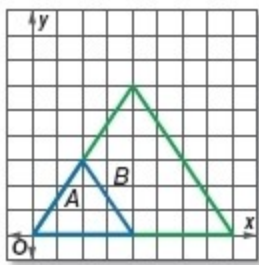


## 7-6 Similarity Transformations

Determine whether the dilation from  $A$  to  $B$  is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.



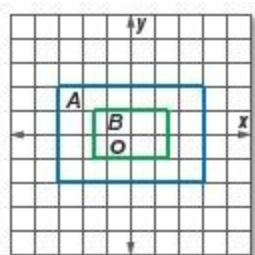
1.

**SOLUTION:**

Triangle  $B$  is larger than triangle  $A$ , so the dilation is an enlargement. The distance between the vertices at  $(0, 0)$  and  $(4, 0)$  for  $A$  is 4 and between the vertices at  $(0, 0)$  and  $(8, 0)$  for  $B$  is 8. So the scale factor is  $\frac{8}{4}$  or 2.

**ANSWER:**

enlargement; 2



2.

**SOLUTION:**

Rectangle  $B$  is smaller than  $A$ , so the dilation is a reduction. The distance between the vertices at  $(-3, -2)$  and  $(3, -2)$  for  $A$  is 6 and between the vertices at  $(-1.5, -1)$  and  $(1.5, -1)$  for  $B$  is 3. So the scale factor is  $\frac{3}{6}$  or  $\frac{1}{2}$ .

**ANSWER:**

reduction;  $\frac{1}{2}$

## **7-6 Similarity Transformations**

3. **GAMES** The dimensions of a regulation tennis court are 27 feet by 78 feet. The dimensions of a table tennis table are 152.5 centimeters by 274 centimeters. Is a table tennis table a dilation of a tennis court? If so, what is the scale factor? Explain.



**SOLUTION:**

See if a proportion is formed using the lengths and widths of the table and the court.

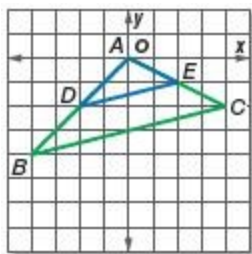
No; sample answer: Since  $\frac{152.5}{27} \neq \frac{274}{78}$ , a table tennis table is not a dilation of a tennis court.

**ANSWER:**

No; sample answer: Since  $\frac{152.5}{27} \neq \frac{274}{78}$ , a table tennis table is not a dilation of a tennis court.

## 7-6 Similarity Transformations

**CCSS ARGUMENTS** Verify that the dilation is a similarity transformation.



4.

**SOLUTION:**

Original:  $A(0, 0)$ ,  $D(-2, -2)$ ,  $E(2, -1)$

Image:  $A(0, 0)$ ,  $B(-4, -4)$ ,  $C(4, -2)$

By the Reflexive Property,  $\angle A \cong \angle A$ .

Use the distance formula to find the length of each segment:

$$\begin{aligned} AC &= \sqrt{(4-0)^2 + (-2-0)^2} \\ &= \sqrt{16+4} \\ &= 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} AE &= \sqrt{(2-0)^2 + (-1-0)^2} \\ &= \sqrt{4+1} \\ &= \sqrt{5} \end{aligned}$$

$$\begin{aligned} AB &= \sqrt{(-4-0)^2 + (-4-0)^2} \\ &= \sqrt{16+16} \\ &= 4\sqrt{2} \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{(-2-0)^2 + (-2-0)^2} \\ &= \sqrt{4+4} \\ &= 2\sqrt{2} \end{aligned}$$

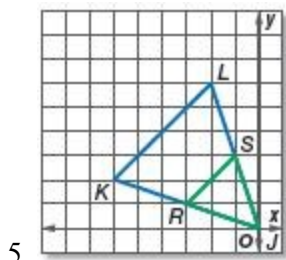
$$\frac{AC}{AE} = \frac{2\sqrt{5}}{\sqrt{5}} = 2$$

$$\frac{AB}{AD} = \frac{4\sqrt{2}}{2\sqrt{2}} = 2 \text{ and } \angle A \cong \angle A \text{ by the Reflexive Property, so } \triangle ADE \sim \triangle ABC \text{ by SAS Similarity.}$$

**ANSWER:**

$$\frac{AC}{AE} = \frac{AB}{AD} = 2 \text{ and } \angle A \cong \angle A \text{ by the Reflexive Property, so } \triangle ADE \sim \triangle ABC \text{ by SAS Similarity.}$$

## 7-6 Similarity Transformations



**SOLUTION:**

Original:  $J(0, 0)$ ,  $K(-6, 2)$ ,  $L(-2, 6)$

Image:  $J(0, 0)$ ,  $R(-3, 1)$ ,  $S(-1, 3)$

By the Reflexive Property,  $\angle J \cong \angle J$ .

Use the distance formula to find the length of each segment:

$$\begin{aligned} RJ &= \sqrt{(0 - (-3))^2 + (0 - 1)^2} \\ &= \sqrt{9 + 1} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} KJ &= \sqrt{(0 - (-6))^2 + (0 - 2)^2} \\ &= \sqrt{36 + 4} \\ &= 2\sqrt{10} \end{aligned}$$

$$\begin{aligned} SJ &= \sqrt{(0 - (-1))^2 + (0 - 3)^2} \\ &= \sqrt{1 + 9} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} LJ &= \sqrt{(0 - (-2))^2 + (0 - 6)^2} \\ &= \sqrt{4 + 36} \\ &= 2\sqrt{10} \end{aligned}$$

$$\frac{RJ}{KJ} = \frac{\sqrt{10}}{2\sqrt{10}} = \frac{1}{2}$$

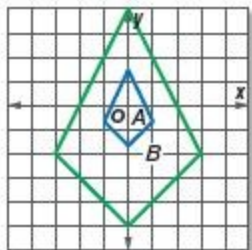
$$\frac{SJ}{LJ} = \frac{\sqrt{10}}{2\sqrt{10}} = \frac{1}{2} \quad \text{and} \quad \angle J \cong \angle J \quad \text{by the Reflexive Property, so } \triangle RSJ \sim \triangle KLJ \quad \text{by SAS Similarity.}$$

**ANSWER:**

$$\frac{RJ}{KJ} = \frac{SJ}{LJ} = \frac{RS}{KL} = \frac{1}{2}, \text{ so } \triangle RSJ \sim \triangle KLJ \text{ by SSS Similarity.}$$

## 7-6 Similarity Transformations

Determine whether the dilation from  $A$  to  $B$  is an *enlargement* or a *reduction*. Then find the scale factor of the dilation.



6.

