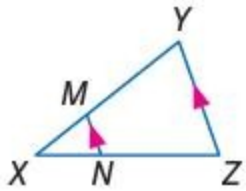


7-4 Parallel Lines and Proportional Parts

1. If $XM = 4$, $XN = 6$, and $NZ = 9$, find XY .



SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Use the Triangle Proportionality Theorem.

$$\frac{XM}{MY} = \frac{XN}{NZ}$$

Substitute.

$$\frac{4}{MY} = \frac{6}{9}$$

Solve for MY .

$$\begin{aligned} MY &= \frac{36}{6} \\ &= 6 \end{aligned}$$

Find XY .

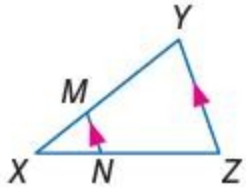
$$\begin{aligned} XY &= XM + MY \\ &= 4 + 6 \\ &= 10 \end{aligned}$$

ANSWER:

10

7-4 Parallel Lines and Proportional Parts

2. If $XN = 6$, $XM = 2$, and $XY = 10$, find NZ .



SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

$$XY = 10, \text{ So, } MY = 10 - 8 = 2.$$

Use the Triangle Proportionality Theorem.

$$\frac{2}{8} = \frac{6}{NZ}$$

Solve for NZ .

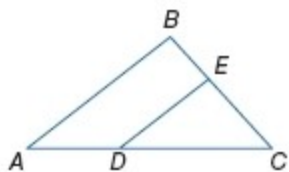
$$\begin{aligned} NZ &= \frac{48}{2} \\ &= 24 \end{aligned}$$

ANSWER:

24

7-4 Parallel Lines and Proportional Parts

3. In $\triangle ABC$, $BC = 15$, $BE = 6$, $DC = 12$, and $AD = 8$. Determine whether $\overline{DE} \parallel \overline{AB}$. Justify your answer.



SOLUTION:

If $BC = 15$, then $EC = 15 - 6 = 9$.

Use the Converse of the Triangle Proportionality Theorem.

$$\frac{AD}{DC} = \frac{8}{12} = \frac{2}{3}$$

$$\frac{BE}{EC} = \frac{6}{9} = \frac{2}{3}$$

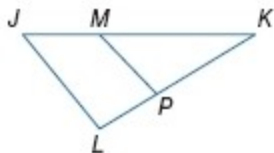
$$\text{so } \frac{AD}{DC} = \frac{BE}{EC} = \frac{2}{3}$$

Therefore, $\overline{DE} \parallel \overline{AB}$.

ANSWER:

yes; $\frac{AD}{DC} = \frac{BE}{EC} = \frac{2}{3}$, so $\overline{DE} \parallel \overline{AB}$.

4. In $\triangle JKL$, $JK = 15$, $JM = 5$, $LK = 13$, and $PK = 9$. Determine whether $\overline{JL} \parallel \overline{MP}$. Justify your answer.



SOLUTION:

$JK = 15$ and $LK = 13$. Therefore, $MK = 15 - 5 = 10$ and $LP = 13 - 9 = 4$.

Use the Converse of the Triangle Proportionality Theorem.

If $BC = 15$, then $EC = 15 - 6 = 9$.

Use the Converse of the Triangle Proportionality Theorem.

$$\frac{JM}{MK} = \frac{5}{10} = \frac{1}{2}$$

$$\frac{LP}{PK} = \frac{4}{9}$$

$$\frac{1}{2} \neq \frac{4}{9}$$

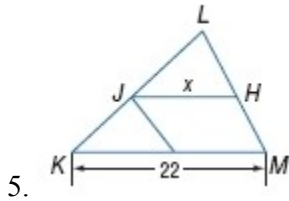
So, \overline{JL} and \overline{MP} are not parallel.

ANSWER:

no; $\frac{1}{2} \neq \frac{4}{9}$

7-4 Parallel Lines and Proportional Parts

\overline{JH} is a midsegment of $\triangle KLM$. Find the value of x .



SOLUTION:

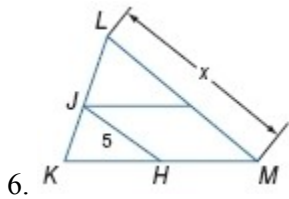
By the Triangle Midsegment Theorem, $JH = \frac{1}{2}KM$.

Substitute.

$$\begin{aligned}x &= \frac{1}{2}(22) \\ &= 11\end{aligned}$$

ANSWER:

11



SOLUTION:

By the Triangle Midsegment Theorem, $JH = \frac{1}{2}LM$.

Substitute.

$$\begin{aligned}5 &= \frac{1}{2}x \\ x &= 10\end{aligned}$$

ANSWER:

10

7-4 Parallel Lines and Proportional Parts

7. **MAPS** Refer to the map. 3rd Avenue and 5th Avenue are parallel. If the distance from 3rd Avenue to City Mall along State Street is 3201 feet, find the distance between 5th Avenue and City Mall along Union Street. Round to the nearest tenth.



SOLUTION:

The distance between 5th Avenue and City Mall along State Street is $3201 - 1056$ or 2145 feet.

Let x be the distance between 5th Avenue and City Mall along Union Street.

Use the Triangle Proportionality Theorem.

$$\frac{1056}{2145} = \frac{1162}{x}$$
$$x \approx 2360.3$$

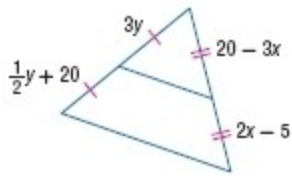
The distance between 5th Avenue and City Mall along Union Street is 2360.3 ft.

ANSWER:

2360.3 ft

7-4 Parallel Lines and Proportional Parts

ALGEBRA Find x and y .



8.

SOLUTION:

We are given that $3y = \frac{1}{2}y + 20$ and $2x - 5 = 20 - 3x$.

Solve for x .

$$2x - 5 = 20 - 3x$$

$$5x - 5 = 20$$

$$5x = 25$$

$$x = 5$$

Solve for y .

$$3y = \frac{1}{2}y + 20$$

$$3y - \frac{1}{2}y = 20$$

$$2\left(3y - \frac{1}{2}y\right) = 2(20)$$

$$6y - y = 40$$

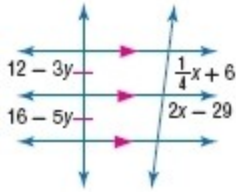
$$5y = 40$$

$$y = 8$$

ANSWER:

$$x = 5; y = 8$$

7-4 Parallel Lines and Proportional Parts



9.

SOLUTION:

We are given that $12 - 3y = 16 - 5y$.

Solve for y .

$$12 - 3y = 16 - 5y$$

$$12 + 2y = 16$$

$$2y = 4$$

$$y = 2$$

By Corollary 7.2, $2x - 29 = \frac{1}{4}x + 6$.

Solve for x .

$$2x - 29 = \frac{1}{4}x + 6$$

$$2x = \frac{1}{4}x + 35$$

$$2x - \frac{1}{4}x = 35$$

$$4\left(2x - \frac{1}{4}x\right) = 4(35)$$

$$8x - x = 140$$

$$7x = 140$$

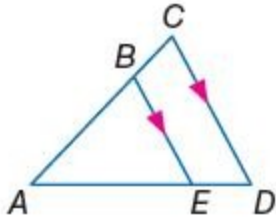
$$x = 20$$

ANSWER:

$$x = 20; y = 2$$

7-4 Parallel Lines and Proportional Parts

10. If $AB = 6$, $BC = 4$, and $AE = 9$, find ED .



SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Use the Triangle Proportionality Theorem.

$$\frac{AB}{BC} = \frac{AE}{ED}$$

Substitute.

$$\frac{6}{4} = \frac{9}{ED}$$

Solve for ED .

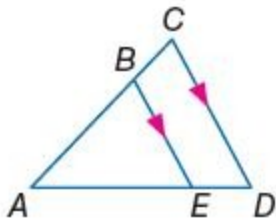
$$\begin{aligned} ED &= \frac{36}{6} \\ &= 6 \end{aligned}$$

ANSWER:

6

7-4 Parallel Lines and Proportional Parts

11. If $AB = 12$, $AC = 16$, and $ED = 5$, find AE .



SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

$AB = 12$. So, $BC = 16 - 12 = 4$.

Use the Triangle Proportionality Theorem.

$$\frac{AB}{BC} = \frac{AE}{ED}$$

Substitute.

$$\frac{12}{4} = \frac{AE}{5}$$

Solve for AE .

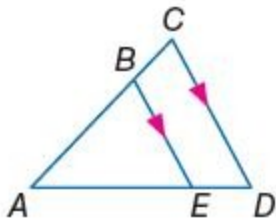
$$\begin{aligned} AE &= \frac{60}{4} \\ &= 15 \end{aligned}$$

ANSWER:

15

7-4 Parallel Lines and Proportional Parts

12. If $AC = 14$, $BC = 8$, and $AD = 21$, find ED .



SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Here, $BC = 8$. So, $AB = 14 - 8 = 6$. Let x be the length of the segment AE . So, $ED = 21 - x$.

Use the Triangle Proportionality Theorem.

$$\frac{AB}{BC} = \frac{AE}{ED}$$

Substitute.

$$\frac{6}{8} = \frac{x}{21-x}$$

Solve for x .

$$6(21-x) = 8x$$

$$126 - 6x = 8x$$

$$-14x = -126$$

$$x = 9$$

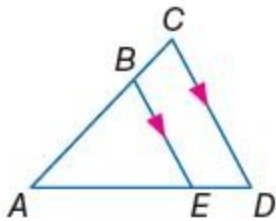
So, $AE = 9$ and $ED = 21 - 9 = 12$.

ANSWER:

12

7-4 Parallel Lines and Proportional Parts

13. If $AD = 27$, $AB = 8$, and $AE = 12$, find BC .



SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Here, $AE = 12$. So, $ED = 27 - 12 = 15$.

Use the Triangle Proportionality Theorem.

$$\frac{AB}{BC} = \frac{AE}{ED}$$

Substitute in values and solve for BC .

$$\frac{8}{BC} = \frac{12}{15}$$

$$BC = \frac{120}{12}$$

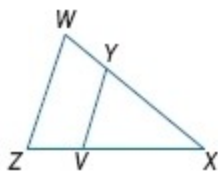
$$BC = 10$$

ANSWER:

10

7-4 Parallel Lines and Proportional Parts

Determine whether $\overline{VY} \parallel \overline{ZW}$. Justify your answer.



14. $ZX = 18$, $ZV = 6$, $WX = 24$, and $YX = 16$

SOLUTION:

$ZV = 6$ and $YX = 16$. Therefore, $VX = 18 - 6 = 12$ and $WY = 24 - 16 = 8$.

Use the Converse of the Triangle Proportionality Theorem.

$$\frac{ZV}{VX} = \frac{6}{12} = \frac{1}{2}$$
$$\frac{WY}{YX} = \frac{8}{16} = \frac{1}{2}$$

Since $\frac{ZV}{VX} = \frac{WY}{YX} = \frac{1}{2}$, then $\overline{VY} \parallel \overline{ZW}$.

