the length of the hypotenuse of the triangle.
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 12^2 + 16^2 &= c^2 \\ 144 + 256 &= c^2 \\ 400 &= c^2 \\ \sqrt{400} &= \sqrt{c^2} \\ 20 &= c \end{aligned}$$

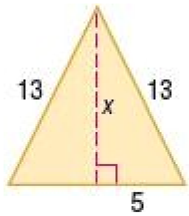
The length of the hypotenuse is 20 units. Therefore, the perimeter is 12 + 16 + 20 = 48 units.

ANSWER:

P = 48 units; *A* = 96 units^2

39. 

SOLUTION:



The area of a triangle is given by the formula

$A = \frac{1}{2}bh$ where b is the base and h is the height of the triangle. The altitude to the base of an isosceles triangle bisects the base. So, we have two right triangles with one of the legs equal to 5 units and the hypotenuse is 13 units each. Use the Pythagorean Theorem to find the length of the common leg of the triangles.

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 + 5^2 &= 13^2 \\ x^2 + 25 &= 169 \\ x^2 + 25 - 25 &= 169 - 25 \\ x^2 &= 144 \\ \sqrt{x^2} &= \sqrt{144} \\ x &= 12 \end{aligned}$$

The altitude is 12 units.

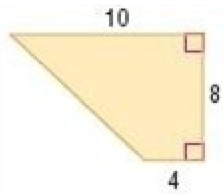
Therefore,

$A = \frac{1}{2}(12)(10) = 60$ sq. unit.

The perimeter is the sum of the lengths of the three sides. Therefore, the perimeter is 13 + 13 + 10 = 36 units.

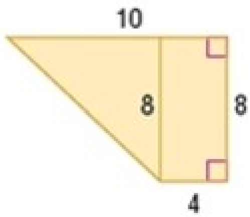
ANSWER:

P = 36 units; A = 60 units^2

40. 

SOLUTION:

The given figure can be divided as a right triangle and a rectangle as shown.



The total are of the figure is the sum of the areas of the right triangle and the rectangle.

The area of a triangle is given by the formula

$A = \frac{1}{2}bh$ where *b* is the base and *h* is the height of the triangle. Since the triangle is a right triangle the base and the height are the legs of the triangle. So,

$A_{triangle} = \frac{1}{2}(6)(8) = 24$ sq. unit.

The area of a rectangle of length *l* and width *w* is given by the formula $A = l \times w$. So,

$A_{rec} = (4)(8) = 32$ sq. units.

Therefore, the total area is 24 + 32 = 56 sq. units.

The perimeter is the sum of the lengths of the four boundaries. Use the Pythagorean Theorem to find the length of the hypotenuse of the triangle.

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = c^2$$
$$36 + 64 = c^2$$
$$100 = c^2$$
$$\sqrt{100} = \sqrt{c^2}$$
$$10 = c$$

The hypotenuse is 10 units.

Therefore, the perimeter is 4 + 8 + 10 + 10 = 32 units

ANSWER:

P = 32 units; *A* = 56 units^2

41. **ALGEBRA** The sides of a triangle have lengths x, $x + 5$, and 25. If the length of the longest side is 25, what value of x makes the triangle a right triangle?

SOLUTION:

By the converse of the Pythagorean Theorem, if the square of the longest side of a triangle is the sum of squares of the other two sides then the triangle is a right triangle.

$$a^2 + b^2 = c^2$$
$$x^2 + (x+5)^2 = 25^2$$
$$x^2 + x^2 + 10x + 25 = 625$$
$$2x^2 + 10x + 25 = 625$$
$$2x^2 + 10x + 25 - 625 = 625 - 625$$
$$2x^2 + 10x - 600 = 0$$
$$2\left(x^2 + 5x - 300\right) = 0$$
$$x^2 + 5x - 300 = 0$$

Use the Quadratic Formula to find the roots of the equation.

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-300)}}{2(1)}$$
$$= \frac{-5 \pm 35}{2}$$
$$= -20, 15$$

Since x is a length, it cannot be negative. Therefore, $x = 15$.

ANSWER:

15

42. **ALGEBRA** The sides of a triangle have lengths $2x$, 8, and 12. If the length of the longest side is $2x$, what values of x make the triangle acute?

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$$8+12>2x \Rightarrow 20>2x \Rightarrow x<10$$

$$2x+12>8 \Rightarrow 2x>-4 \Rightarrow x>-2$$

$$2x+8>12 \Rightarrow 2x>4 \Rightarrow x>2$$

So, the value of x should be between 2 and 10.

By the Pythagorean Inequality Theorem, if the square of the longest side of a triangle is less than the sum of squares of the other two sides then the triangle is an acute triangle.

$$(2x)^2<8^2+12^2$$

$$4x^2<64+144$$

$$4x^2<208$$

$$\frac{4x^2}{4}<\frac{208}{4}$$

$$x^2<52$$

$$\sqrt{x^2}<\sqrt{52}$$

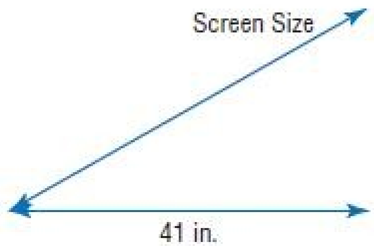
$$x<2\sqrt{13}$$

Therefore, for the triangle to be acute, $2<x<2\sqrt{13}$.