**SOLUTION:**

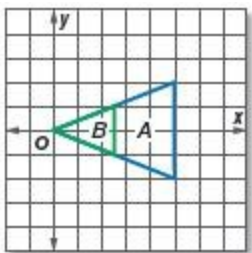
Kite  $B$  is larger than kite  $A$ , so the dilation is an enlargement.

The distance between the vertices at  $(0, 0)$  and  $(0, 4)$  for  $B$  is  $4 - 0 = 4$  units and between the vertices at  $(0, 0)$  and  $(0, \frac{4}{3})$  for  $A$  is  $\frac{4}{3}$ .

So, the scale factor is  $\frac{4}{\frac{4}{3}} = 4 \cdot \frac{3}{4} = 3$ .

**ANSWER:**

enlargement; 3



7.

**SOLUTION:**

Triangle  $B$  is smaller than triangle  $A$ , so the dilation is a reduction.

The distance between the vertices at  $(0, 0)$  and  $(5, 2)$  for  $A$  is

$$\sqrt{(2-0)^2 + (5-0)^2} = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$$

The distance between the vertices at  $(0, 0)$  and  $(2.5, 1)$  for  $B$  is

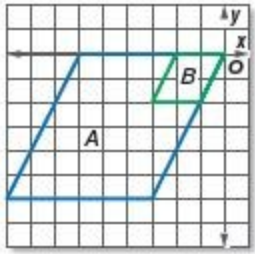
$$\sqrt{(1-0)^2 + (2.5-0)^2} = \sqrt{1^2 + 2.5^2} = \sqrt{1 + 6.25} = \sqrt{7.25}$$

So, the scale factor is  $\frac{\sqrt{7.25}}{\sqrt{29}} \approx \frac{2.69}{5.39} = \frac{1}{2}$ .

**ANSWER:**

reduction;  $\frac{1}{2}$

## 7-6 Similarity Transformations



8.

**SOLUTION:**

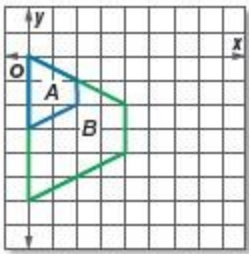
Parallelogram  $B$  is smaller than parallelogram  $A$ , so the dilation is a reduction.

The distance between the vertices at  $(0, 0)$  and  $(-6, 0)$  for  $A$  is 6 and between the vertices at  $(0, 0)$  and  $(-2, 0)$  for  $B$  is 2.

So the scale factor is  $\frac{2}{6}$  or  $\frac{1}{3}$ .

**ANSWER:**

reduction;  $\frac{1}{3}$



9.

**SOLUTION:**

Trapezoid  $B$  is larger than trapezoid  $A$ , so the dilation is an enlargement.

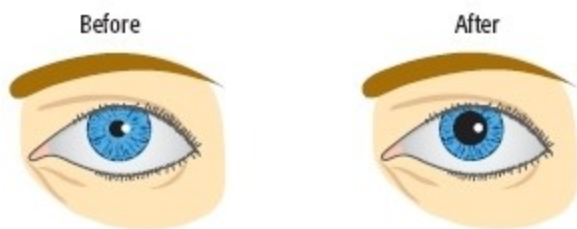
The distance between the vertices at  $(0, 0)$  and  $(0, -6)$  for  $B$  is 6 and between the vertices at  $(0, 0)$  and  $(0, -3)$  for  $A$  is 3. So the scale factor is  $\frac{6}{3}$  or 2.

**ANSWER:**

enlargement; 2

## 7-6 Similarity Transformations

Determine whether each dilation is an *enlargement* or *reduction*.



10.

**SOLUTION:**

Since the "after" image of the pupil is larger than the "before" image, this is an enlargement.

**ANSWER:**

enlargement

11. Refer to Page 513.

**SOLUTION:**

This is a reduction because the postcard is a smaller and similar version of the painting.

**ANSWER:**

reduction

12. **YEARBOOK** Jordan is putting a photo of the lacrosse team in a full-page layout in the yearbook. The original photo is 4 inches by 6 inches. If the photo in the yearbook is  $6\frac{2}{3}$  inches by 10 inches, is the yearbook photo a dilation of the original photo? If so, what is the scale factor? Explain.

**SOLUTION:**

Yes;  $1\frac{1}{2}$ , sample answer: The dimensions of the photo are 4 inches by 6 inches, the ratio of the sides is  $\frac{4}{6} = \frac{2}{3}$ .

The dimensions of the yearbook are  $6\frac{2}{3}$  by 10 inches, which results in a ratio of  $\frac{6\frac{2}{3}}{10} = \frac{\frac{20}{3}}{10} = \frac{20}{3} \cdot \frac{1}{10} = \frac{20}{30} = \frac{2}{3}$ .

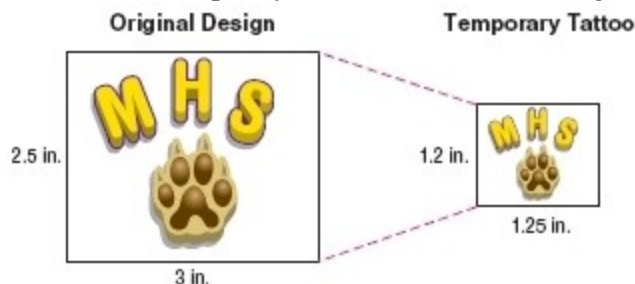
The photo in the yearbook is a dilation of the original photo. The scale factor is  $\frac{6}{4} = \frac{3}{2}$ , or  $1\frac{1}{2}$ .

**ANSWER:**

Yes;  $1\frac{1}{2}$ , sample answer: Since  $\frac{4}{6} = \frac{6\frac{2}{3}}{10} = \frac{2}{3}$ , the photo in the yearbook is a dilation of the original photo. The scale factor is  $\frac{6}{4}$ , or  $1\frac{1}{2}$ .

## 7-6 Similarity Transformations

13. **CCSS MODELING** Candace created a design to be made into temporary tattoos for a homecoming game as shown. Is the temporary tattoo a dilation of the original design? If so, what is the scale factor? Explain.



**SOLUTION:**

See if a proportion is formed using the lengths and widths of the design and tattoo.

No; sample answer: Since  $\frac{1.2}{2.5} \neq \frac{1.25}{3}$ , the design and the actual tattoo are not proportional. Therefore, the tattoo is not a dilation of the design.

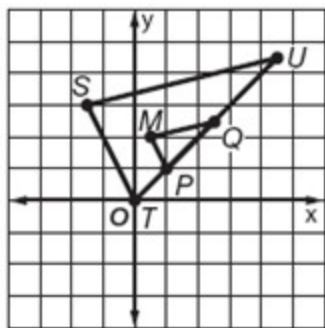
**ANSWER:**

No; sample answer: Since  $\frac{1.2}{2.5} \neq \frac{1.25}{3}$ , the design and the actual tattoo are not proportional. Therefore, the tattoo is not a dilation of the design.