ANSWER:

yes; $\frac{ZV}{VX} = \frac{WY}{YX} = \frac{1}{2}$

15. $VX = 7.5$, $ZX = 24$, $WY = 27.5$, and $WX = 40$

SOLUTION:

$VX = 7.5$ and $WY = 27.5$. So, $ZV = 24 - 7.5 = 16.5$ and $YX = 40 - 27.5 = 12.5$.

Use the Converse of the Triangle Proportionality Theorem.

$$\frac{ZV}{VX} = \frac{16.5}{7.5} = \frac{11}{5}$$
$$\frac{WY}{YX} = \frac{27.5}{12.5} = \frac{11}{5}$$

Since $\frac{ZV}{VX} = \frac{WY}{YX} = \frac{11}{5}$, so $\overline{VY} \parallel \overline{ZW}$.

ANSWER:

yes; $\frac{ZV}{VX} = \frac{WY}{YX} = \frac{11}{5}$

7-4 Parallel Lines and Proportional Parts

16. $ZV = 8$, $VX = 2$, and $YX = \frac{1}{2} WY$

SOLUTION:

Use the Converse of the Triangle Proportionality Theorem.

$$\frac{ZV}{VX} = \frac{8}{2} = 4$$

$$\frac{WY}{YX} = \frac{WY}{\frac{WY}{2}} = WY \cdot \frac{2}{WY} = 2$$

Because $\frac{ZV}{VX} \neq \frac{WY}{YX}$, \overline{VY} and \overline{ZW} are not parallel.

ANSWER:

no; $\frac{ZV}{VX} \neq \frac{WY}{YX}$

17. $WX = 31$, $YX = 21$, and $ZX = 4ZV$

SOLUTION:

$YX = 21$, so $WY = 31 - 21 = 10$ and since $ZX = 4ZV$, then $VX = 3ZV$.

Use the Converse of the Triangle Proportionality Theorem.

$$\frac{ZV}{VX} = \frac{ZV}{3ZV} = \frac{1}{3}$$

$$\frac{WY}{YX} = \frac{10}{21}$$

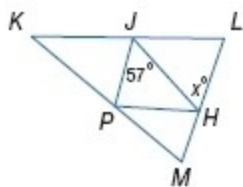
Because $\frac{ZV}{VX} \neq \frac{WY}{YX}$, we can say that \overline{VY} and \overline{ZW} are not parallel.

ANSWER:

no; $\frac{ZV}{VX} \neq \frac{WY}{YX}$

7-4 Parallel Lines and Proportional Parts

\overline{JH} , \overline{JP} , and \overline{PH} are midsegments of $\triangle KLM$. Find the value of x .



18.

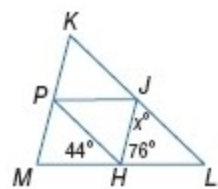
SOLUTION:

By the Triangle Midsegment Theorem, $\overline{JP} \parallel \overline{LM}$.

By the Alternate Interior Angles Theorem, $x = 57$.

ANSWER:

57



19.

SOLUTION:

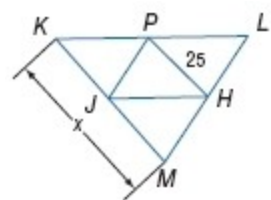
By the Triangle Midsegment Theorem, $\overline{PH} \parallel \overline{KL}$.

$$\begin{aligned} m\angle PHJ &= 180 - (44 + 76) \\ &= 180 - 120 \\ &= 60 \end{aligned}$$

By the Alternate Interior Angles Theorem, $x = m\angle PHJ = 60$.

ANSWER:

60



20.

SOLUTION:

By the Triangle Midsegment Theorem, $PH = \frac{1}{2}KM$.

Substitute.

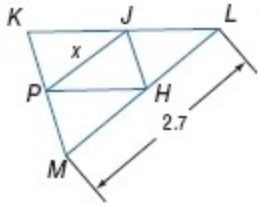
$$25 = \frac{1}{2}(KM)$$

$$KM = 50$$

ANSWER:

50

7-4 Parallel Lines and Proportional Parts



21.

SOLUTION:

By the Triangle Midsegment Theorem, $PJ = \frac{1}{2}ML$.

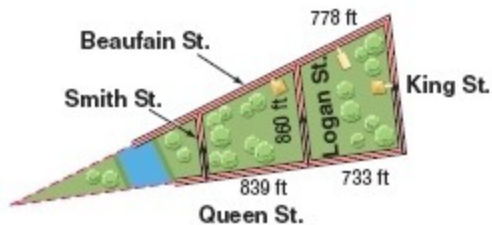
Substitute.

$$\begin{aligned} x &= \frac{1}{2}(2.7) \\ &= 1.35 \end{aligned}$$

ANSWER:

1.35

22. **CCSS MODELING** In Charleston, South Carolina, Logan Street is parallel to both King Street and Smith Street between Beaufain Street and Queen Street. What is the distance from Smith to Logan along Beaufain? Round to the nearest foot.



SOLUTION:

Let x be the distance from Smith to Logon along Beaufain. Use the Triangle Proportionality Theorem.

$$\frac{778}{x} = \frac{733}{839}$$

Solve for x .

$$\begin{aligned} x &= \frac{653742}{733} \\ &= 890.5075... \\ &= 891 \end{aligned}$$

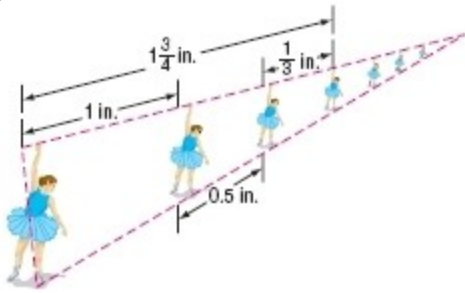
So, the distance from Smith to Logan is 891 ft.

ANSWER:

about 891 ft

7-4 Parallel Lines and Proportional Parts

23. **ART** Tonisha drew the line of dancers shown below for her perspective project in art class. Each of the dancers is parallel. Find the lower distance between the first two dancers.



SOLUTION:

$$\text{Distance between second dancer and third dancer} = 1\frac{3}{4} - \left(1 + \frac{1}{3}\right)$$

$$\begin{aligned} 1\frac{3}{4} - \left(1 + \frac{1}{3}\right) &= \frac{7}{4} - \left(\frac{3}{3} + \frac{1}{3}\right) \\ &= \frac{7}{4} - \frac{4}{3} \\ &= \frac{7 \cdot 3}{4 \cdot 3} - \frac{4 \cdot 4}{3 \cdot 4} \\ &= \frac{21}{12} - \frac{16}{12} \\ &= \frac{5}{12} \end{aligned}$$

Let x be the lower distance between the first two dancers.

Use the Triangle Proportionality Theorem.

$$\frac{1}{\left(\frac{5}{12}\right)} = \frac{x}{0.5}$$

$$\frac{0.5}{1} = \frac{5x}{12}$$

$$0.5 \cdot 12 = 5x$$

$$6 = 5x$$

$$\frac{6}{5} = x$$

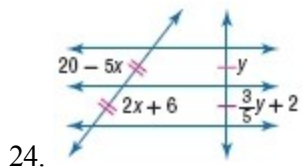
So, the lower distance between the first two dancers is $\frac{6}{5}$ or 1.2 inches.

ANSWER:

1.2 in.

7-4 Parallel Lines and Proportional Parts

ALGEBRA Find x and y .



SOLUTION:

We are given that $y = \frac{3}{5}y + 2$ and $2x + 6 = 20 - 5x$.

Solve for x .

$$2x + 6 = 20 - 5x$$

$$7x = 14$$

$$x = 2$$

Solve for y .

$$y = \frac{3}{5}y + 2$$

$$y - \frac{3}{5}y = 2$$

$$5\left(y - \frac{3}{5}y\right) = 5(2)$$

$$5y - 3y = 10$$

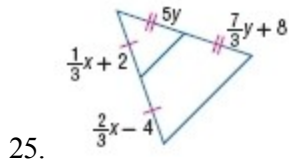
$$2y = 10$$

$$y = 5$$

ANSWER:

$$x = 2; y = 5$$

7-4 Parallel Lines and Proportional Parts



SOLUTION:

We are given that $5y = \frac{7}{3}y + 8$ and $\frac{2}{3}x - 4 = \frac{1}{3}x + 2$.

Solve for x .

$$\frac{2}{3}x - 4 = \frac{1}{3}x + 2.$$

$$\frac{2}{3}x - \frac{1}{3}x = 2 + 4$$

$$\frac{1}{3}x = 6$$

$$3 \cdot \frac{1}{3}x = 3 \cdot 6$$

$$x = 18$$

Solve for y .

$$5y = \frac{7}{3}y + 8$$

$$5y - \frac{7}{3}y = 8$$

$$3\left(5y - \frac{7}{3}y\right) = 3(8)$$

$$15y - 7y = 24$$

$$8y = 24$$

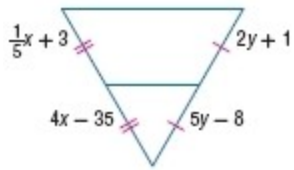
$$y = 3$$

ANSWER:

$$x = 18; y = 3$$

7-4 Parallel Lines and Proportional Parts

ALGEBRA Find x and y .



26.

SOLUTION:

It is given that $5y - 8 = 2y + 1$ and $\frac{1}{5}x + 3 = 4x - 35$.

Solve for x .

$$\frac{1}{5}x + 3 = 4x - 35$$

$$\frac{1}{5}x + 38 = 4x$$

$$5\left(\frac{1}{5}x + 38\right) = 5(4x)$$

$$x + 190 = 20x$$

$$-19x = -190$$

$$x = 10$$

Solve for y .

$$5y - 8 = 2y + 1$$

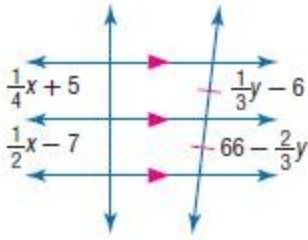
$$3y = 9$$

$$y = 3$$

ANSWER:

$$x = 10; y = 3$$

7-4 Parallel Lines and Proportional Parts



27.

SOLUTION:

We are given that $\frac{1}{3}y - 6 = 66 - \frac{2}{3}y$.

Solve for y .

$$\frac{1}{3}y - 6 = 66 - \frac{2}{3}y$$

$$\frac{1}{3}y + \frac{2}{3}y = 66 + 6$$

$$\frac{3}{3}y = 72$$

$$y = 72$$

By Corollary 7.2, $\frac{1}{2}x - 7 = \frac{1}{4}x + 5$.