ANSWER:

$$6<x<2\sqrt{13}$$

43. **TELEVISION** The screen aspect ratio, or the ratio of the width to the length, of a high-definition television is 16:9. The size of a television is given by the diagonal distance across the screen. If an HDTV is 41 inches wide, what is its screen size?
Refer to the photo on page 547.



SOLUTION:

Use the ratio to find the length of the television.
Let x be the length of the television. Then,

$$\frac{\text{width}}{\text{height}} = \frac{\text{width}}{\text{height}}$$

$$\frac{16}{9} = \frac{41}{x}.$$

Solve the proportion for x.

$$\frac{16}{9} = \frac{41}{x}$$
$$16x = 369$$
$$\frac{16x}{16} = \frac{369}{16}$$
$$x \approx 23$$

The two adjacent sides and the diagonal of the television form a right triangle.
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

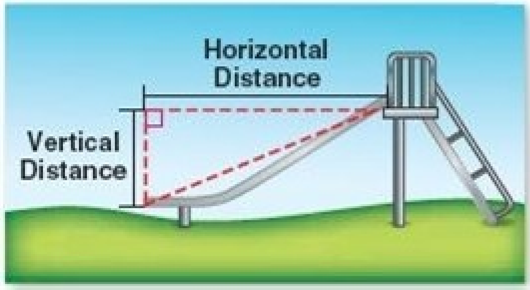
$$a^2 + b^2 = c^2$$
$$(41)^2 + (23)^2 = c^2$$
$$1681 + 529 = c^2$$
$$2210 = c^2$$
$$\sqrt{2210} = \sqrt{c^2}$$
$$47 \approx c$$

Therefore, the screen size is about 47 inches.

ANSWER:

47 in.

44. **PLAYGROUND** According to the Handbook for Public Playground Safety, the ratio of the vertical distance to the horizontal distance covered by a slide should not be more than about 4 to 7. If the horizontal distance allotted in a slide design is 14 feet, approximately how long should the slide be?



SOLUTION:

Use the ratio to find the vertical distance.
Let *x* be the vertical distance. Then,

$$\frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{\text{vertical distance}}{\text{horizontal distance}}$$

$$\frac{4}{7} = \frac{x}{14}.$$

Solve the proportion for *x*.

$$\frac{4}{7} = \frac{x}{14}$$
$$7x = 56$$
$$\frac{7x}{7} = \frac{56}{7}$$
$$x = 8$$

The vertical distance, the horizontal distance, and the slide form a right triangle.
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

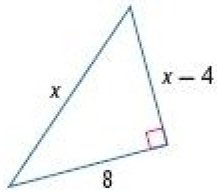
$$a^2 + b^2 = c^2$$
$$(14)^2 + (8)^2 = c^2$$
$$196 + 64 =$$
$$260 = c^2$$
$$\sqrt{260} = \sqrt{c^2}$$
$$16 \approx c$$

Therefore, the slide will be about 16 ft long.

ANSWER:

about 16 ft

Find *x*.

45. 

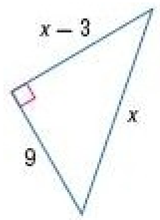
SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is x and the lengths of the legs are $x – 4$ and 8.
Solve for x.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (x-4)^2 + 8^2 &= x^2 \\ x^2 - 8x + 16 + 64 &= x^2 \\ x^2 - x^2 - 8x + 16 + 64 &= x^2 - x^2 \\ -8x + 80 &= 0 \\ -8x + 80 - 80 &= -80 \\ -8x &= -80 \\ \frac{-8x}{-8} &= \frac{-80}{-8} \\ x &= 10 \end{aligned}$$

ANSWER:

10

46. 

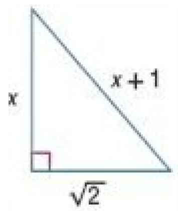
SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is *x* and the lengths of the legs are *x* – 3 and 9.
Solve for *x*.

$$\begin{aligned} a^2+b^2&=c^2\\ x-3^2+9^2&=x^2\\ x^2-6x+9+81&=x^2\\ x^2-6x+90&=x^2\\ x^2-x^2-6x+90&=x^2-x^2\\ -6x+90&=0\\ -6x+90-90&=-90\\ -6x&=-90\\ \frac{-6x}{-6}&=\frac{-90}{-6}\\ x&=15 \end{aligned}$$

ANSWER:

15

47. 

SOLUTION:

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. The length of the hypotenuse is $x + 1$ and the lengths of the legs are x and $\sqrt{2}$.
Solve for x.

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
x^2 + \left(\sqrt{2}\right)^2 &= (x+1)^2 \\
x^2 + 2 &= x^2 + 2x + 1 \\
x^2 - x^2 + 2 &= x^2 - x^2 + 2x + 1 \\
2 &= 2x + 1 \\
2 - 1 &= 2x + 1 - 1 \\
1 &= 2x \\
\frac{1}{2} &= \frac{2x}{2} \\
\frac{1}{2} &= x
\end{aligned}
$$

ANSWER:

$\mathbf{\frac{1}{2}}$

48. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate special right triangles.

a. GEOMETRIC Draw three different isosceles right triangles that have whole-number side lengths. Label the triangles *ABC*, *MNP*, and *XYZ* with the right angle located at vertex *A*, *M*, and *X*, respectively. Label the leg lengths of each side and find the length of the hypotenuse in simplest radical form.

b. TABULAR Copy and complete the table below.