**Graph the original figure and its dilated image. Then verify that the dilation is a similarity transformation.**

14.  $M(1, 4)$ ,  $P(2, 2)$ ,  $Q(5, 5)$ ;  $S(-3, 6)$ ,  $T(0, 0)$ ,  $U(9, 9)$

**SOLUTION:**



Use the distance formula to find the lengths of the sides.

$$MP = \sqrt{(2-1)^2 + (2-4)^2}$$

$$= \sqrt{1+4}$$

$$= \sqrt{5}$$

$$PQ = \sqrt{(5-2)^2 + (5-2)^2}$$

$$= \sqrt{9+9}$$

$$= 3\sqrt{2}$$

$$MQ = \sqrt{(5-1)^2 + (5-4)^2}$$

$$= \sqrt{16+1}$$

$$= \sqrt{17}$$

## 7-6 Similarity Transformations

$$ST = \sqrt{(0 - (-3))^2 + (0 - 6)^2}$$

$$= \sqrt{9 + 36}$$

$$= 3\sqrt{5}$$

$$TU = \sqrt{(9 - 0)^2 + (9 - 0)^2}$$

$$= \sqrt{81 + 81}$$

$$= 9\sqrt{2}$$

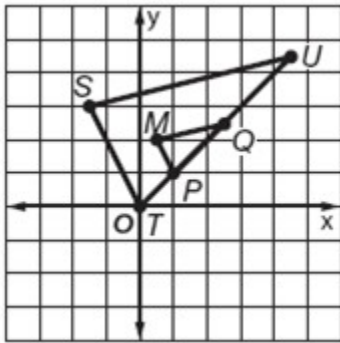
$$SU = \sqrt{(9 - (-3))^2 + (9 - 6)^2}$$

$$= \sqrt{144 + 9}$$

$$= 3\sqrt{17}$$

$$\frac{MP}{ST} = \frac{PQ}{TU} = \frac{MQ}{SU} = \frac{1}{3}, \text{ so } \triangle MPQ \sim \triangle STU \text{ by SSS Similarity.}$$

ANSWER:

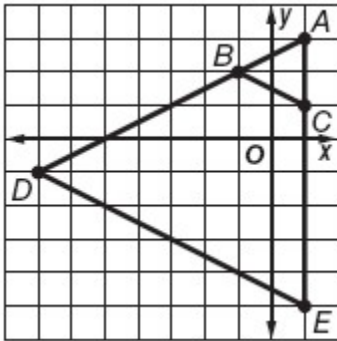


$$\frac{MP}{ST} = \frac{PQ}{TU} = \frac{MQ}{SU} = \frac{1}{3}, \text{ so } \triangle MPQ \sim \triangle STU \text{ by SSS Similarity.}$$

## 7-6 Similarity Transformations

15.  $A(1, 3)$ ,  $B(-1, 2)$ ,  $C(1, 1)$ ;  $D(-7, -1)$ ,  $E(1, -5)$

**SOLUTION:**



Use the distance formula to find the lengths of the sides.

$$\begin{aligned} AB &= \sqrt{(-1-1)^2 + (2-3)^2} \\ &= \sqrt{4+1} \\ &= \sqrt{5} \end{aligned}$$

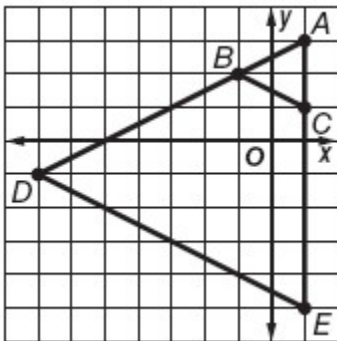
$$\begin{aligned} AC &= \sqrt{(1-1)^2 + (1-3)^2} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{(-7-1)^2 + (-1-3)^2} \\ &= \sqrt{64+16} \\ &= \sqrt{80} \\ &= 4\sqrt{5} \end{aligned}$$

$$\begin{aligned} AE &= \sqrt{(1-1)^2 + (-5-3)^2} \\ &= \sqrt{64} \\ &= 8 \end{aligned}$$

By the Reflexive Property  $\angle A \cong \angle A$  and  $\frac{AB}{AD} = \frac{AC}{AE} = \frac{1}{4}$ , so  $\triangle ABC \sim \triangle ADE$  by SAS Similarity.

**ANSWER:**

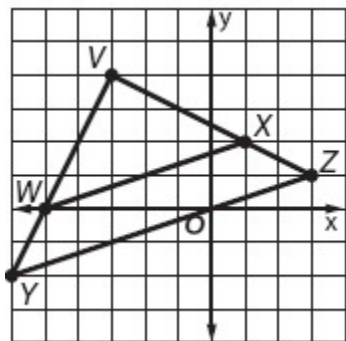


$\angle A \cong \angle A$  and  $\frac{AB}{AD} = \frac{AC}{AE} = \frac{1}{4}$ , so  $\triangle ABC \sim \triangle ADE$  by SAS Similarity.

## 7-6 Similarity Transformations

16.  $V(-3, 4)$ ,  $W(-5, 0)$ ,  $X(1, 2)$ ;  $Y(-6, -2)$ ,  $Z(3, 1)$

**SOLUTION:**



Use the distance formula to find the lengths of the sides.

$$\begin{aligned} VW &= \sqrt{(-5 - (-3))^2 + (0 - 4)^2} \\ &= \sqrt{4 + 16} \\ &= 2\sqrt{5} \end{aligned}$$

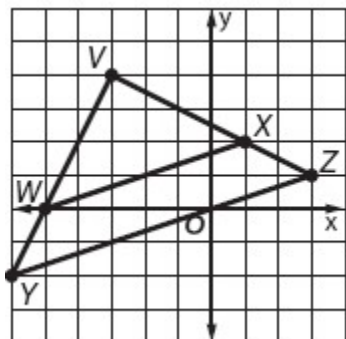
$$\begin{aligned} VX &= \sqrt{(1 - (-3))^2 + (2 - 4)^2} \\ &= \sqrt{16 + 4} \\ &= 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} VY &= \sqrt{(-6 - (-3))^2 + (-2 - 4)^2} \\ &= \sqrt{9 + 36} \\ &= 3\sqrt{5} \end{aligned}$$

$$\begin{aligned} VZ &= \sqrt{(3 - (-3))^2 + (1 - 4)^2} \\ &= \sqrt{36 + 9} \\ &= 3\sqrt{5} \end{aligned}$$

$$\frac{VW}{VY} = \frac{VX}{VZ} = \frac{2}{3} \text{ and } \angle V \cong \angle V \text{ by the Reflexive Property, so } \triangle VWX \sim \triangle VYZ \text{ by SAS Similarity.}$$