Solve for x .

$$\frac{1}{2}x - 7 = \frac{1}{4}x + 5$$

$$4\left(\frac{1}{2}x - 7\right) = 4\left(\frac{1}{4}x + 5\right)$$

$$2x - 28 = x + 20$$

$$2x - x = 20 + 28$$

$$x = 48$$

ANSWER:

$$x = 48; y = 72$$

7-4 Parallel Lines and Proportional Parts

CCSS ARGUMENTS Write a paragraph proof.

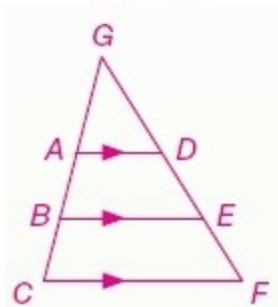
28. Corollary 7.1

SOLUTION:

In Corollary 7.1, it is stated that, if three or more parallel lines intersect two transversals, then they cut off the transversals proportionally. A good approach to this proof is to apply the Triangle Proportionality theorem, one triangle at a time.

Given: $\overline{AD} \parallel \overline{BE} \parallel \overline{CF}$

Prove: $\frac{AB}{BC} = \frac{DE}{EF}$



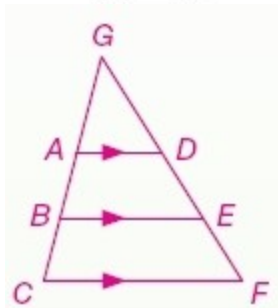
Proof: In $\triangle GBE$, $\overline{AD} \parallel \overline{BE}$. By the Triangle Proportionality Theorem, AB and DE are proportional. In

$\triangle GCF$, $\overline{BE} \parallel \overline{CF}$. By the Triangle Proportionality Theorem, BC and EF are proportional. Therefore, $\frac{AB}{BC} = \frac{DE}{EF}$.

ANSWER:

Given: $\overline{AD} \parallel \overline{BE} \parallel \overline{CF}$

Prove: $\frac{AB}{BC} = \frac{DE}{EF}$



Proof: In $\triangle GBE$, $\overline{AD} \parallel \overline{BE}$. By the Triangle Proportionality Theorem, AB and DE are proportional. In

$\triangle GCF$, $\overline{BE} \parallel \overline{CF}$. By the Triangle Proportionality Theorem, BC and EF are proportional. Therefore, $\frac{AB}{BC} = \frac{DE}{EF}$.

7-4 Parallel Lines and Proportional Parts

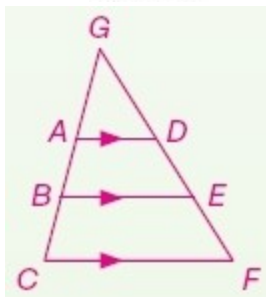
29. Corollary 7.2

SOLUTION:

Corollary 7.2 states, if three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal. This proof can be approached by using Corollary 7.1 to establish that, since we have three parallel lines, then we know they cut off the transversals proportionally. If the ratio of one side of this proportion is equal to 1, since both parts are equal, then the other side of the proportion must also equal 1. Therefore, they are also equal, or congruent, parts.

Given: $\overline{AD} \parallel \overline{BE} \parallel \overline{CF}$, $\overline{AB} \cong \overline{BC}$

Prove: $\overline{DE} \cong \overline{EF}$



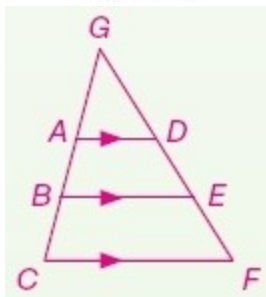
Proof: From Corollary 7.1, $\frac{AB}{BC} = \frac{DE}{EF}$. Since $\overline{AB} \cong \overline{BC}$, $AB = BC$ by definition of congruence. Therefore, $\frac{AB}{BC} = 1$.

By substitution, $1 = \frac{DE}{EF}$. Thus, $DE = EF$. By definition of congruence, $\overline{DE} \cong \overline{EF}$.

ANSWER:

Given: $\overline{AD} \parallel \overline{BE} \parallel \overline{CF}$, $\overline{AB} \cong \overline{BC}$

Prove: $\overline{DE} \cong \overline{EF}$



Proof: From Corollary 7.1, $\frac{AB}{BC} = \frac{DE}{EF}$. Since $\overline{AB} \cong \overline{BC}$, $AB = BC$ by definition of congruence. Therefore, $\frac{AB}{BC} = 1$.

By substitution, $1 = \frac{DE}{EF}$. Thus, $DE = EF$. By definition of congruence, $\overline{DE} \cong \overline{EF}$.

30. Theorem 7.5

SOLUTION:

Theorem 7.5 states, if a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths. In order to prove that $\frac{BA}{CB} = \frac{DE}{CD}$, we first need to establish that

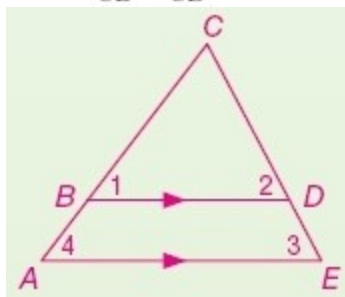
$\frac{CA}{CB} = \frac{CE}{CD}$, which can be accomplished by showing that $\triangle ACE \sim \triangle BCD$. Then, by Segment Addition Postulate, we

7-4 Parallel Lines and Proportional Parts

can state that $CA = BA + CB$ and $CE = DE + CD$. Substitute these values in for CA and CE in the previous proportion and the simplify.

Given: $\overline{BD} \parallel \overline{AE}$

Prove: $\frac{BA}{CB} = \frac{DE}{CD}$

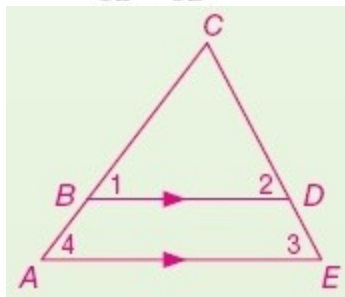


Proof: $\overline{BD} \parallel \overline{AE}$, $\angle 4 \cong \angle 1$ and $\angle 3 \cong \angle 2$ because they are corresponding angles. By AA Similarity, $\triangle ACE \sim \triangle BCD$. From the definition of similar polygons, $\frac{CA}{CB} = \frac{CE}{CD}$. By the Segment Addition Postulate, $CA = BA + CB$ and $CE = DE + CD$. By substitution, $\frac{BA+CB}{CB} = \frac{DE+CD}{CD}$. Rewriting as a sum, $\frac{BA}{CB} + \frac{CB}{CB} = \frac{DE}{CD} + \frac{CD}{CD}$. From simplifying, $\frac{BA}{CB} + 1 = \frac{DE}{CD} + 1$. Thus, $\frac{BA}{CB} = \frac{DE}{CD}$ by subtracting one from each side.

ANSWER:

Given: $\overline{BD} \parallel \overline{AE}$

Prove: $\frac{BA}{CB} = \frac{DE}{CD}$



Proof: $\overline{BD} \parallel \overline{AE}$, $\angle 4 \cong \angle 1$ and $\angle 3 \cong \angle 2$ because they are corresponding angles. By AA Similarity, $\triangle ACE \sim \triangle BCD$. From the definition of similar polygons, $\frac{CA}{CB} = \frac{CE}{CD}$. By the Segment Addition Postulate, $CA = BA + CB$ and $CE = DE + CD$. By substitution, $\frac{BA+CB}{CB} = \frac{DE+CD}{CD}$. Rewriting as a sum, $\frac{BA}{CB} + \frac{CB}{CB} = \frac{DE}{CD} + \frac{CD}{CD}$. From simplifying, $\frac{BA}{CB} + 1 = \frac{DE}{CD} + 1$. Thus, $\frac{BA}{CB} = \frac{DE}{CD}$ by subtracting one from each side.

CCSS ARGUMENTS Write a two-column proof.

31. Theorem 7.6

SOLUTION:

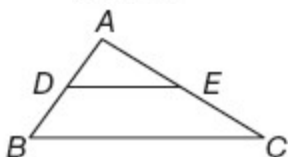
Theorem 7.6 states, if a line intersects two sides of a triangle and separates the sides into proportional corresponding segments, then the line is parallel to the third side of the triangle. Thinking backwards, how can we prove that two

7-4 Parallel Lines and Proportional Parts

lines are parallel to each other? We can prove that $\overline{DE} \parallel \overline{BC}$ by proving that a pair of corresponding angles, formed by these parallel lines, are congruent to each other. Using SAS Similarity theorem, prove that . Then, you can use congruent corresponding angles as a result of similar triangles.

Given: $\frac{DB}{AD} = \frac{EC}{AE}$

Prove: $\overline{DE} \parallel \overline{BC}$



Proof:

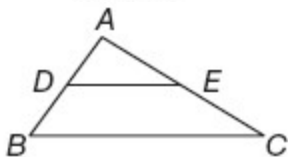
Statements (Reasons)

1. $\frac{DB}{AD} = \frac{EC}{AE}$ (Given)
2. $\frac{AD}{AD} + \frac{DB}{AD} = \frac{AE}{AE} + \frac{EC}{AE}$ (Add. Prop.)
3. $\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$ (Subst.)
4. $AB = AD + DB, AC = AE + EC$ (Seg. Add. Post.)
5. $\frac{AB}{AD} = \frac{AC}{AE}$ (Subst.)
6. $\angle A \cong \angle A$ (Refl. Prop.)
7. $\triangle ADE \sim \triangle ABC$ (SAS Similarity)
8. $\angle ADE \cong \angle ABC$ (Def. of \sim polygons)
9. $\overline{DE} \parallel \overline{BC}$ (If corr. angles are \cong , then the lines are \parallel .)

ANSWER:

Given: $\frac{DB}{AD} = \frac{EC}{AE}$

Prove: $\overline{DE} \parallel \overline{BC}$



Proof:

Statements (Reasons)

1. $\frac{DB}{AD} = \frac{EC}{AE}$ (Given)
2. $\frac{AD}{AD} + \frac{DB}{AD} = \frac{AE}{AE} + \frac{EC}{AE}$ (Add. Prop.)
3. $\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$ (Subst.)
4. $AB = AD + DB, AC = AE + EC$ (Seg. Add. Post.)

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5. $\frac{AB}{AD} = \frac{AC}{AE}$ (Subst.)
6. $\angle A \cong \angle A$ (Refl. Prop.)
7. $\triangle ADE \sim \triangle ABC$ (SAS Similarity)
8. $\angle ADE \cong \angle ABC$ (Def. of \sim polygons)
9. $\overline{DE} \parallel \overline{BC}$ (If corr. angles are \cong , then the lines are \parallel .)