Triangle	Length				Ratio	
ABC	*BC*		*AB*		$\frac{BC}{AB}$	
MNP	*NP*		*MN*		$\frac{NP}{MN}$	
XYZ	*YZ*		*XY*		$\frac{YZ}{XY}$	

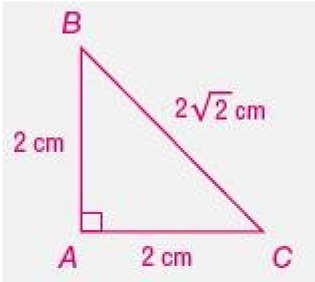
c. VERBAL Make a conjecture about the ratio of the hypotenuse to a leg of an isosceles right triangle.

SOLUTION:

a. Let $AB = AC$. Use the Pythagorean Theorem to find BC.

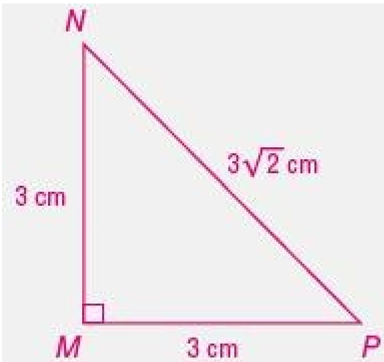
Sample answer:

$$AB^2 + AC^2 = BC^2$$
$$2^2 + 2^2 = BC^2$$
$$4 + 4 = BC^2$$
$$8 = BC^2$$
$$BC = \sqrt{8} = 2\sqrt{2}$$



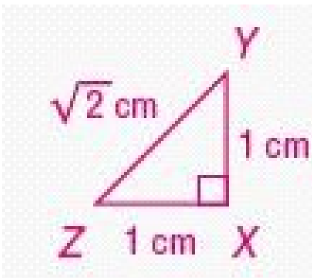
Let $MN = PM$. Use the Pythagorean Theorem to find NP.

$$MN^2 + PM^2 = NP^2$$
$$3^2 + 3^2 = NP^2$$
$$9 + 9 = NP^2$$
$$18 = NP^2$$
$$NP = \sqrt{18} = 3\sqrt{2}$$



Let $ZX = XY$. Use the Pythagorean Theorem to find ZY.

$$ZX^2 + XY^2 = ZY^2$$
$$1^2 + 1^2 = ZY^2$$
$$1 + 1 = ZY^2$$
$$2 = ZY^2$$
$$ZY = \sqrt{2}$$



b. Complete the table below with the side lengths calculated for each triangle. Then, compute the given ratios.

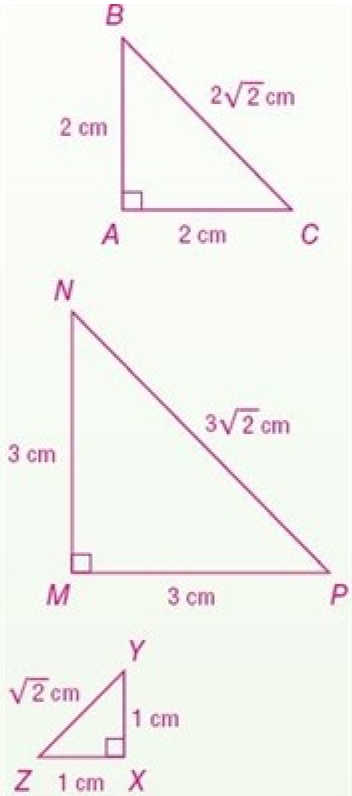
Triangle	Length				Ratio	
ABC	BC	$2\sqrt{2}$	AB	2	$\frac{BC}{AB}$	$\sqrt{2}$
MNP	NP	$3\sqrt{2}$	MN	3	$\frac{NP}{MN}$	$\sqrt{2}$
XYZ	YZ	$\sqrt{2}$	XY	1	$\frac{YZ}{XY}$	$\sqrt{2}$

c. Summarize any observations made based on the patterns in the table.

Sample answer: The ratio of the hypotenuse to a leg of an isosceles right triangle is $\sqrt{2}$.

ANSWER:

a.

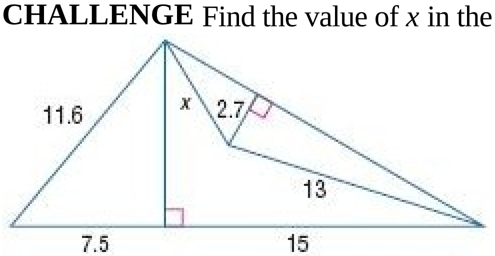


b.

Triangle	Length				Ratio	
ABC	BC	$2\sqrt{2}$	AB	2	$\frac{BC}{AB}$	$\sqrt{2}$
MNP	NP	$3\sqrt{2}$	MN	3	$\frac{NP}{MN}$	$\sqrt{2}$
XYZ	YZ	$\sqrt{2}$	XY	1	$\frac{YZ}{XY}$	$\sqrt{2}$

c. Sample answer: The ratio of the hypotenuse to a leg of an isosceles right triangle is $\sqrt{2}$.

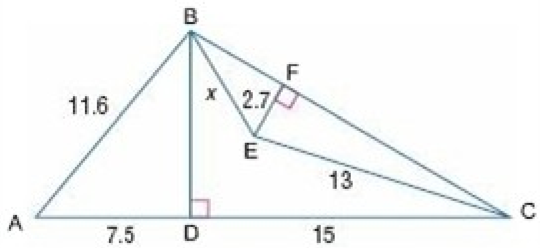
49. **CHALLENGE** Find the value of x in the figure.



SOLUTION:

Label the vertices to make the sides easier to identify. Find each side length, one at a time, until you get to the value of x.