**ANSWER:**

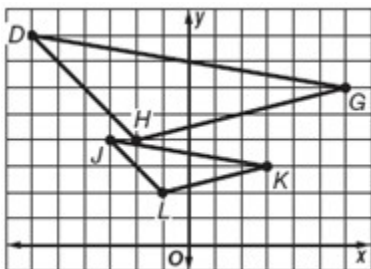


$$\frac{VW}{VY} = \frac{VX}{VZ} = \frac{2}{3} \text{ and } \angle V \cong \angle V \text{ by the Reflexive Property, so } \triangle VWX \sim \triangle VYZ \text{ by SAS Similarity.}$$

17.  $J(-6, 8)$ ,  $K(6, 6)$ ,  $L(-2, 4)$ ;  $D(-12, 16)$ ,  $G(12, 12)$ ,  $H(-4, 8)$

## 7-6 Similarity Transformations

SOLUTION:



Use the distance formula to find the lengths of the sides.

$$\begin{aligned} JK &= \sqrt{(6 - (-6))^2 + (6 - 8)^2} \\ &= \sqrt{144 + 4} \\ &= 2\sqrt{37} \end{aligned}$$

$$\begin{aligned} KL &= \sqrt{(-2 - 6)^2 + (4 - 6)^2} \\ &= \sqrt{64 + 4} \\ &= 2\sqrt{17} \end{aligned}$$

$$\begin{aligned} JL &= \sqrt{(-2 - (-6))^2 + (4 - 8)^2} \\ &= \sqrt{16 + 16} \\ &= 4\sqrt{2} \end{aligned}$$

$$\begin{aligned} DG &= \sqrt{(12 - (-12))^2 + (12 - 16)^2} \\ &= \sqrt{576 + 16} \\ &= \sqrt{592} \\ &= 4\sqrt{37} \end{aligned}$$

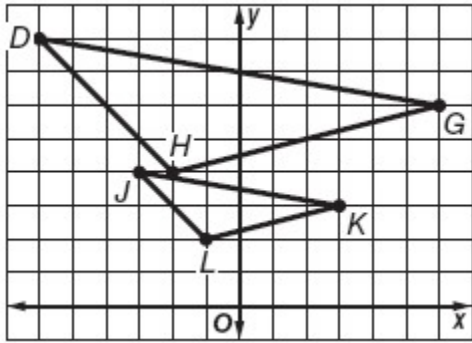
$$\begin{aligned} GH &= \sqrt{(-4 - 12)^2 + (8 - 12)^2} \\ &= \sqrt{256 + 16} \\ &= \sqrt{272} \\ &= 4\sqrt{17} \end{aligned}$$

$$\begin{aligned} DH &= \sqrt{(-4 - (-12))^2 + (8 - 16)^2} \\ &= \sqrt{64 + 64} \\ &= \sqrt{128} \\ &= 8\sqrt{2} \end{aligned}$$

$$\frac{JK}{DG} = \frac{KL}{GH} = \frac{JL}{DH} = \frac{1}{2}, \text{ so } \triangle JKL \sim \triangle DGH \text{ by SSS Similarity.}$$

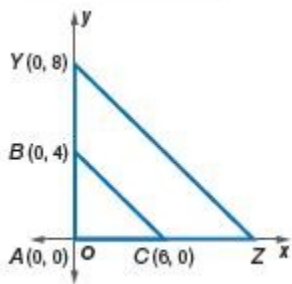
ANSWER:

## 7-6 Similarity Transformations



$\frac{JK}{DG} = \frac{KL}{GH} = \frac{JL}{DH} = \frac{1}{2}$ , so  $\Delta JKL \sim \Delta DGH$  by SSS Similarity.

If  $\Delta ABC \sim \Delta AYZ$ , find the missing coordinate.



18.

**SOLUTION:**

Since  $\Delta ABC \sim \Delta AYZ$ ,  $\frac{AB}{AY} = \frac{AC}{AZ}$ .

$AB = 4$ ,  $AY = 8$ , and  $AC = 6$

Substitute.

$$\frac{4}{8} = \frac{6}{AZ}$$

$$4AZ = 48$$

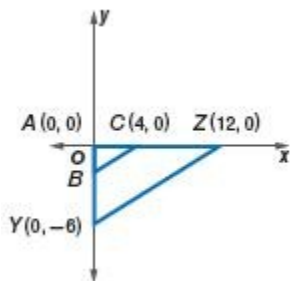
$$AZ = 12$$

The coordinates of  $Z$  are  $(12, 0)$  because the point  $Z$  lies on the right side of the  $x$ -axis,  $A$  lies on the origin, and  $AZ = 12$ .

**ANSWER:**

$(12, 0)$

## 7-6 Similarity Transformations



19.

**SOLUTION:**

Since  $\triangle ABC \sim \triangle AYZ$ ,  $\frac{AB}{AY} = \frac{AC}{AZ}$ .

$AZ = 12$ ,  $AY = 6$ , and  $AC = 4$

Substitute.

$$\frac{AB}{6} = \frac{4}{12}$$

$$12AB = 24$$

$$AB = 2$$

Since the point  $B$  lies below the  $y$  axis, the coordinates of  $B$  are  $(0, -2)$ ,  $A$  lies on the origin, and  $AB = 2$ .

**ANSWER:**

$(0, -2)$

## 7-6 Similarity Transformations

20. **GRAPHIC ART** Aimee painted the sample sign shown using  $\frac{1}{2}$  bottle of glass paint. The actual sign she will paint in a shop window is to be 3 feet by  $7\frac{1}{2}$  feet.



- Explain why the actual sign is a dilation of her sample.
- How many bottles of paint will Aimee need to complete the actual sign?

**SOLUTION:**

- Since  $\frac{6\text{ in.}}{15\text{ in.}} = \frac{2}{5}$  and  $\frac{3\text{ ft}}{7.5\text{ ft}} = \frac{6}{15} = \frac{2}{5}$ , the new sign is a dilation of her sample.
- For 6 inches by 15 inches sign, or 0.5 foot by 1.25 feet sign, she used  $\frac{1}{2}$  bottle of glass paint.

$$\begin{aligned}\text{Area of the sample sign} &= 0.5 \times 1.25 \\ &= 0.625\end{aligned}$$

$$\begin{aligned}\text{Area of the actual sign} &= 3 \times 7.5 \\ &= 22.5 \\ &= 36(0.625)\end{aligned}$$

Therefore, Aimee needs 36(0.5) bottles or 18 bottles of paint to complete the actual sign.