32. Theorem 7.7

SOLUTION:

Theorem 7.7 states that a midsegment of a triangle is parallel to one side of the triangle, and its length is half the length of that side. For this proof, use the given information that $\overline{DE} \parallel \overline{BC}$ to prove that $\triangle ADE \sim \triangle ABC$ by AA

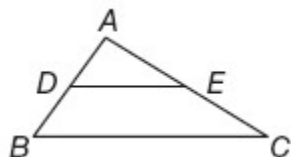
Similarity. Then, since you know that D and E are both midpoints, then you can prove eventually prove that

$\frac{AB}{AD} = 2, \frac{AC}{AE} = 2$, using midpoint relationships and substitution. Then, using $\frac{BC}{DE} = \frac{AB}{AD}$ as a result of proving

$\triangle ADE \sim \triangle ABC$, then you can substitute into $\frac{AB}{AD} = 2$ into $\frac{BC}{DE} = \frac{AB}{AD}$ and prove that $DE = \frac{1}{2}BC$, using algebra.

Given: D is the midpoint of \overline{AB} .

E is the midpoint of \overline{AC} .



Prove: $\overline{DE} \parallel \overline{BC}$; $DE = \frac{1}{2}BC$

Proof:

Statements (Reasons)

1. D is the midpoint of \overline{AB} ; E is the midpoint of \overline{AC} . (Given)
2. $\overline{AD} \cong \overline{DB}$, $\overline{AE} \cong \overline{EC}$ (Midpoint Thm.)
3. $AD = DB, AE = EC$ (Def. of \cong segs.)
4. $AB = AD + DB, AC = AE + EC$ (Seg. Add. Post.)
5. $AB = AD + AD, AC = AE + AE$ (Subst.)
6. $AB = 2AD, AC = 2AE$ (Subst.)
7. $\frac{AB}{AD} = 2, \frac{AC}{AE} = 2$ (Div. Prop.)
8. $\frac{AB}{AD} = \frac{AC}{AE}$ (Trans. Prop.)
9. $\angle A \cong \angle A$ (Refl. Prop.)
10. $\triangle ADE \sim \triangle ABC$ (SAS Similarity)
11. $\angle ADE \cong \angle ABC$ (Def. of \sim polygons)
12. $\overline{DE} \parallel \overline{BC}$ (If corr. angles are \cong , the lines are parallel.)
13. $\frac{BC}{DE} = \frac{AB}{AD}$ (Def. of \sim polygons)

7-4 Parallel Lines and Proportional Parts

14. $\frac{BC}{DE} = 2$ (Substitution Prop.)

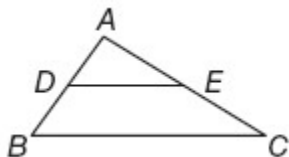
15. $2DE = BC$ (Mult. Prop.)

16. $DE = \frac{1}{2}BC$ (Division Prop.)

ANSWER:

Given: D is the midpoint of \overline{AB} .

E is the midpoint of \overline{AC} .



Prove: $\overline{DE} \parallel \overline{BC}$; $DE = \frac{1}{2}BC$

Proof:

Statements (Reasons)

1. D is the midpoint of \overline{AB} ; E is the midpoint of \overline{AC} . (Given)

2. $\overline{AD} \cong \overline{DB}$, $\overline{AE} \cong \overline{EC}$ (Midpoint Thm.)

3. $AD = DB$, $AE = EC$ (Def. of \cong segs.)

4. $AB = AD + DB$, $AC = AE + EC$ (Seg. Add. Post.)

5. $AB = AD + AD$, $AC = AE + AE$ (Subst.)

6. $AB = 2AD$, $AC = 2AE$ (Subst.)

7. $\frac{AB}{AD} = 2$, $\frac{AC}{AE} = 2$ (Div. Prop.)

8. $\frac{AB}{AD} = \frac{AC}{AE}$ (Trans. Prop.)

9. $\angle A \cong \angle A$ (Refl. Prop.)

10. $\triangle ADE \sim \triangle ABC$ (SAS Similarity)

11. $\angle ADE \cong \angle ABC$ (Def. of \sim polygons)

12. $\overline{DE} \parallel \overline{BC}$ (If corr. angles are \cong , the lines are parallel.)

13. $\frac{BC}{DE} = \frac{AB}{AD}$ (Def. of \sim polygons)

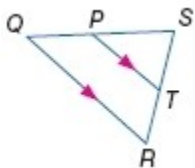
14. $\frac{BC}{DE} = 2$ (Substitution Prop.)

15. $2DE = BC$ (Mult. Prop.)

16. $DE = \frac{1}{2}BC$ (Division Prop.)

7-4 Parallel Lines and Proportional Parts

Refer to $\triangle QRS$.



33. If $ST = 8$, $TR = 4$, and $PT = 6$, find QR .

SOLUTION:

Since $\overline{PT} \parallel \overline{QR}$, we know that $\angle SPT \cong \angle SQR$ and $\angle STP \cong \angle SRQ$. Therefore, by AA Similarity, $\triangle SPT \sim \triangle SQR$.

Use the definition of similar polygons to create a proportion:

$$\frac{QR}{PT} = \frac{SR}{ST}$$

We know that $SR = 8 + 4 = 12$. Substitute values and solve for QR .

$$\begin{aligned}\frac{QR}{6} &= \frac{12}{8} \\ QR &= \frac{72}{8} \\ QR &= 9\end{aligned}$$

ANSWER:

9

34. If $SP = 4$, $PT = 6$, and $QR = 12$, find SQ .

SOLUTION:

Since $\overline{PT} \parallel \overline{QR}$, we know that $\angle SPT \cong \angle SQR$ and $\angle STP \cong \angle SRQ$. Therefore, by AA Similarity, $\triangle SPT \sim \triangle SQR$.

Use the definition of similar polygons to set up a proportion:

$$\frac{QR}{PT} = \frac{SQ}{SP}$$

Substitute and solve for SQ :

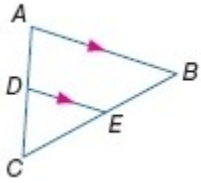
$$\begin{aligned}\frac{12}{6} &= \frac{SQ}{4} \\ 2 &= \frac{SQ}{4} \\ SQ &= 8\end{aligned}$$

ANSWER:

8

7-4 Parallel Lines and Proportional Parts

35. If $CE = t - 2$, $EB = t + 1$, $CD = 2$, and $CA = 10$, find t and CE .



SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Use the Triangle Proportionality Theorem.

$$\frac{CE}{EB} = \frac{CD}{DA}$$

Since $CA = 10$ and $CD = 2$, then $DA = 10 - 2 = 8$.

Substitute and solve for t .

$$\frac{t-2}{t+1} = \frac{2}{8}$$

$$8(t-2) = 2(t+1)$$

$$8t - 16 = 2t + 2$$

$$6t = 18$$

$$t = 3$$

Find CE .

$$CE = t - 2$$

$$= 3 - 2$$

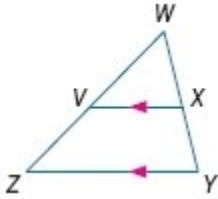
$$= 1$$

ANSWER:

3, 1

7-4 Parallel Lines and Proportional Parts

36. If $WX = 7$, $WY = a$, $WV = 6$, and $VZ = a - 9$, find WY .



SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Use the Triangle Proportionality Theorem.

$$\frac{WX}{XY} = \frac{WV}{VZ}$$

Since $WY = a$ and $WX = 7$, $XY = a - 7$.

Substitute and solve for a .

$$\frac{7}{a-7} = \frac{6}{a-9}$$

$$7(a-9) = 6(a-7)$$

$$7a - 63 = 6a - 42$$

$$a = 21$$

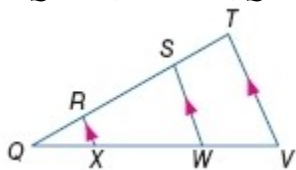
So, $a = WY = 21$.

ANSWER:

21

7-4 Parallel Lines and Proportional Parts

37. If $QR = 2$, $XW = 12$, $QW = 15$, and $ST = 5$, find RS and WV .



SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Use the Triangle Proportionality Theorem.

$$\frac{QR}{RS} = \frac{QX}{XW}$$

Since $QW = 15$ and $WX = 12$, then $QX = 3$.

Substitute and solve for RS .

$$\frac{2}{RS} = \frac{3}{12}$$

$$3RS = 24$$

$$RS = 8$$

Additionally, we know that $\frac{QS}{ST} = \frac{QW}{WV}$.

Substitute and solve for WV .

$$\frac{10}{5} = \frac{15}{WV}$$

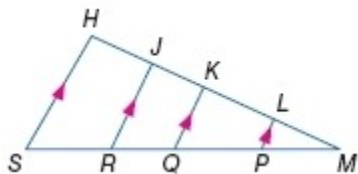
$$10WV = 75$$

$$WV = 7.5$$

ANSWER:

8, 7.5

38. If $LK = 4$, $MP = 3$, $PQ = 6$, $KJ = 2$, $RS = 6$, and $LP = 2$, find ML , QR , QK , and JH .



SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

7-4 Parallel Lines and Proportional Parts

Use the Triangle Proportionality Theorem.

$$\frac{ML}{LK} = \frac{MP}{PQ}$$

Substitute and solve for ML .

$$\frac{ML}{4} = \frac{3}{6}$$

$$6ML = 12$$

$$ML = 2$$

Also, we know that $\frac{LK}{JK} = \frac{PQ}{QR}$.

Substitute and solve for

$$\frac{4}{2} = \frac{6}{QR}$$

$$4QR = 12$$

$$QR = 3$$

Because $\triangle MLP \sim \triangle MKQ$, by AA Similarity, we know that $\frac{MP}{PL} = \frac{MQ}{QK}$.

Substitute and solve for QK .

$$\frac{MP}{PL} = \frac{MQ}{QK}$$

$$\frac{3}{2} = \frac{9}{QK}$$

$$3QK = 18$$

$$QK = 6$$

Finally, by Triangle Proportionality Theorem, $\frac{KJ}{JH} = \frac{QR}{RS}$. Substitute and solve for JH .

$$\frac{2}{JH} = \frac{3}{6}$$

$$3JH = 12$$

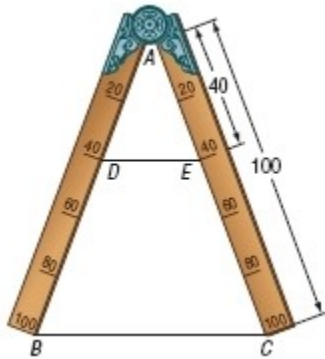
$$JH = 4$$

ANSWER:

2, 3, 6, 4

7-4 Parallel Lines and Proportional Parts

39. **MATH HISTORY** The sector compass was a tool perfected by Galileo in the sixteenth century for measurement. To draw a segment two-fifths the length of a given segment, align the ends of the arms with the given segment. Then draw a segment at the 40 mark. Write a justification that explains why the sector compass works for proportional measurement.