By the Pythagorean Theorem, in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. Use the theorem to find the lengths of the required segments.



$$AD^2 + DB^2 = AB^2$$
$$7.5^2 + DB^2 = 11.6^2$$
$$56.25 + DB^2 = 134.56$$
$$DB^2 = 78.31$$
$$DB = \sqrt{78.31}$$

BC is the hypotenuse of the right triangle BDC.

$$BD^2 + DC^2 = BC^2$$
$$\sqrt{78.31}^2 + 15^2 = BC^2$$
$$78.31 + 225 = BC^2$$
$$BC^2 = 303.31$$
$$BC = \sqrt{303.31} \approx 17.4$$

Find FC.

$$EF^2 + FC^2 = EC^2$$
$$2.7^2 + FC^2 = 13^2$$
$$7.29 + FC^2 = 169$$
$$FC^2 = 161.71$$
$$FC = \sqrt{161.71} \approx 12.7$$

We have $BF = BC - FC = 17.4 - 12.7 = 4.7$.

Use the value of BF to find x in the right triangle BEF.

$$BF^2 + EF^2 = x^2$$
$$4.7^2 + 2.7^2 = x^2$$
$$22.09 + 7.29 = x^2$$
$$x^2 = 29.38$$
$$x = \sqrt{29.38} \approx 5.4$$

ANSWER:

5.4

50. **CCSS ARGUMENTS** *True* or *false*? Any two right triangles with the same hypotenuse have the same area. Explain your reasoning.

SOLUTION:

A good approach to this problem is to come up with a couple different sets of three numbers that form a right triangle, but each have a hypotenuse of 5. You could pick a leg length (less that 5) and then solve for the other leg. Leave any non-integers in radical form, for accuracy.

For example:

$$\begin{aligned} 2^2 + x^2 &= 5^2 \\ 4 + x^2 &= 25 \\ x^2 &= 21 \\ x &= \sqrt{21} \end{aligned}$$

Then, find the area of a right triangle with legs of 2 and $\sqrt{2}$.

$$\begin{aligned} A &= \frac{1}{2}\left(\sqrt{21}\right)(2) \\ A &= \sqrt{21} \end{aligned}$$

$$\begin{aligned} 3^2 + x^2 &= 5^2 \\ 9 + x^2 &= 25 \\ x^2 &= 16 \\ x &= 4 \end{aligned}$$

Then, find the area of a right triangle with legs of 3 and 4.

$$\begin{aligned} A &= \frac{1}{2}(4)(3) \\ A &= 6 \end{aligned}$$

Since these areas are not equivalent, the statement is False.

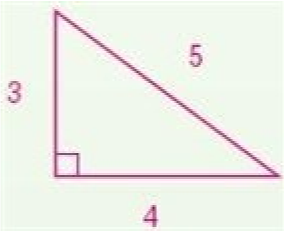
ANSWER:

False; sample answer: A right triangle with legs measuring 3 in. and 4 in. has a hypotenuse of 5 in. and an area of $\frac{1}{2} \times 3 \times 4$

or 6 in^2. A right triangle with legs measuring 2 in. and $\sqrt{21}$ in. also has a hypotenuse of 5 in., but its area is $\frac{1}{2} \times 2 \times \sqrt{21}$ or $\sqrt{21}$ in^2, which is not equivalent to 6 in^2.

51. **OPEN ENDED** Draw a right triangle with side lengths that form a Pythagorean triple. If you double the length of each side, is the resulting triangle acute, right, or obtuse? if you halve the length of each side? Explain.

SOLUTION:



If you double the side lengths, you get 6, 8, and 10. Determine the type of triangle by using the Converse of the Pythagorean Theorem:

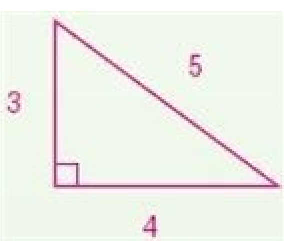
$$6^2 + 8^2 \stackrel{?}{=} 10^2$$
$$36 + 64 \stackrel{?}{=} 100$$
$$100 = 100$$

If you halve the side lengths, you get 1.5, 2, and 2.5. Determine the type of triangle by using the Converse of the Pythagorean Theorem:

$$1.5^2 + 2^2 \stackrel{?}{=} 2.5^2$$
$$2.25 + 4 \stackrel{?}{=} 6.25$$
$$6.25 = 6.25$$

Right; sample answer: If you double or halve the side lengths, all three sides of the new triangles are proportional to the sides of the original triangle. Using the Side- Side-Side Similarity Theorem, you know that both of the new triangles are similar to the original triangle, so they are both right.

ANSWER:

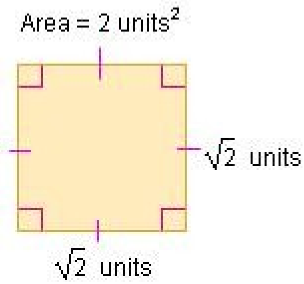


Right; sample answer: If you double or halve the side lengths, all three sides of the new triangles are proportional to the sides of the original triangle. Using the Side- Side-Side Similarity Theorem, you know that both of the new triangles are similar to the original triangle, so they are both right.

52. **WRITING IN MATH** Research *incommensurable magnitudes,* and describe how this phrase relates to the use of irrational numbers in geometry. Include one example of an irrational number used in geometry.

SOLUTION:

Sample answer: Incommensurable magnitudes are magnitudes of the same kind that do not have a common unit of measure. Irrational numbers were invented to describe geometric relationships, such as ratios of incommensurable magnitudes that cannot be described using rational numbers. For example, to express the measures of the sides of a square with an area of 2 square units, the irrational number $\sqrt{2}$ is needed.