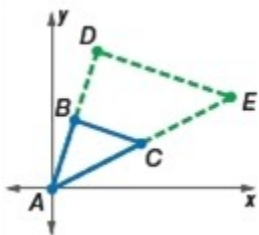
**ANSWER:**

- $\frac{6\text{ in.}}{15\text{ in.}} = \frac{3\text{ ft}}{7.5\text{ ft}}$  is a true proportion.
  - 18 bottles
21. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate similarity of triangles on the coordinate plane.
- GEOMETRIC** Draw a triangle with vertex  $A$  at the origin. Make sure that the two additional vertices  $B$  and  $C$  have whole-number coordinates. Draw a similar triangle that is twice as large as  $\triangle ABC$  with its vertex also located at the origin. Label the triangle  $\triangle ADE$ .
  - GEOMETRIC** Repeat the process in part a two times. Label the second pair of triangles  $\triangle MNP$  and  $\triangle MQR$  and the third pair  $\triangle TWX$  and  $\triangle TYZ$ . Use different scale factors than part a.
  - TABULAR** Complete the table below with the appropriate values.

Coordinates									
$\triangle ABC$		$\triangle ADE$		$\triangle MNP$		$\triangle MQR$		$\triangle TWX$	
$A$		$A$		$M$		$M$		$T$	
$B$		$D$		$N$		$Q$		$W$	
$C$		$E$		$P$		$R$		$X$	
								$Z$	

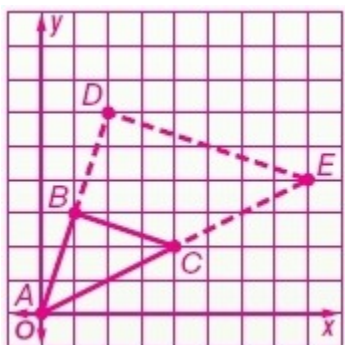
- VERBAL** Make a conjecture about how you could predict the coordinates of a dilated triangle with a scale factor of  $n$  if the two similar triangles share a corresponding vertex at the origin.

## 7-6 Similarity Transformations

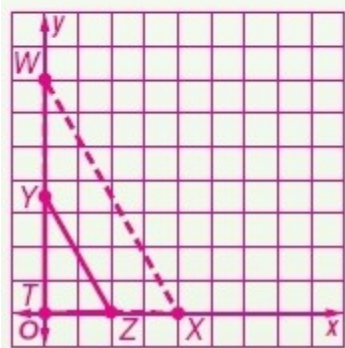
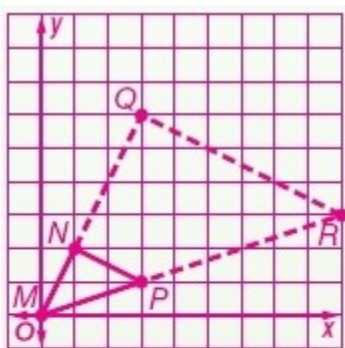


**SOLUTION:**

a. When drawing your two triangles, where one is twice as large as the other, you will need to make each side length of the new triangle ( $\triangle ADE$ ) twice as long as the smaller triangle ( $\triangle ABC$ ).



b. When drawing your two triangles, where one is twice as large as the other, you will need to make each side length of the new triangle twice as long as the smaller triangle. Try making figures in different quadrants to obtain a variety of coordinates.



c. Carefully record the coordinates of your triangles in the provided table. Pay attention to any patterns noticed when comparing the coordinates of the small triangle and the enlarged triangle.

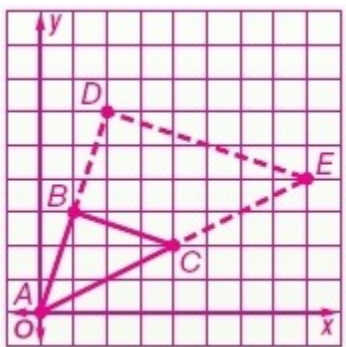
## 7-6 Similarity Transformations

Coordinates											
$\triangle ABC$		$\triangle ADE$		$\triangle MNP$		$\triangle MQR$		$\triangle TWX$		$\triangle TYZ$	
A	(0, 0)	A	(0, 0)	M	(0, 0)	M	(0, 0)	T	(0, 0)	T	(0, 0)
B	(1, 3)	D	(2, 6)	N	(1, 2)	Q	(3, 6)	W	(0, 7)	Y	(0, 3.5)
C	(4, 2)	E	(8, 4)	P	(3, 1)	R	(9, 3)	X	(4, 0)	Z	(2, 0)

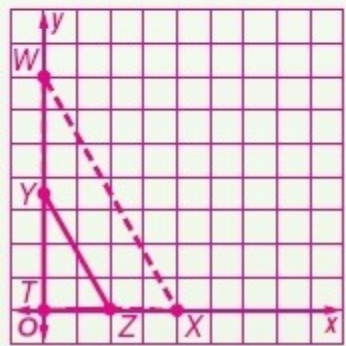
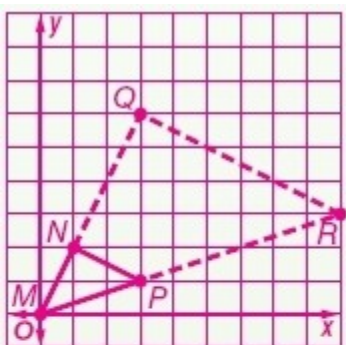
d. Sample answer: Multiply the coordinates of the given triangle by the scale factor to get the coordinates of the dilated triangle.

ANSWER:

a.



b.



c.

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Coordinates									
$\triangle ABC$		$\triangle ADE$		$\triangle MNP$		$\triangle MQR$		$\triangle TWX$	
A	(0, 0)	A	(0, 0)	M	(0, 0)	M	(0, 0)	T	(0, 0)
B	(1, 3)	D	(2, 6)	N	(1, 2)	Q	(3, 6)	W	(0, 7)
C	(4, 2)	E	(8, 4)	P	(3, 1)	R	(9, 3)	X	(4, 0)
								Y	(0, 3.5)
								Z	(2, 0)

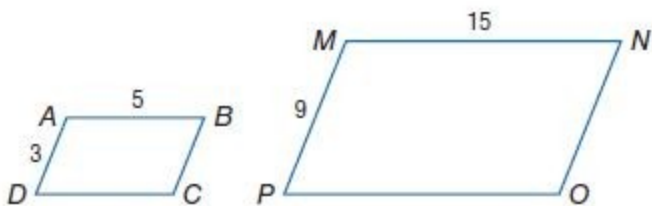
d. Sample answer: Multiply the coordinates of the given triangle by the scale factor to get the coordinates of the dilated triangle.