SOLUTION:

To prove that two corresponding sides of two triangles are the same ratio as another pair of corresponding sides, you need to first establish that the triangles are similar. Once this is completed, a proportion statement can be written, relating the proportional sides. Substitute in given values from the diagram to prove that

$$\frac{2}{5}BC = DE$$

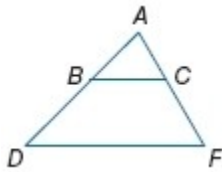
$\triangle ABC \sim \triangle ADE$	SAS Similarity
$\frac{AD}{AB} = \frac{DE}{BC}$	Def. of $\sim \Delta$ s
$\frac{40}{100} = \frac{DE}{BC}$	Substitution
$\frac{2}{5} = \frac{DE}{BC}$	Simplify.
$\frac{2}{5}BC = DE$	Multiply.

ANSWER:

$\triangle ABC \sim \triangle ADE$	SAS Similarity
$\frac{AD}{AB} = \frac{DE}{BC}$	Def. of $\sim \Delta$ s
$\frac{40}{100} = \frac{DE}{BC}$	Substitution
$\frac{2}{5} = \frac{DE}{BC}$	Simplify.
$\frac{2}{5}BC = DE$	Multiply.

7-4 Parallel Lines and Proportional Parts

Determine the value of x so that $\overline{BC} \parallel \overline{DF}$.



40. $AB = x + 5$, $BD = 12$, $AC = 3x + 1$, and $CF = 15$

SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Use the Triangle Proportionality Theorem.

$$\frac{AB}{BC} = \frac{AC}{CF}$$

Substitute.

$$\frac{x+5}{12} = \frac{3x+1}{15}$$

$$15x + 75 = 36x + 12$$

$$21x = 63$$

$$x = 3$$

ANSWER:

3

7-4 Parallel Lines and Proportional Parts

41. $AC = 15$, $BD = 3x - 2$, $CF = 3x + 2$, and $AB = 12$

SOLUTION:

Triangle Proportionality Theorem:

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Use the Triangle Proportionality Theorem.

$$\frac{AB}{BC} = \frac{AC}{CF}$$

Substitute.

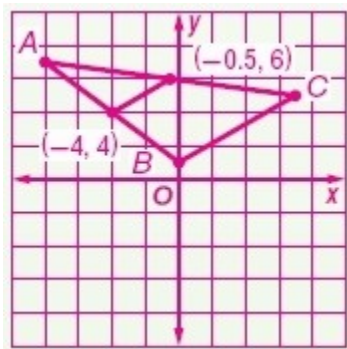
$$\begin{aligned}\frac{12}{3x-2} &= \frac{15}{3x+2} \\ 36x+24 &= 45x-30 \\ 9x &= 54 \\ x &= 6\end{aligned}$$

ANSWER:

6

42. **COORDINATE GEOMETRY** $\triangle ABC$ has vertices $A(-8, 7)$, $B(0, 1)$, and $C(7, 5)$. Draw $\triangle ABC$. Determine the coordinates of the midsegment of $\triangle ABC$ that is parallel to \overline{BC} . Justify your answer.

SOLUTION:



Use the midpoint formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ to determine the midpoints of \overline{AB} and \overline{AC} .

$$\text{The midpoint of } \overline{AB} = \left(\frac{-8+0}{2}, \frac{7+1}{2}\right) = (-4, 4).$$

$$\text{The midpoint of } \overline{AC} = \left(\frac{-8+7}{2}, \frac{7+5}{2}\right) = (-0.5, 6).$$

$$\text{Slope of } BC = \frac{5-1}{7-0} = \frac{4}{7}$$

7-4 Parallel Lines and Proportional Parts

$$\begin{aligned}\text{Slope of the midsegment} &= \frac{4 - 6}{-4 - (-0.5)} \\ &= \frac{-2}{-3.5} = \frac{4}{7}\end{aligned}$$

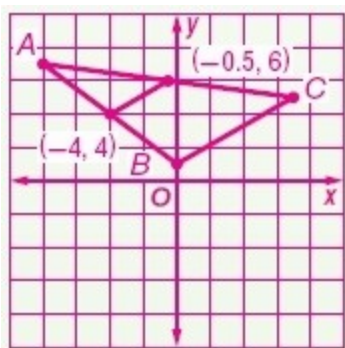
Use the distance formula.

$$\begin{aligned}BC &= \sqrt{(7 - 0)^2 + (5 - 1)^2} \\ &= \sqrt{49 + 16} \\ &= \sqrt{65} \approx 8.06\end{aligned}$$

$$\begin{aligned}\text{Length of the segment} &= \sqrt{(-4 + 0.5)^2 + (6 - 4)^2} \\ &= \sqrt{12.25 + 4} \\ &= \sqrt{16.25}\end{aligned}$$

The segment is parallel to \overline{BC} because the slopes are both $\frac{4}{7}$ and the segment length is half of \overline{BC} . Thus, the segment is the midsegment of $\triangle ABC$.

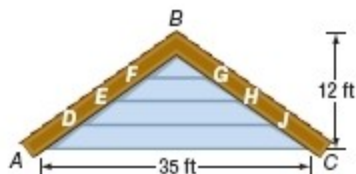
ANSWER:



The endpoints of the midsegment are $(-4, 4)$ and $(-0.5, 6)$. Sample answer: The segment is parallel to \overline{BC} because the slopes are both $\frac{4}{7}$ and the segment length is half of \overline{BC} . Thus, the segment is the midsegment of $\triangle ABC$.

7-4 Parallel Lines and Proportional Parts

43. **HOUSES** Refer to the diagram of the gable. Each piece of siding is a uniform width. Find the lengths of \overline{FG} , \overline{EH} , and \overline{DJ} .



SOLUTION:

All the triangles are isosceles. Segment \overline{EH} is the midsegment of triangle ABC . Therefore, segment \overline{EH} is the half of the length of \overline{AC} , which is $35 \div 2$ or 17.5 feet. Similarly, \overline{FG} is the midsegment of triangle BEH , so $\overline{FG} = 17.5 \div 2$ or 8.75 feet.

To find \overline{DJ} , use the vertical altitude which is 12 feet. Let the altitude from B to the segment \overline{AC} meet the segment \overline{DJ} at K . Find \overline{BC} using the Pythagorean Theorem.

$$BC^2 = BK^2 + KC^2$$

$$BC^2 = 12^2 + 17.5^2$$

$$BC = \sqrt{12^2 + 17.5^2}$$

$$BC \approx 21.22 \text{ in.}$$

Since the width of each piece of siding is the same, $\overline{BJ} = \frac{3}{4}\overline{BC}$, which is about $\frac{3}{4}(21.22)$ or 15.92 in.

Now, use the Triangle Proportionality Theorem.

$$\frac{AC}{BC} = \frac{DJ}{BJ}$$

$$\frac{35}{21.22} = \frac{DJ}{15.92}$$

$$21.22(DJ) = (15.92)(35)$$

$$21.22(DJ) = 557.2$$

$$DJ \approx 26.25 \text{ in.}$$

ANSWER:

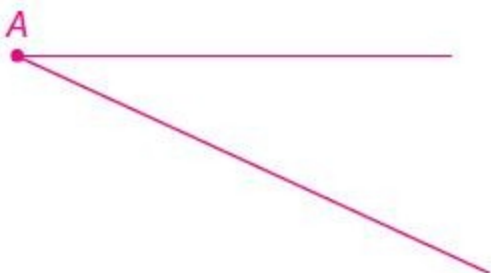
8.75 in., 17.5 in., 26.25 in.

CONSTRUCTIONS Construct each segment as directed.

44. a segment separated into five congruent segments

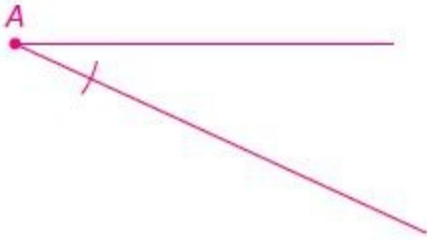
SOLUTION:

Step 1: Construct an angle with vertex A , as shown below:

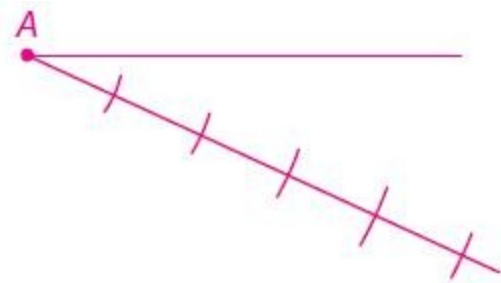


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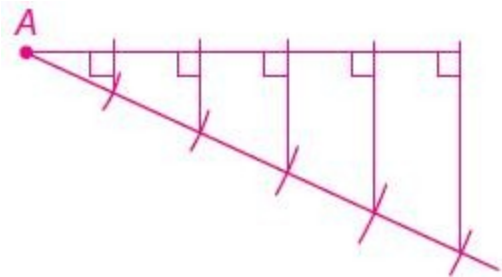
Step 2: With your compass on vertex A, choose a radius and make an arc on the diagonal, as shown below:



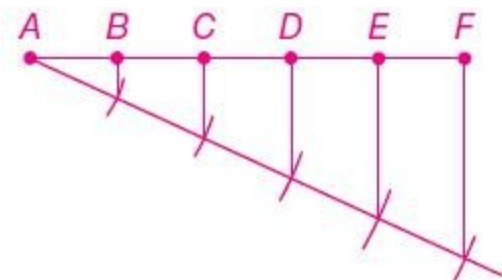
Step 3: With your compass on the new point formed on the diagonal, keep the same radius and make another arc further down the diagonal side of the angle. Continue this process until you have five arcs, like below:



Step 4: Using a straight edge, draw a segment that connects each new point back to the horizontal side of the angle, perpendicular to that side, as shown below:



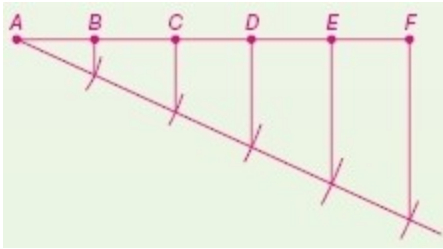
Step 5. Label the points formed on the horizontal side of the angle and erase any extra length beyond the last point.



ANSWER:

Sample answer:

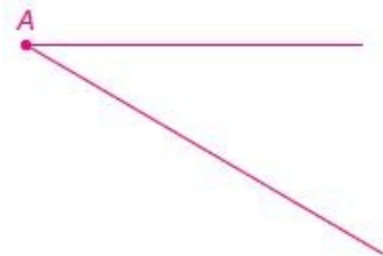
7-4 Parallel Lines and Proportional Parts



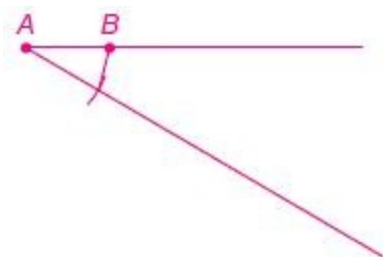
45. a segment separated into two segments in which their lengths have a ratio of 1 to 3

SOLUTION:

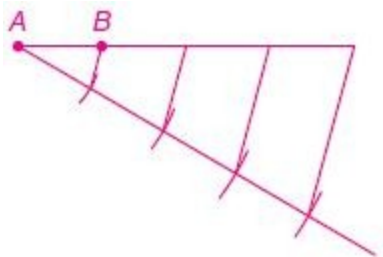
Step 1: Make an angle, with vertex A, as shown below:



Step 2: With your compass on vertex A, make an arc that passes through the diagonal side of the angle. Connect this new point back to the horizontal side of the angle. Label B as the new point made on the horizontal side of the angle, as shown below.