ANSWER:

Sample answer: Incommensurable magnitudes are magnitudes of the same kind that do not have a common unit of measure. Irrational numbers were invented to describe geometric relationships, such as ratios of incommensurable magnitudes that cannot be described using rational numbers. For example, to express the measures of the sides of a square with an area of 2 square units, the irrational number $\sqrt{2}$ is needed.

53. Which set of numbers cannot be the measures of the sides of a triangle?

A 10, 11, 20
B 14, 16, 28
C 35, 45, 75
D 41, 55, 98

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$10+11>20$ ✓	$14+16>28$ ✓	$35+45>75$ ✓	$41+55>98$ ✗
$11+20>10$ ✓	$16+28>14$ ✓	$45+75>35$ ✓	$55+98>41$ ✓
$10+20>11$ ✓	$14+28>16$ ✓	$35+75>45$ ✓	$41+98>55$ ✓

In option D, the sum of the lengths of two sides is less than that of the third side, the set of numbers cannot be measures of a triangle.
Therefore, the correct choice is D.

ANSWER:

D

54. A square park has a diagonal walkway from one corner to another. If the walkway is 120 meters long, what is the approximate length of each side of the park?
F 60 m
G 85 m
H 170 m
J 240 m

SOLUTION:

Let *x* be the length of each side of the square. The diagonal of a square divides the square into two congruent isosceles right triangles.

Use Pythagorean Theorem to solve for *x*:

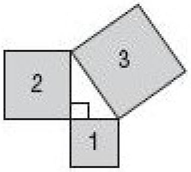
$$x^2 + x^2 = 120^2$$
$$2x^2 = 14400$$
$$x^2 = 7200$$
$$x = \sqrt{7200}$$
$$x \approx 84.9$$

Therefore, the correct choice is G.

ANSWER:

G

55. **SHORT RESPONSE** If the perimeter of square 2 is 200 units and the perimeter of square 1 is 150 units, what is the perimeter of square 3?



SOLUTION:

The length of each side of square 2 is $\frac{200}{4} = 50$ units and that of square 1 is $\frac{150}{4} = 37.5$ units.

The side lengths of the squares 1 and 2 are the lengths of the legs of the right triangle formed. Use the Pythagorean Theorem to find the length of the hypotenuse.

$$50^2 + 37.5^2 = hypotenuse^2$$
$$2500 + 1406.25 = hypotenuse^2$$
$$3906.25 = hypotenuse^2$$
$$62.5 \approx hypotenuse$$

Therefore, the perimeter of the square 3 is 4(62.5) = 250 units.

ANSWER:

250 units

56. **SAT/ACT** In ΔABC, $\angle B$ is a right angle and $\angle A$ is $20°$ greater than $\angle C$. What is the measure of $\angle C$?
A 30
B 35
C 40
D 45
E 70

SOLUTION:

Let x be the measure $\angle C$. Then $m\angle A = x + 20$. The sum of the three angles of a triangle is 180.

$$m\angle A + m\angle B + m\angle C = 180$$
$$(x + 20) + 90 + x = 180$$
$$2x + 110 = 180$$
$$2x = 70$$
$$x = 35$$

Therefore, the correct choice is B.

ANSWER:

B

Find the geometric mean between each pair of numbers.

57. 9 and 4

SOLUTION:

By the definition, the geometric mean *x* of any two numbers *a* and *b* is given by $x = \sqrt{ab}$.
Therefore, the geometric mean of 9 and 4 is
$\sqrt{(9)(4)} = \sqrt{36} = 6.$

ANSWER:

6

58. 45 and 5

SOLUTION:

By the definition, the geometric mean *x* of any two numbers *a* and *b* is given by $x = \sqrt{ab}$.
Therefore, the geometric mean of 45 and 5 is
$\sqrt{(45)(5)} = \sqrt{225} = 15.$

ANSWER:

15

59. 12 and 15

SOLUTION:

By the definition, the geometric mean *x* of any two numbers *a* and *b* is given by $x = \sqrt{ab}$.
Therefore, the geometric mean of 12 and 15 is
$\sqrt{(12)(15)} = \sqrt{180} \approx 13.4.$

ANSWER:

$6\sqrt{5} \approx 13.4$

60. 36 and 48

SOLUTION:

By the definition, the geometric mean *x* of any two numbers *a* and *b* is given by $x = \sqrt{ab}$.
Therefore, the geometric mean of 36 and 48 is
$\sqrt{(36)(48)} = \sqrt{1728} \approx 41.6.$

ANSWER:

$24\sqrt{3} \approx 41.6$

61. **SCALE DRAWING** Teodoro is creating a scale model of a skateboarding ramp on a 10-by-8-inch sheet of graph paper. If the real ramp is going to be 12 feet by 8 feet, find an appropriate scale for the drawing and determine the ramp's dimensions.

SOLUTION:

The real ramp is 12 feet by 8 feet. We need a reduced version of this ratio to fit on a 10 by 8 inch paper. To begin, consider the ratio of the sides: $\frac{height\,of\,real\,ramp}{width\,of\,real\,ramp}=\frac{12}{8}$.