22. **CHALLENGE**  $MNOP$  is a dilation of  $ABCD$ . How is the scale factor of the dilation related to the similarity ratio of  $ABCD$  to  $MNOP$ ? Explain your reasoning.

**SOLUTION:**

Reciprocals; sample answer: The similarity ratio of  $ABCD$  to  $MNOP$  can be expressed using the ratio  $\frac{AB}{MN}$ . The scale factor is the ratio  $\frac{MN}{AB}$ . Therefore, the similarity ratio of  $ABCD$  to  $MNOP$  and the scale factor are reciprocals.

For example, in the similar parallelograms  $ABCD$  and  $MNOP$  below, the similarity ratio of the corresponding sides is  $\frac{AB}{MN} = \frac{AD}{MP} = \frac{3}{9} = \frac{5}{15} = \frac{1}{3}$ . Since  $MNOP$  is larger than  $ABCD$ , it is an enlargement and the scale factor is  $\frac{MN}{AB} = \frac{9}{3} = 3$ .



**ANSWER:**

Reciprocals; sample answer: The similarity ratio of  $ABCD$  to  $MNOP$  can be expressed using the ratio  $\frac{AB}{MN}$ . The scale factor is the ratio  $\frac{MN}{AB}$ . Therefore, the similarity ratio of  $ABCD$  to  $MNOP$  and the scale factor are reciprocals.

## 7-6 Similarity Transformations

23. **CCSS REASONING** The coordinates of two triangles are provided in the table. Is  $\triangle XYZ$  a dilation of  $\triangle PQR$ ? Explain.

$\triangle PQR$		$\triangle XYZ$	
$P$	$(a, b)$	$X$	$(3a, 2b)$
$Q$	$(c, d)$	$Y$	$(3c, 2d)$
$R$	$(e, f)$	$Z$	$(3e, 2f)$

**SOLUTION:**

No; sample answer: For one triangle to be a dilation of the other, their scale factor must enlarge or reduce the transformation proportionally. Since the  $x$ -coordinates are multiplied by 3 and the  $y$ -coordinates are multiplied by 2,  $\triangle XYZ$  is 3 times as wide and only 2 times as tall as  $\triangle PQR$ . Therefore, the transformation is not a dilation.

**ANSWER:**

No; sample answer: Since the  $x$ -coordinates are multiplied by 3 and the  $y$ -coordinates are multiplied by 2,  $\triangle XYZ$  is 3 times as wide and only 2 times as tall as  $\triangle PQR$ . Therefore, the transformation is not a dilation.

**OPEN ENDED** Describe a real-world example of each transformation other than those given in this lesson.

24. enlargement

**SOLUTION:**

Choose a real-world example in which a new image created is proportionally larger than the original. Sample answer: An image formed using a digital projector is an enlargement.

**ANSWER:**

Sample answer: An image formed using a digital projector is an enlargement.

25. reduction

**SOLUTION:**

Choose a real-world example in which a new image created is proportionally smaller than the original. Sample answer: Architectural plans are reductions.

**ANSWER:**

Sample answer: Architectural plans are reductions.

26. congruence transformation

**SOLUTION:**

Choose a real-world example in which a new image created is exactly the same size and dimensions as the original image. Sample answer: Stamps are congruence transformations.

**ANSWER:**

Sample answer: Stamps are congruence transformations.

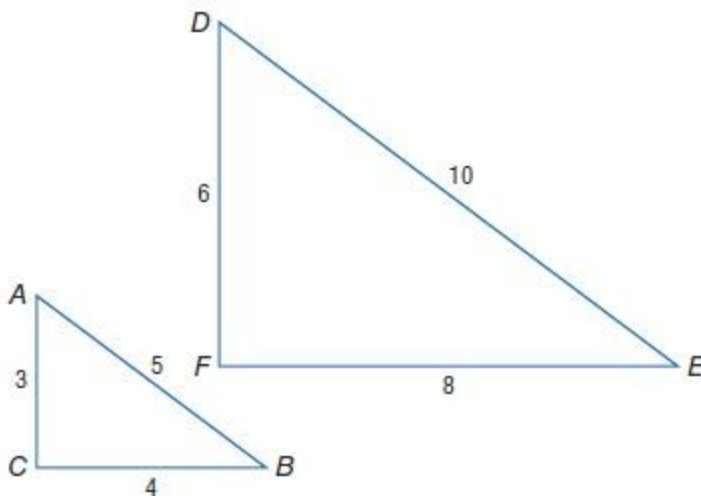
27. **WRITING IN MATH** Explain how you can use scale factor to determine whether a transformation is an enlargement, a reduction, or a congruence transformation.

**SOLUTION:**

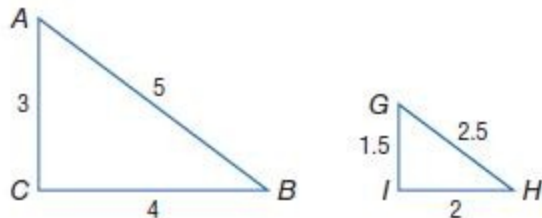
Sample answer: If a transformation is an enlargement, the lengths of the transformed object will be greater than the original object, so the scale factor will be greater than 1. In the example below,  $\triangle DEF$  is an enlargement of

## 7-6 Similarity Transformations

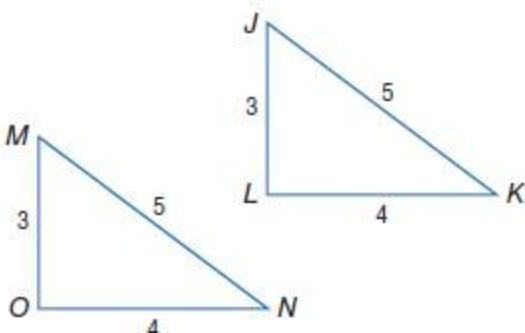
$\triangle ABC$  with a scale factor of  $\frac{2}{1}$ .



If a transformation is a reduction, the lengths of the transformed object will be less than the original object, so the scale factor will be less than 1, but greater than 0. In the example below,  $\triangle GHI$  is a reduction of  $\triangle ABC$  with a scale factor of  $\frac{1}{2}$ .



If the transformation is a congruence transformation, the scale factor is 1, because the lengths of the transformed object are equal to the lengths of the original object. In the example below,  $\triangle DEF$  is congruent to  $\triangle ABC$  with a scale factor of  $\frac{1}{1}$ .