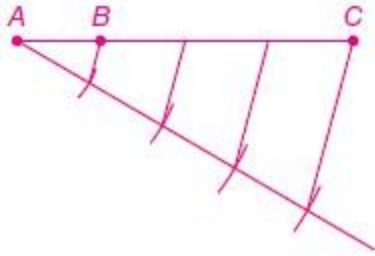


Step 3: Continue this process until you have four arcs. When you connect the points on the diagonal back to the horizontal, make sure the connecting lines are all parallel to each other. (Since you want segment lengths at a ratio of 1 to 3, this can be created by 4 equal smaller segments, where three can be pieced together to make one that is $\frac{3}{4}$ the original length.)



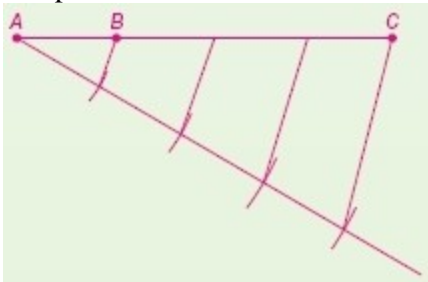
Step 4: Label the first point B and the last point C. $\frac{BC}{AC} = \frac{3}{4}$.

7-4 Parallel Lines and Proportional Parts



ANSWER:

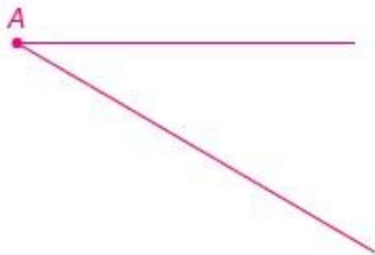
Sample answer:



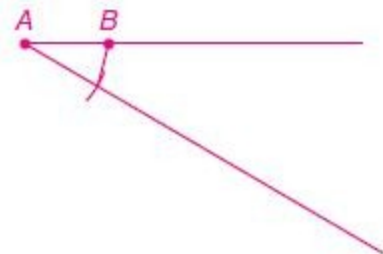
46. a segment 3 inches long, separated into four congruent segments

SOLUTION:

Step 1: Copy a 3 inch segment. horizontally. Then, make an angle, with vertex A, as shown below:

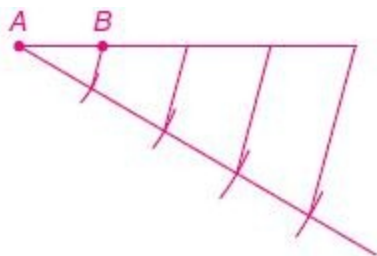


Step 2: With your compass on vertex A, make an arc that passes through the diagonal side of the angle. Connect this new point back to the horizontal side of the angle. Label B as the new point made on the horizontal side of the angle, as shown below.

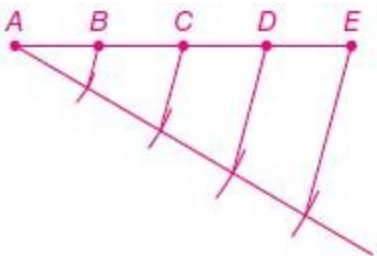


Step 3: Continue this process until you have four arcs. When you connect the points on the diagonal back to the horizontal, make sure the connecting lines are all parallel to each other.

7-4 Parallel Lines and Proportional Parts

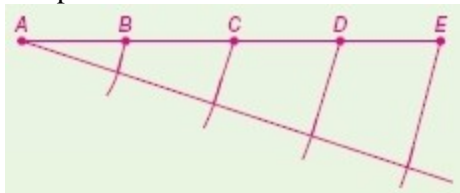


Step 4: Label the points as shown. $AB = BC = CD = DE$



ANSWER:

Sample answer:



47. **MULTIPLE REPRESENTATIONS** In this problem, you will explore angle bisectors and proportions.
- GEOMETRIC** Draw three triangles, one acute, one right, and one obtuse. Label one triangle ABC and draw angle bisector \overline{BD} . Label the second MNP with angle bisector \overline{NQ} and the third WXY with angle bisector \overline{XZ} .
 - TABULAR** Complete the table at the right with the appropriate values.
 - VERBAL** Make a conjecture about the segments of a triangle created by an angle bisector.

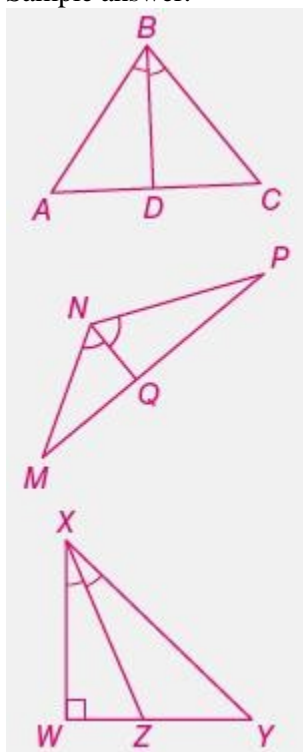
Triangle	Length	Ratio
ABC	AD	$\frac{AD}{CD}$
	CD	
	AB	$\frac{AB}{CB}$
	CB	
MNP	MQ	$\frac{MQ}{PQ}$
	PQ	
	MN	$\frac{MN}{PN}$
	PN	
WXY	WZ	$\frac{WZ}{YZ}$
	YZ	
	WX	$\frac{WX}{YX}$
	YX	

SOLUTION:

7-4 Parallel Lines and Proportional Parts

a. When drawing the triangles, pay close attention to the directions and labeling instructions. Use a protractor, or construction tool, when making the angle bisectors, to ensure accurate measurement values for the table.

Sample answer:



b. Carefully measure the indicated lengths in centimeters.

Triangle	Length	Ratio
ABC	AD	1.1 cm
	CD	1.1 cm
	AB	2.0 cm
	CB	2.0 cm
MNP	MQ	1.4 cm
	PQ	1.7 cm
	MN	1.6 cm
	PN	2.0 cm
WXY	WZ	0.8 cm
	YZ	1.2 cm
	WX	2.0 cm
	YX	2.9 cm

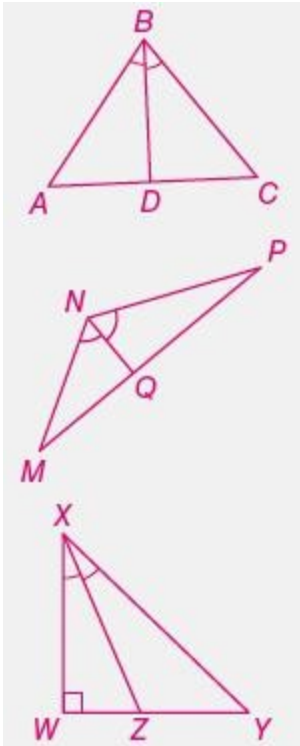
c. Look for a pattern in the table, specifically comparing the lengths of the ratios of sides for each triangle.

Sample answer: The proportion of the segments created by the angle bisector of a triangle is equal to the proportion of their respective consecutive sides.

ANSWER:

a. Sample answer:

7-4 Parallel Lines and Proportional Parts



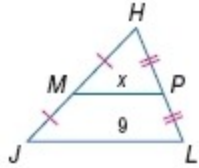
b.

Triangle	Length	Ratio
ABC	AD	1.1 cm
	CD	1.1 cm
	AB	2.0 cm
	CB	2.0 cm
MNP	MQ	1.4 cm
	PQ	1.7 cm
	MN	1.6 cm
	PN	2.0 cm
WXY	WZ	0.8 cm
	YZ	1.2 cm
	WX	2.0 cm
	YX	2.9 cm

c. Sample answer: The proportion of the segments created by the angle bisector of a triangle is equal to the proportion of their respective consecutive sides.

7-4 Parallel Lines and Proportional Parts

48. **CCSS CRITIQUE** Jacob and Sebastian are finding the value of x in $\triangle JHL$. Jacob says that MP is one half of JL , so x is 4.5. Sebastian says that JL is one half of MP , so x is 18. Is either of them correct? Explain.



SOLUTION:

Jacob; sample answer: Since M is the midpoint of \overline{JH} and P is the midpoint of \overline{HL} , then \overline{MP} is the midsegment of $\triangle JHL$. Therefore,

$$MP = \frac{1}{2}JL$$

Substitute:

$$x = \frac{1}{2}(9)$$

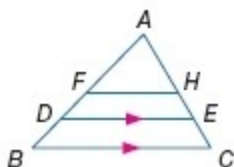
$$x = 4.5$$

ANSWER:

Jacob; sample answer: \overline{MP} is the midsegment, so $MP = \frac{1}{2}JL$.

7-4 Parallel Lines and Proportional Parts

49. **REASONING** In $\triangle ABC$, $AF = FB$ and $AH = HC$. If D is $\frac{3}{4}$ of the way from A to B and E is $\frac{3}{4}$ of the way from A to C , is DE sometimes, always, or never $\frac{3}{4}$ of BC ? Explain.



SOLUTION:

Always; sample answer: Since $FA=FB$, then F is a midpoint of \overline{AB} . Similarly, since $AH=HC$ and H is the midpoint of \overline{AC} .

Therefore, FH is a midsegment of $\triangle BAC$ so $\overline{FH} \parallel \overline{BC}$ and $FH = \frac{1}{2}BC$.

Let $BC = x$, then $FH = \frac{1}{2}x$.

Because $\overline{FH} \parallel \overline{BC}$, we know that $FHCB$ is a trapezoid, so

$$DE = \frac{1}{2}(BC + FH)$$

Substitute:

$$DE = \frac{1}{2}\left(x + \frac{1}{2}x\right)$$

$$DE = \frac{1}{2}x + \frac{1}{4}x$$

$$DE = \frac{3}{4}x$$

ANSWER:

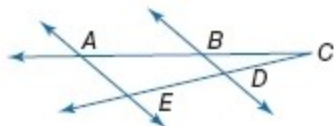
Always; sample answer: FH is a midsegment. Let $BC = x$, then $FH = \frac{1}{2}x$. $FHCB$ is a trapezoid, so

$$DE = \frac{1}{2}(BC + FH) = \frac{1}{2}\left(x + \frac{1}{2}x\right) = \frac{1}{2}x + \frac{1}{4}x = \frac{3}{4}x. \text{ Therefore, } DE = \frac{3}{4}BC.$$

CHALLENGE Write a two-column proof.