Now divide each dimension by the same scale factor to create dimensions of a similar figure:

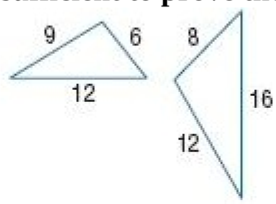
$$\frac{12\div 2}{8\div 2}=\frac{6}{4}$$

Since 6<10 and 4<8, these dimensions will fit on a 10 by 8 inch paper. Therefore, the dimensions of the model will be $6\text{ in.}\times 4\text{ in.}$ and, since we divided the original dimensions by 2, the scale for the drawing will be 1 in = 2 ft.

ANSWER:

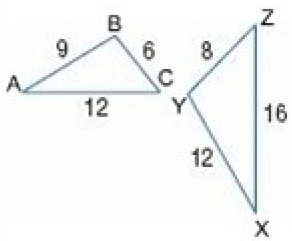
1 in. = 2 ft; $6\text{in.}\times 4\text{in.}$

Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning.

62. 

SOLUTION:

Find the ratio of the lengths of the corresponding sides (short with short, medium with medium, and long with long).

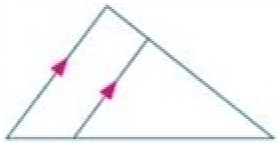


$\frac{AB}{XY} = \frac{9}{12} = \frac{3}{4}$
$\frac{BC}{YZ} = \frac{6}{8} = \frac{3}{4}$
$\frac{AC}{XZ} = \frac{12}{16} = \frac{3}{4}$.

Since $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} = \frac{3}{4}$, then $\Delta ABC \sim \Delta XYZ$ by SSS Similarity.

ANSWER:

Yes; SSS

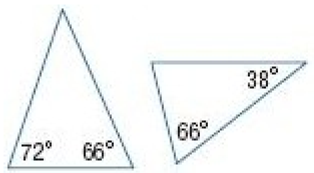
63. 

SOLUTION:

The corresponding angles formed when two parallel lines are cut by a transversal are congruent. So, the two triangles are similar by AA Similarity.

ANSWER:

Yes; AA

64. 

SOLUTION:

Using the triangle angle sum theorem, we can find the missing angles of each triangle.

In the left triangle, the missing angle is $180 - (72+66) = 42$ degrees.

In the right triangle, the missing angle is $180 - (66+38)=76$ degrees.

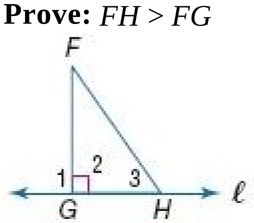
Since one triangle has angles whose measures are 72, 66, and 42 degrees and the other has angles whose measures are 76, 66, and 38 degrees. Therefore, the corresponding angles are not congruent and the two triangles are not similar.

ANSWER:

No; corresponding angles are not congruent.

Write a two-column proof.

65. **Given**: $\overline{FG} \perp \ell \overline{FH}$ is any non-perpendicular segment from F to ℓ.
Prove: $FH > FG$

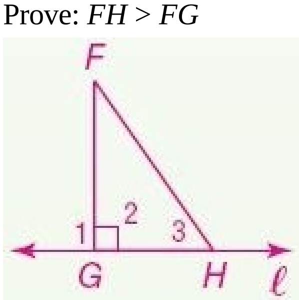


SOLUTION:

This proof involves knowledge of perpendicular lines, right angles, and the Exterior Angle Inequality theorem. If you can prove that an exterior angle of a triangle is greater than a remote interior angle, then any angle congruent to that exterior angle would also be greater than that exterior angle, as well.

Given: $\overline{FG} \perp \ell$; $\overline{FH}$ is any non-perpendicular segment from F to ℓ.

Prove: $FH > FG$



Proof:

Statements (Reasons)

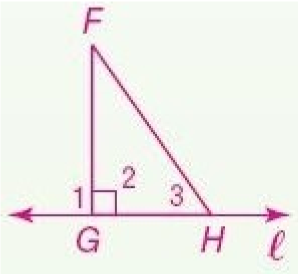
1. $\overline{FG} \perp \ell$ (Given)

2. $\angle 1$ and $\angle 2$ are right angles. ($\perp$ lines form right angles.)
3. $\angle 1 \cong \angle 2$ (All right angles are congruent.)
4. $m\angle 1 = m\angle 2$ (Definition of congruent angles)
5. $m\angle 1 > m\angle 3$ (Exterior Angle Inequality Theorem)
6. $m\angle 2 > m\angle 3$ (Substitution Property)
7. $FH > FG$ (If an angle of a triangle is greater than a second angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.)

ANSWER:

Given: $\overline{FG} \perp \ell$ $\overline{FH}$ is any non-perpendicular segment from F to ℓ.

Prove: $FH > FG$

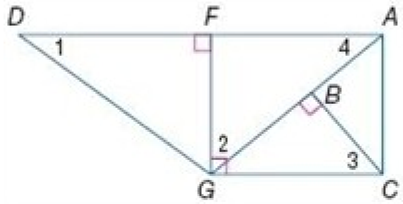


Proof:

Statements (Reasons)

1. $\overline{FG} \perp \ell$ (Given)
2. $\angle 1$ and $\angle 2$ are right angles. ($\perp$ lines form right angles.)
3. $\angle 1 \cong \angle 2$ (All right angles are congruent.)
4. $m\angle 1 = m\angle 2$ (Definition of congruent angles)
5. $m\angle 1 > m\angle 3$ (Exterior Angle Inequality Theorem)
6. $m\angle 2 > m\angle 3$ (Substitution Property)
7. $FH > FG$ (If an angle of a triangle is greater than a second angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.)

Find each measure if $m\angle DGF = 53$ and $m\angle AGC = 40$.