### ANSWER:

Sample answer: If a transformation is an enlargement, the lengths of the transformed object will be greater than the original object, so the scale factor will be greater than 1. If a transformation is a reduction, the lengths of the transformed object will be less than the original object, so the scale factor will be less than 1, but greater than 0. If the transformation is a congruence transformation, the scale factor is 1, because the lengths of the transformed object are equal to the lengths of the original object.

## 7-6 Similarity Transformations

28. **ALGEBRA** Which equation describes the line that passes through  $(-3, 4)$  and is perpendicular to  $3x - y = 6$ ?

**A**  $y = -\frac{1}{3}x + 4$

**B**  $y = -\frac{1}{3}x + 3$

**C**  $y = 3x + 4$

**D**  $y = 3x + 3$

**SOLUTION:**

The slope of the line  $3x - y = 6$  is 3. So, the slope of the perpendicular line of  $3x - y = 6$  is  $-\frac{1}{3}$ .

Use the point-slope formula.

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{3}(x + 3)$$

$$y - 4 = -\frac{1}{3}x - 1$$

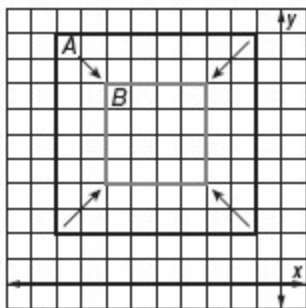
$$y = -\frac{1}{3}x + 3$$

So, the correct option is B.

**ANSWER:**

B

29. **SHORT RESPONSE** What is the scale factor of the dilation shown below?



**SOLUTION:**

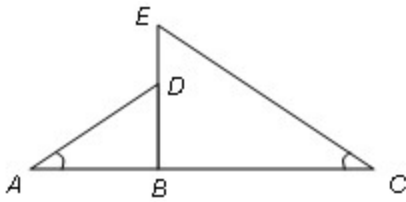
The length of a side of square B is  $\frac{1}{2}$  the length of a side of square A. So, the scale factor is  $\frac{1}{2}$ .

**ANSWER:**

$$\frac{1}{2}$$

## 7-6 Similarity Transformations

30. In the figure below,  $\angle A \cong \angle C$ .



Which additional information would *not* be enough to prove that  $\triangle ADB \sim \triangle CEB$ ?

F.  $\frac{AB}{AD} = \frac{CB}{CE}$

G.  $\angle ADB \cong \angle CEB$

H.  $\overline{ED} \cong \overline{DB}$

J.  $\overline{EB} \perp \overline{AC}$

**SOLUTION:**

If  $\angle A \cong \angle C$  and  $\frac{AB}{AD} = \frac{CB}{CE}$ , then  $\triangle ADB \sim \triangle CEB$  by SAS Similarity.

If  $\angle A \cong \angle C$  and  $\angle ADB \cong \angle CEB$ , then  $\triangle ADB \sim \triangle CEB$  by AA Similarity.

If  $\overline{EB} \perp \overline{AC}$ , then  $\angle ABD$  and  $\angle CBE$  are both right angles. Since all right angles have a measure of 90, then  $\angle ABD \cong \angle CBE$ . If  $\angle A \cong \angle C$  and  $\angle ABD \cong \angle CBE$ , then  $\triangle ADB \sim \triangle CEB$  by AA Similarity.

If  $\angle A \cong \angle C$  and  $\overline{ED} \cong \overline{DB}$ , then there is not enough information to prove that  $\triangle ADB \sim \triangle CEB$ .

The correct answer is H.

**ANSWER:**

H

## 7-6 Similarity Transformations

31. **SAT/ACT** If  $x = \frac{6}{4p+3}$  and  $xy = \frac{3}{4p+3}$ , then  $y =$

- A 4
- B 2
- C 1
- D  $\frac{3}{4}$
- E  $\frac{1}{2}$

**SOLUTION:**

$$x = \frac{6}{4p+3} \text{ and } xy = \frac{3}{4p+3}.$$

$$\left(\frac{6}{4p+3}\right)y = \frac{3}{4p+3}$$

$$(4p+3)\left(\frac{6}{4p+3}\right)y = (4p+3)\frac{3}{4p+3}$$

$$6y = 3$$

$$y = \frac{1}{2}$$

So, the correct choice is E.

**ANSWER:**

E

32. **LANDSCAPING** Shea is designing two gardens shaped like similar triangles. One garden has a perimeter of 53.5 feet, and the longest side is 25 feet. She wants the second garden to have a perimeter of 32.1 feet. Find the length of the longest side of this garden.

**SOLUTION:**

If two triangles are similar, the lengths of corresponding medians are proportional to the lengths of corresponding sides.

Form a proportion for the given situation. Let  $x$  be the length of the longest side of the second garden.

$$\frac{x}{32.1} = \frac{25}{53.5}$$

$$x = \frac{25 \times 32.1}{53.5}$$

$$x = 15$$

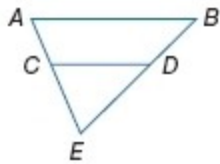
So, the length of the longest side of the second garden is 15 feet.

**ANSWER:**

15 ft

## 7-6 Similarity Transformations

Determine whether  $\overline{AB} \parallel \overline{CD}$ . Justify your answer.



33.  $AC = 8.4$ ,  $BD = 6.3$ ,  $DE = 4.5$ , and  $CE = 6$

**SOLUTION:**

$$\frac{AC}{BD} = \frac{8.4}{6.3} = \frac{84}{63} = \frac{4}{3}$$

yes;  $\frac{DE}{CE} = \frac{4.5}{6} = \frac{45}{60} = \frac{3}{4}$

**ANSWER:**

yes;  $\frac{AC}{BD} = \frac{DE}{CE} = \frac{4}{3}$

34.  $AC = 7$ ,  $BD = 10.5$ ,  $BE = 22.5$ , and  $AE = 15$

**SOLUTION:**

$$\frac{AC}{BD} = \frac{7}{10.5} = \frac{14}{21} = \frac{2}{3}$$

yes;  $\frac{AE}{BE} = \frac{15}{22.5} = \frac{30}{45} = \frac{2}{3}$

**ANSWER:**

yes;  $\frac{AC}{BD} = \frac{AE}{BE} = \frac{2}{3}$

35.  $AB = 8$ ,  $AE = 9$ ,  $CD = 4$ , and  $CE = 4$

**SOLUTION:**

$$\frac{AB}{CD} = \frac{8}{4} = \frac{2}{1}$$

no;  $\frac{AE}{CE} = \frac{9}{4}$ ,  $\frac{AB}{CD} \neq \frac{AE}{CE}$

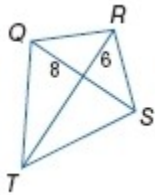
**ANSWER:**

no;  $\frac{AB}{CD} \neq \frac{AE}{CE}$

## 7-6 Similarity Transformations

If each figure is a kite, find each measure.