50. **Given:** $AB = 4$, $BC = 4$, and $CD = DE$

Prove: $\overline{BD} \parallel \overline{AE}$



SOLUTION:

An effective strategy for this proof is to think of a way to get $\triangle ACE \sim \triangle BCD$, by SAS Similarity. We already know

7-4 Parallel Lines and Proportional Parts

that $\angle C \cong \angle C$, so we need to establish that $\frac{AC}{BC} = \frac{EC}{DC}$. You can show that $2BC = AC$ and $2DC = EC$, through the given information and substitution into Segment Addition Postulate statements. Once this is done, you can prove that $\frac{AC}{BC} = \frac{EC}{DC}$ by transitive property. Once the triangles are proven similar, then the lines can be proven parallel by choosing a pair of congruent corresponding angles from the similar triangles.

,

Proof:

Statements (Reasons)

1. $AB = 4, BC = 4$ (Given)
2. $AB = BC$ (Subst.)
3. $AB + BC = AC$ (Seg. Add. Post.)
4. $BC + BC = AC$ (Subst.)
5. $2BC = AC$ (Subtraction property.)
6. $AC = 2BC$ (Symm. Prop.)
7. $\frac{AC}{BC} = 2$ (Div. Prop.)
8. $ED = DC$ (Given)
9. $ED + DC = EC$ (Seg. Add. Post.)
10. $DC + DC = EC$ (Subst.)
11. $2DC = EC$ (Subst.)
12. $2 = \frac{EC}{DC}$ (Div. Prop.)
13. $\frac{AC}{BC} = \frac{EC}{DC}$ (Trans. Prop.)
14. $\angle C \cong \angle C$ (Reflexive Prop.)
15. $\triangle ACE \sim \triangle BCD$ (SAS Similarity)
16. $\angle CAE \cong \angle CBD$ (Def. of \sim polygons)
17. $\overline{BD} \parallel \overline{AE}$ (If corr. angles are \cong , lines are \parallel .)

ANSWER:

Proof:

Statements (Reasons)

1. $AB = 4, BC = 4$ (Given)
2. $AB = BC$ (Sub.)
3. $AB + BC = AC$ (Seg. Add. Post.)
4. $BC + BC = AC$ (Subst.)
5. $2BC = AC$ (Sub.)
6. $AC = 2BC$ (Symm. Prop.)
7. $\frac{AC}{BC} = 2$ (Div. Prop.)
8. $ED = DC$ (Given)
9. $ED + DC = EC$ (Seg. Add. Post.)
10. $DC + DC = EC$ (Sub.)
11. $2DC = EC$ (Sub.)
12. $2 = \frac{EC}{DC}$ (Div. Prop.)

7-4 Parallel Lines and Proportional Parts

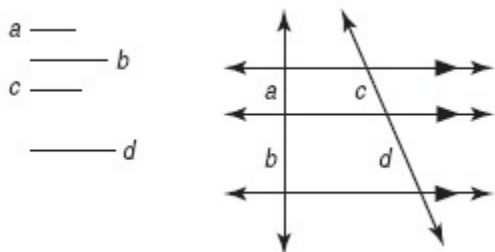
13. $\frac{AC}{BC} = \frac{EC}{DC}$ (Trans. Prop.)
14. $\angle C \cong \angle C$ (Reflexive Prop.)
15. $\triangle ACE \sim \triangle BCD$ (SAS Similarity)
16. $\angle CAE \cong \angle CBD$ (Def. of \sim polygons)
17. $\overline{BD} \parallel \overline{AE}$ (If corr. angles are \cong , lines are \parallel .)

51. **OPEN ENDED** Draw three segments, a , b , and c , of all different lengths. Draw a fourth segment, d , such that

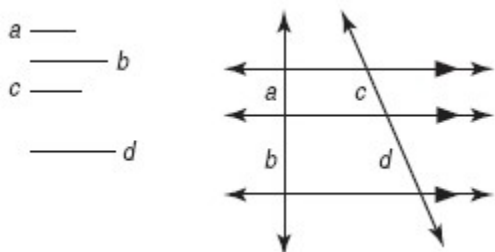
$$\frac{a}{b} = \frac{c}{d}.$$

SOLUTION:

By Corollary 7.1, we know that if we draw three parallel lines intersected by two transversals, then they will cut the transversals proportionally or $\frac{a}{b} = \frac{c}{d}$, as seen in the diagram below.



ANSWER:



By Corollary 7.1, $\frac{a}{b} = \frac{c}{d}$.

52. **WRITING IN MATH** Compare the Triangle Proportionality Theorem and the Triangle Midsegment Theorem.

SOLUTION:

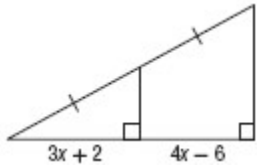
Both theorems deal with a parallel line inside the triangle. The Midsegment Theorem is a special case of the Converse of the Proportionality Theorem.

ANSWER:

Both theorems deal with a parallel line inside the triangle. The Midsegment Theorem is a special case of the Converse of the Proportionality Theorem.

7-4 Parallel Lines and Proportional Parts

53. **SHORT RESPONSE** What is the value of x ?



SOLUTION:

By Corollary 7.2, $4x - 6 = 3x + 2$.

Solve for x .

$$4x - 6 = 3x + 2$$

$$x - 6 = 2$$

$$x = 8$$

ANSWER:

8

54. If the vertices of triangle JKL are $(0, 0)$, $(0, 10)$ and $(10, 10)$ then the area of triangle JKL is

A 20 units²

B 30 units²

C 40 units²

D 50 units²

SOLUTION:

$$\begin{aligned}\text{Area of triangle } JKL &= \frac{1}{2}(\text{base})(\text{height}) \\ &= \frac{1}{2} \times 10 \times 10 \\ &= 50\end{aligned}$$

So, the correct choice is D.

ANSWER:

D

7-4 Parallel Lines and Proportional Parts

55. **ALGEBRA** A breakfast cereal contains wheat, rice, and oats in the ratio 2 : 4: 1. If the manufacturer makes a mixture using 110 pounds of wheat, how many pounds of rice will be used?

F 120 lb

G 220 lb

H 240 lb

J 440 lb

SOLUTION:

Since the ratio of rice to wheat is 4: 2, we can set up a proportion to find the amount of rice needed for 110 pounds of wheat.

$$\frac{\text{rice ratio}}{\text{wheat ratio}} = \frac{\text{amount of rice (lbs)}}{\text{amount of wheat (lbs)}}$$

$$\frac{4}{2} = \frac{x}{110}$$

$$440 = 2x$$

$$220 = x$$

The correct answer is G, 220 lb.

ANSWER:

G

7-4 Parallel Lines and Proportional Parts

56. **SAT/ACT** If the area of a circle is 16 square meters, what is its radius in meters?

A $\frac{4\sqrt{\pi}}{\pi}$

B $\frac{8}{\pi}$

C $\frac{16}{\pi}$

D 12π

E 16π

SOLUTION:

Since the area of a circle can be found with $A = \pi r^2$, we can substitute in 16 for the area (A) and solve for r .

$$16 = \pi r^2$$

$$\frac{16}{\pi} = r^2$$

$$\sqrt{\frac{16}{\pi}} = r$$

$$\frac{4}{\sqrt{\pi}} = r$$

$$\frac{4 \cdot \sqrt{\pi}}{\sqrt{\pi} \cdot \sqrt{\pi}} = r$$

$$\frac{4\sqrt{\pi}}{\pi} = r$$

Therefore, the answer is A.

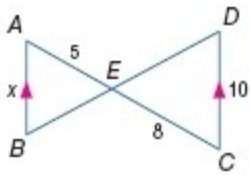
ANSWER:

A

7-4 Parallel Lines and Proportional Parts

ALGEBRA Identify the similar triangles. Then find the measure(s) of the indicated segment(s).

57. \overline{AB}



SOLUTION:

$\angle AEB \cong \angle CED$ by the Vertical Angles Theorem.

Since $\overline{AB} \parallel \overline{CD}$, $\angle ABE \cong \angle CDE$ by the Alternate Interior Angles Theorem.

Therefore, by AA Similarity, $\triangle ABE \sim \triangle CDE$.

To find AB or x , write a proportion using the definition of similar polygons.

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{x}{10} = \frac{5}{8}$$

$$8x = 50$$

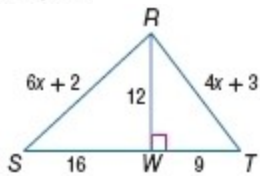
$$x = \frac{50}{8} \text{ or } 6.25$$

ANSWER:

$\triangle ABE \sim \triangle CDE$ by AA Similarity; 6.25

7-4 Parallel Lines and Proportional Parts

58. $\overline{RT}, \overline{RS}$



SOLUTION:

$\angle SWR \cong \angle RWT$, since right angles are congruent. $\frac{SW}{RW} = \frac{RW}{TW}$, since $\frac{16}{12} = \frac{12}{9}$.

Therefore, by SAS Similarity, $\triangle RSW \sim \triangle TRW$.

Write a proportion using the definition of similar polygons to find the value of x .

$$\frac{SR}{SW} = \frac{RT}{RW}$$
$$\frac{6x + 2}{16} = \frac{4x + 3}{12}$$

$$9(6x + 2) = 12(4x + 3)$$

$$54x + 18 = 48x + 36$$

$$6x + 18 = 36$$

$$6x = 18$$

$$x = 3$$

Substitute this value for x to find RT and RS .

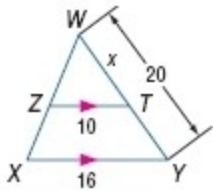
$$\begin{array}{ll} RT = 4x + 3 & RS = 6x + 2 \\ = 4(3) + 3 \text{ or } 15 & = 6(3) + 2 \text{ or } 20 \end{array}$$

ANSWER:

$\triangle RSW \sim \triangle TRW$ by SAS Similarity; 15, 20

7-4 Parallel Lines and Proportional Parts

59. \overline{TY}



SOLUTION:

$\angle W \cong \angle W$ by the Reflexive Property of Congruence.

Since $\overline{ZT} \parallel \overline{XY}$, $\angle WZT \cong \angle WXY$ by the Corresponding Angles Theorem.

Therefore, by AA Similarity, $\Delta WZT \sim \Delta WXY$.

Write a proportion using the definition of similar polygons to find the value of x .

$$\begin{aligned}\frac{x}{20} &= \frac{10}{16} \\ x &= \frac{200}{16} \\ x &= 12.5\end{aligned}$$

So, $WT = 12.5$. $WT + TY = WY$ by the Segment Addition Postulate. Since $WY = 20$, you can solve for TY .

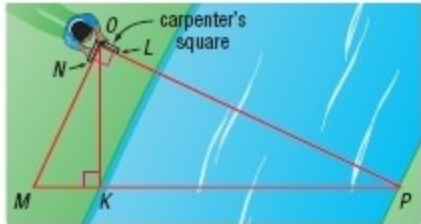
$$\begin{aligned}WT + TY &= WY \\ 12.5 + TY &= 20 \\ TY &= 7.5\end{aligned}$$

ANSWER:

$\Delta WZT \sim \Delta WXY$ by AA Similarity; 7.5

7-4 Parallel Lines and Proportional Parts

60. **SURVEYING** Mr. Turner uses a carpenter's square to find the distance across a stream. The carpenter's square models right angle NOL . He puts the square on top of a pole that is high enough to sight along \overline{OL} to point P across the river. Then he sights along \overline{ON} to point M . If MK is 1.5 feet and OK is 4.5 feet, find the distance KP across the stream.