66. $m\angle 1$

SOLUTION:

The sum of the measures of the three angles of a triangle is 180.

$m\angle GFD = 90$ and $m\angle DGF = 53$.

Therefore, $m\angle 1 = m\angle GDF = 180 - (90 + 53) = 37$.

ANSWER:

37

67. $m\angle 2$

SOLUTION:

$m\angle FGC = 90$ and $m\angle AGC = 40$.

Therefore, $m\angle 2 = m\angle FGA = 90 - 40 = 50$.

ANSWER:

50

68. $m\angle 3$

SOLUTION:

The sum of the measures of the three angles of a triangle is 180.

$m\angle CBG = 90$ and $m\angle AGC = 40$.

Therefore, $m\angle 3 = m\angle BCG = 180 - (90 + 40) = 50$.

ANSWER:

50

69. $m\angle 4$

SOLUTION:

$m\angle FGC = 90$ and $m\angle AGC = 40$.

Therefore, $m\angle 2 = m\angle FGA = 90 - 40 = 50$.

The sum of the measures of the three angles of a triangle is 180.

$m\angle GFA = 90$ and $m\angle FGA = 50$.

Therefore, $m\angle 4 = m\angle FAG = 180 - (90 + 50) = 40$.

ANSWER:

40

Find the distance between each pair of parallel lines with the given equations.

70. $y = 4x$
$y = 4x - 17$

SOLUTION:

The slope of a line perpendicular to both the lines will be $m = -\frac{1}{4}$.

Consider the y-intercept of any of the two lines and write the equation of the perpendicular line through it. The y-intercept of the line $y = 4x$ is (0, 0). So, the equation of a line with slope –¼ and a y-intercept of 0 is $y = -\frac{1}{4}x$.

The perpendicular meets the first line at (0, 0). To find the point of intersection of the perpendicular and the second line, solve the two equations: $y = -\frac{1}{4}x$ and $y = 4x - 17$.

The left sides of the equations are the same so you can equate the right sides and solve for x.

$$4x - 17 = -\frac{1}{4}x$$
$$4x + \frac{1}{4}x = 17$$
$$\frac{17}{4}x = 17$$
$$x = 4$$

Use the value of x to find the value of y.

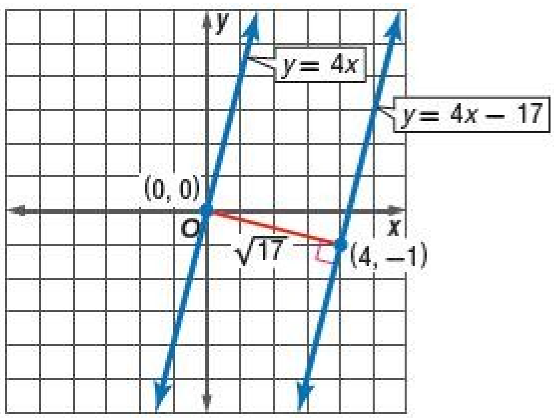
$$y = 4(4) - 17$$
$$= -1$$

So, the point of intersection is (4, –1).

Use the Distance Formula to find the distance between the points (0, 0) and (4, –1).

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4-0)^2 + (-1-0)^2} \\ &= \sqrt{16+1} \\ &= \sqrt{17} \end{aligned}$$

Therefore, the distance between the two lines is $\sqrt{17}$ units.



ANSWER:

$\sqrt{17}$

71. $\begin{aligned} y &= 2x - 3 \\ 2x - y &= -4 \end{aligned}$

SOLUTION:

First, write the second equation also in the slope-intercept form.

$$\begin{aligned} 2x - y &= -4 \\ -y &= -2x - 4 \\ y &= 2x + 4 \end{aligned}$$

The slope of a line perpendicular to both the lines will be $m = -\frac{1}{2}$.

Consider the y-intercept of any of the two lines and write the equation of the perpendicular line through it. The y-intercept of the line $y = 2x - 3$ is (0, –3). So, the equation of a line with slope –½ and a y-intercept of –3 is

$$y = -\frac{1}{2}x - 3$$

The perpendicular meets the first line at (0, –3). To find the point of intersection of the perpendicular and the second line, solve the two equations: $y = 2x + 4 \; and \; y = -\frac{1}{2}x - 3$.

The left sides of the equations are the same so equate the right sides and solve for x.

$$\begin{aligned} 2x+4 &= -\frac{1}{2}x-3 \\ 2x+\frac{1}{2}x &= -7 \\ \frac{5}{2}x &= -7 \\ x &= \frac{-14}{5} \end{aligned}$$

Use the value of x to find the value of y.

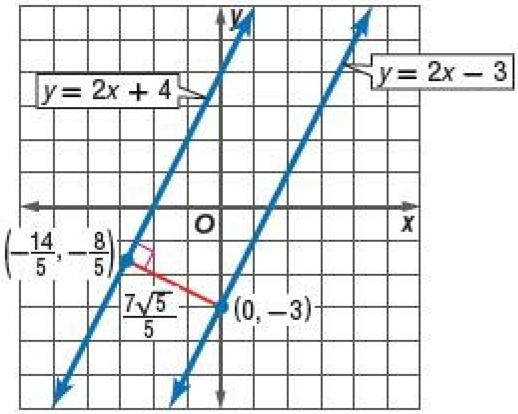
$$\begin{aligned} y &= 2\left(-\frac{14}{5}\right)+4 \\ &= -\frac{28}{5}+4 \\ &= -\frac{8}{5} \end{aligned}$$

So, the point of intersection is $\left(-\frac{14}{5},-\frac{8}{5}\right)$.