36.  $QR$



**SOLUTION:**

By the Pythagorean Theorem,

$$QR^2 = 8^2 + 6^2 = 100$$

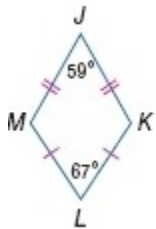
$$QR = \pm 10$$

Since the length must be positive,  $QR = 10$ .

**ANSWER:**

10

37.  $m\angle K$



**SOLUTION:**

A kite can only have one pair of opposite congruent angles and  $\angle J \neq \angle L$ , so  $\angle M \cong \angle K$ .

Let  $m\angle M = m\angle K = x$ .

The sum of the measures of the angles of a quadrilateral is 360.

$$m\angle J + m\angle L + m\angle M + m\angle K = 360$$

$$59 + 67 + x + x = 360$$

$$2x = 234$$

$$x = 117$$

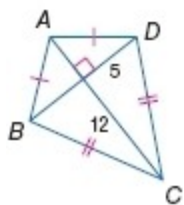
So,  $m\angle K = 117$ .

**ANSWER:**

117

## 7-6 Similarity Transformations

38.  $BC$



**SOLUTION:**

By the Pythagorean Theorem,

$$DC^2 = 12^2 + 5^2 = 169$$

$$DC = \pm 13$$

Since the length must be positive,  $DC = 13$ .

Here,  $DC = BC$ .

So,  $BC = 13$ .

**ANSWER:**

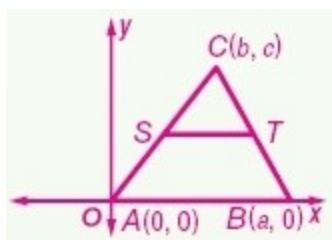
13

39. **PROOF** Write a coordinate proof for the following statement.

*If a line segment joins the midpoints of two sides of a triangle, then it is parallel to the third side.*

**SOLUTION:**

The "if" statement contains the given information for your proof; namely that you know the midpoints of two sides of a triangle. The "then" part of the sentence contains what you are trying to prove; that the midsegment formed by the midpoints of two sides is parallel to the third side of the triangle. It is helpful to place one of the sides of the triangle on the  $x$ -axis, for ease of calculation of coordinates and midpoints. Create a triangle and label the vertices  $A(0,0)$ ,  $B(a, 0)$ , and  $C(b, c)$ . Calculate the midpoint of each side, in terms of  $a$ ,  $b$ , and  $c$ . Then, compare the slope of the midsegment and the third side of the triangle, to prove that they are parallel.



Given:  $\triangle ABC$

$S$  is the midpoint of  $\overline{AC}$ .

$T$  is the midpoint of  $\overline{BC}$ .

Prove:  $\overline{ST} \parallel \overline{AB}$

Proof:

Midpoint  $S$  is  $\left(\frac{b+0}{2}, \frac{c+0}{2}\right)$  or  $\left(\frac{b}{2}, \frac{c}{2}\right)$ .

Midpoint  $T$  is  $\left(\frac{a+b}{2}, \frac{0+c}{2}\right)$  or  $\left(\frac{a+b}{2}, \frac{c}{2}\right)$ .

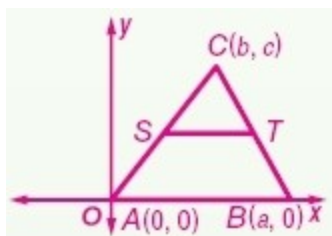
## 7-6 Similarity Transformations

$$\text{Slope of } \overline{ST} = \frac{\frac{c}{2} - \frac{c}{2}}{\frac{a+b}{2} - \frac{b}{2}} = \frac{0}{a} \text{ or } 0.$$

$$\text{Slope of } \overline{AB} = \frac{0-0}{a-0} = \frac{0}{a} \text{ or } 0.$$

$\overline{ST}$  and  $\overline{AB}$  have the same slope so  $\overline{ST} \parallel \overline{AB}$ .

ANSWER:



Given:  $\triangle ABC$

$S$  is the midpoint of  $\overline{AC}$ .

$T$  is the midpoint of  $\overline{BC}$ .

Prove:  $\overline{ST} \parallel \overline{AB}$

Proof:

$$\text{Midpoint } S \text{ is } \left( \frac{b+0}{2}, \frac{c+0}{2} \right) \text{ or } \left( \frac{b}{2}, \frac{c}{2} \right).$$

$$\text{Midpoint } T \text{ is } \left( \frac{a+b}{2}, \frac{0+c}{2} \right) \text{ or } \left( \frac{a+b}{2}, \frac{c}{2} \right).$$

$$\text{Slope of } \overline{ST} = \frac{\frac{c}{2} - \frac{c}{2}}{\frac{a+b}{2} - \frac{b}{2}} = \frac{0}{a} \text{ or } 0.$$

$$\text{Slope of } \overline{AB} = \frac{0-0}{a-0} = \frac{0}{a} \text{ or } 0.$$

$\overline{ST}$  and  $\overline{AB}$  have the same slope so  $\overline{ST} \parallel \overline{AB}$ .

**Solve each equation.**

40.  $145 = 29 \cdot t$

SOLUTION:

$$145 = 29t$$

$$\frac{145}{29} = \frac{29t}{29}$$

$$5 = t$$

ANSWER:

5

## 7-6 Similarity Transformations

41.  $216 = d \cdot 27$

*SOLUTION:*

$$216 = d \cdot 27$$

$$\frac{216}{27} = \frac{d \cdot 27}{27}$$

$$8 = d$$

*ANSWER:*

8

42.  $2r = 67 \cdot 5$

*SOLUTION:*

$$2r = 67 \cdot 5$$

$$2r = 335$$

$$r = \frac{335}{2}$$

$$r = 167.5$$

*ANSWER:*

167.5

43.  $100t = \frac{70}{240}$

*SOLUTION:*

$$100t = \frac{70}{240}$$

$$100t \approx 0.3$$

$$t \approx 0.003$$

*ANSWER:*

0.003

44.  $\frac{80}{4} = 14d$

*SOLUTION:*

$$\frac{80}{4} = 14d$$

$$20 = 14d$$

$$\frac{20}{14} = \frac{14d}{14}$$

$$d \approx 1.43.$$

*ANSWER:*

1.43

## 7-6 Similarity Transformations

$$45. \frac{2t+15}{t} = 92$$

*SOLUTION:*

$$\frac{2t+15}{t} = 92$$

$$2t+15 = 92t$$

$$90t = 15$$

$$t \approx 0.17$$

*ANSWER:*

0.17