SOLUTION:

By AA Similarity, $\triangle MOP \sim \triangle MKO$.

Use the Pythagorean Theorem to find MO .

$$MO^2 = MK^2 + KO^2$$

$$MO^2 = (1.5)^2 + (4.5)^2$$

$$MO^2 = 2.25 + 20.25$$

$$MO^2 = 22.5$$

$$MO = \sqrt{22.5} \approx 4.74 \text{ ft}$$

Write a proportion using corresponding sides of the two triangles:

$$\frac{MP}{MO} = \frac{OM}{MK}$$

$$\frac{MK+KP}{4.74} = \frac{4.74}{1.5}$$

$$\frac{1.5+KP}{4.74} = \frac{4.74}{1.5}$$

$$1.5(1.5 + KP) = 4.74 \cdot 4.74$$

$$2.25 + 1.5KP = 22.47$$

$$1.5KP = 20.22$$

$$KP = 13.48$$

Therefore, the distance KP is about 13.5 feet.

ANSWER:

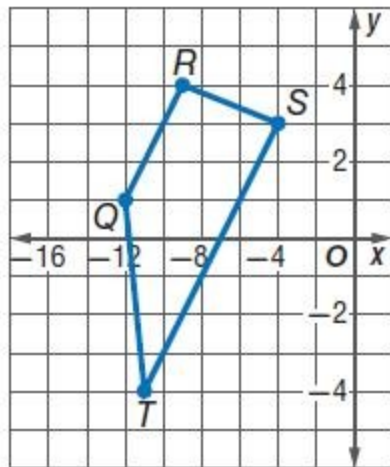
13.5 ft

7-4 Parallel Lines and Proportional Parts

COORDINATE GEOMETRY For each quadrilateral with the given vertices, verify that the quadrilateral is a trapezoid and determine whether the figure is an isosceles trapezoid.

61. $Q(-12, 1)$, $R(-9, 4)$, $S(-4, 3)$, $T(-11, -4)$

SOLUTION:



Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{QR} = \frac{4 - 1}{-9 - (-12)} = \frac{3}{3} = 1$$

$$m_{RS} = \frac{3 - 4}{-4 - (-9)} = -\frac{1}{5}$$

$$m_{ST} = \frac{-4 - 3}{-11 - (-4)} = \frac{-7}{-7} = 1$$

$$m_{TQ} = \frac{-4 - 1}{-11 - (-12)} = \frac{-5}{1}$$

The slopes of exactly one pair of opposite sides are equal. So, this quadrilateral has only one pair of parallel sides. Therefore, the quadrilateral $QRST$ is a trapezoid.

Use the Distance Formula to find the lengths of the legs of the trapezoid.

$$RS = \sqrt{(-4 - (-9))^2 + (3 - 4)^2} = \sqrt{(5)^2 + (-1)^2} = \sqrt{25 + 1} = \sqrt{26}$$

$$QT = \sqrt{(-11 - (-12))^2 + (-4 - 1)^2} = \sqrt{(1)^2 + (-5)^2} = \sqrt{26}$$

The lengths of the legs are equal. Therefore, $QRST$ is an isosceles trapezoid.

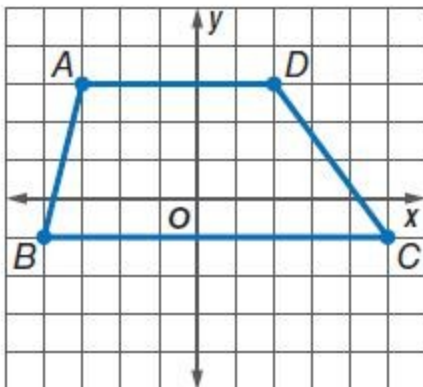
ANSWER:

$\overline{QR} \parallel \overline{TS}$, $\overline{QT} \cong \overline{RS}$; $QRST$ is an isosceles trapezoid since $RS = \sqrt{26} = QT$.

7-4 Parallel Lines and Proportional Parts

62. $A(-3, 3)$, $B(-4, -1)$, $C(5, -1)$, $D(2, 3)$

SOLUTION:



Use the slope formula to find the slope of the sides of the quadrilateral.

$$m_{AB} = \frac{-1 - 3}{-4 - (-3)} = \frac{-4}{-1} = 4$$

$$m_{BC} = \frac{-1 - (-1)}{5 - (-4)} = \frac{0}{9} = 0$$

$$m_{CD} = \frac{3 - (-1)}{2 - 5} = \frac{4}{-3} = -\frac{4}{3}$$

$$m_{DA} = \frac{3 - 3}{2 - (-3)} = \frac{0}{1} = 0$$

The slopes of exactly one pair of opposite sides are equal. So, they are parallel. Therefore, the quadrilateral $ABCD$ is a trapezoid.

Use the Distance Formula to find the lengths of the legs of the trapezoid.

$$AB = \sqrt{(-4 - (-3))^2 + (-1 - 3)^2} = \sqrt{(-1)^2 + (-4)^2} = \sqrt{1 + 16} = \sqrt{17}$$

$$CD = \sqrt{(2 - 5)^2 + (3 - (-1))^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

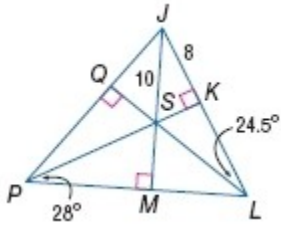
The lengths of the legs are not equal. Therefore, $ABCD$ is not an isosceles trapezoid.

ANSWER:

$\overline{AD} \parallel \overline{BC}$, $\overline{CD} \not\parallel \overline{AB}$; $ABCD$ is a trapezoid, but not isosceles since $AB = \sqrt{17}$ and $CD = 5$.

7-4 Parallel Lines and Proportional Parts

Point S is the incenter of $\triangle JPL$. Find each measure.



63. SQ

SOLUTION:

Since S is the incenter of $\triangle JPL$, then it is formed by the angle bisectors of each vertex. \overline{JS} is the angle bisector of $\angle PLJ$ therefore, by the Angle Bisector Theorem, $SK = SQ$.

Use Pythagorean Theorem in the right triangle JSK .

$$SK^2 + 8^2 = 10^2$$

$$SK^2 + 64 = 100$$

$$SK^2 = 36$$

$$SK = \sqrt{36} = 6$$

$$SK = SQ = 6$$

ANSWER:

6

7-4 Parallel Lines and Proportional Parts

64. QJ

SOLUTION:

Since S is the incenter of $\triangle JPL$, then it is formed by the angle bisectors of each vertex. \overline{JS} is the angle bisector of $\angle PLJ$ therefore, by the Angle Bisector Theorem, $SK = SQ$.

Use Pythagorean Theorem in the right triangle JSK .

$$SK^2 + 8^2 = 10^2$$

$$SK^2 + 64 = 100$$

$$SK^2 = 36$$

$$SK = \sqrt{36} = 6$$

Use Pythagorean Theorem in the right triangle JSQ to find QJ .

$$QJ^2 + 6^2 = 10^2$$

$$QJ^2 + 36 = 100$$

$$QJ^2 = 64$$

$$QJ = \sqrt{64} = 8$$

Therefore $QJ=8$.

ANSWER:

8

65. $m\angle MPQ$

SOLUTION:

Since S is the incenter of $\triangle JPL$, then it is formed by the angle bisectors of each vertex. \overline{PS} is the angle bisector of $\angle MPQ$.

Therefore, $m\angle SPQ = m\angle SPM = 28^\circ$.

$$m\angle MPQ = m\angle SPQ + m\angle SPM = 56$$

ANSWER:

56

7-4 Parallel Lines and Proportional Parts

66. $m\angle SJP$

SOLUTION:

Since S is the incenter of $\triangle JPL$, then it is formed by the angle bisectors of each vertex. \overline{PS} is the angle bisector of $\angle MPQ$.

Therefore, $m\angle SPQ = m\angle SPM = 28^\circ$.

$$m\angle MPQ = m\angle SPQ + m\angle SPM = 56$$

and similarly,

$$\begin{aligned} m\angle JLP &= m\angle JLQ + m\angle QLP \\ &= 24.5 + 24.5 \\ &= 49 \end{aligned}$$

We know that sum of the measures of a triangle is 180.

$$\begin{aligned} m\angle JLP + m\angle JPL + m\angle PJL &= 180 \\ 49 + 56 + m\angle PJL &= 180 \\ 105 + m\angle PJL &= 180 \\ m\angle PJL &= 75 \end{aligned}$$

$$m\angle PJL = m\angle PJM + m\angle MJL$$

We know that $\angle PJM \approx \angle MJL$ because \overline{JM} is an angle bisector of $\angle PJL$. Therefore,

$$\begin{aligned} 75 &= m\angle PJM + m\angle PJM \\ 37.5 &= m\angle PJM \\ m\angle PJM &= m\angle MJL = m\angle SJP = 37.5 \end{aligned}$$

ANSWER:

37.5

7-4 Parallel Lines and Proportional Parts

Solve each proportion.

67. $\frac{1}{3} = \frac{x}{2}$

SOLUTION:

Solve for x .

$$\frac{1}{3} = \frac{x}{2}$$

$$3x = 2$$

$$x = \frac{2}{3}$$

ANSWER:

$$\frac{2}{3}$$

68. $\frac{3}{4} = \frac{5}{x}$

SOLUTION:

Solve for x .

$$\frac{3}{4} = \frac{5}{x}$$

$$3x = 20$$

$$x = \frac{20}{3}$$

$$x \approx 6.7$$

ANSWER:

$$6.7$$

69. $\frac{2.3}{4} = \frac{x}{3.7}$

SOLUTION:

Solve for x .

$$\frac{2.3}{4} = \frac{x}{3.7}$$

$$4x = 8.51$$

$$x = \frac{8.51}{4}$$

$$x \approx 2.1$$

ANSWER:

$$2.1$$

7-4 Parallel Lines and Proportional Parts

$$70. \frac{x-2}{2} = \frac{4}{5}$$

SOLUTION:

Solve for x .

$$\frac{x-2}{2} = \frac{4}{5}$$

$$5(x-2) = 8$$

$$5x - 10 = 8$$

$$5x = 18$$

$$x = 3.6$$

ANSWER:

3.6

$$71. \frac{x}{12-x} = \frac{8}{3}$$

SOLUTION:

Solve for x .

$$\frac{x}{12-x} = \frac{8}{3}$$

$$3x = 8(12 - x)$$

$$3x = 96 - 8x$$

$$11x = 96$$

$$x \approx 8.7$$

ANSWER:

8.7