Use the Distance Formula to find the distance between the points (0, –3) and $\left(-\frac{14}{5},-\frac{8}{5}\right)$.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{\left(-\frac{14}{5}-0\right)^2+\left(-\frac{8}{5}-(-3)\right)^2} \\ &= \sqrt{\frac{196}{25}+\frac{49}{25}} \\ &= \frac{\sqrt{245}}{5} \\ &= \frac{7\sqrt{5}}{5} \end{aligned}$$

Therefore, the distance between the two lines is $\frac{7\sqrt{5}}{5}$ units.



ANSWER:

$\frac{7\sqrt{5}}{5}$

72. $y = -0.75x - 1$
$3x + 4y = 20$

SOLUTION:

First, write the second equation in the slope-intercept form:

$$\begin{aligned}3x + 4y &= 20\\ 4y &= -3x + 20\\ y &= -\frac{3}{4}x + \frac{20}{4}\\ y &= -\frac{3}{4}x + 5\end{aligned}$$

The slope of a line perpendicular to both the lines will be $m = \frac{4}{3}$.

Consider the y-intercept of any of the two lines and write the equation of the perpendicular line through it. The y-intercept of the line $y = -0.75x - 1$ is (0, –1). So, the equation of a line with slope $\frac{4}{3}$ and y-intercept of –1 is $y = \frac{4}{3}x - 1$.

The perpendicular meets the first line at (0, –1). To find the point of intersection of the perpendicular and the second line, solve the two equations: $y = -\frac{3}{4}x + 5$ *and* $y = \frac{4}{3}x - 1$.

The left sides of the equations are the same so you can equate the right sides and solve for x.

$$-\frac{3}{4}x+5=\frac{4}{3}x-1$$
$$-\frac{3}{4}x-\frac{4}{3}x=-6$$
$$-\frac{25}{12}x=-6$$
$$x=\frac{72}{25}$$

Use the value of x to find the value of y.

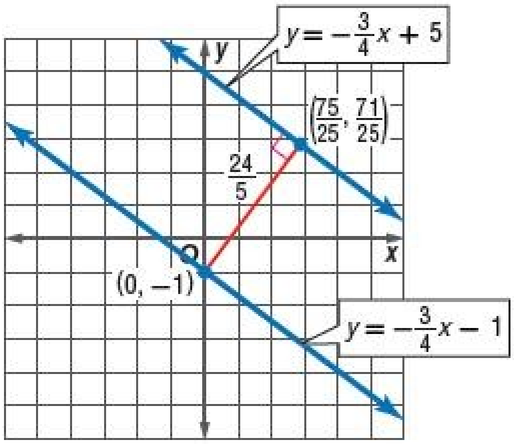
$$y=-\frac{3}{4}\left(\frac{72}{25}\right)+5$$
$$=-\frac{54}{25}+5$$
$$=\frac{71}{25}$$

So, the point of intersection is $\left(\frac{72}{25},\frac{71}{25}\right)$.

Use the Distance Formula to find the distance between the points (0, –1) and $\left(\frac{72}{25},\frac{71}{25}\right)$.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{\left(\frac{72}{25}-0\right)^2+\left(\frac{71}{25}-(-1)\right)^2}$$
$$=\sqrt{\frac{14400}{625}}$$
$$=\frac{120}{25}$$
$$=\frac{24}{5}$$

Therefore, the distance between the two lines is $\frac{24}{5}$ units.



ANSWER:

$\frac{24}{5}$

Find the value of *x*.

73. $18 = 3x\sqrt{3}$

SOLUTION:

Divide each side by $3\sqrt{3}$.

$$\frac{18}{3\sqrt{3}} = \frac{3x\sqrt{3}}{3\sqrt{3}}$$
$$\frac{6}{\sqrt{3}} = x$$

Multiply the numerator and the denominator of the fraction by $\sqrt{3}$ to rationalize the denominator.

$$\frac{6}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = x$$
$$\frac{6\sqrt{3}}{3} = x$$
$$2\sqrt{3} = x$$

ANSWER:

$2\sqrt{3}$

74. $24 = 2x\sqrt{2}$

SOLUTION:

Divide each side by $2\sqrt{2}$.

$$\frac{24}{2\sqrt{2}} = \frac{2x\sqrt{2}}{2\sqrt{2}}$$
$$\frac{12}{\sqrt{2}} = x$$

Multiply the numerator and the denominator of the fraction by $\sqrt{2}$ to rationalize the denominator.

$$\frac{12}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = x$$
$$\frac{12\sqrt{2}}{2} = x$$
$$6\sqrt{2} = x$$

ANSWER:

$6\sqrt{2}$

75. $9\sqrt{2} \cdot x = 18\sqrt{2}$

SOLUTION:

Divide each side by $9\sqrt{2}$.

$$\frac{9\sqrt{2}x}{9\sqrt{2}} = \frac{18\sqrt{2}}{9\sqrt{2}}$$
$$x = 2$$

ANSWER:

2

76. $2 = x \cdot \frac{4}{\sqrt{3}}$

SOLUTION:

Multiply each side by $\frac{\sqrt{3}}{4}$.

$$2\left(\frac{\sqrt{3}}{4}\right) = x \cdot \frac{4}{\sqrt{3}}\left(\frac{\sqrt{3}}{4}\right)$$
$$\frac{\sqrt{3}}{2} = x$$

ANSWER:

$\frac{\sqrt{3}}{2}$