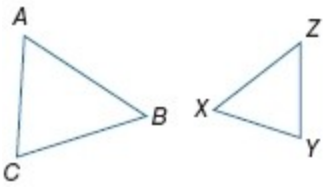
7-2 Similar Polygons

List all pairs of congruent angles, and write a proportion that relates the corresponding sides for each pair of similar polygons.

1. $\Delta ABC \sim \Delta ZYX$



SOLUTION:

The order of vertices in a similarity statement identifies the corresponding angles and sides. Since we know that $\Delta ABC \sim \Delta ZYX$, we can take the corresponding angles of this statement and set them congruent to each other. Then, since the corresponding sides of similar triangles are proportional to each other, we can write a proportion that relates the corresponding sides to each other.

$$\angle A \cong \angle Z, \angle B \cong \angle Y, \angle C \cong \angle X; \frac{AC}{ZX} = \frac{BC}{YX} = \frac{AB}{ZY}$$

ANSWER:

$$\angle A \cong \angle Z, \angle B \cong \angle Y, \angle C \cong \angle X; \frac{AC}{ZX} = \frac{BC}{YX} = \frac{AB}{ZY}$$

2. $JKLM \sim TSRQ$

SOLUTION:

The order of vertices in a similarity statement identifies the corresponding angles and sides. Since we know that $JKLM \sim TSRQ$, we can take the corresponding angles of this statement and set them congruent to each other. Then, since the corresponding sides of similar polygons are proportional to each other, we can write a proportion that relates the corresponding sides to each other.

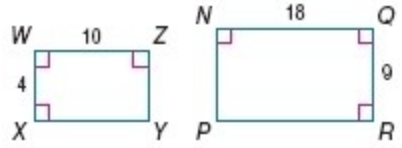
$$\angle J \cong \angle T, \angle K \cong \angle S, \angle M \cong \angle Q, \angle L \cong \angle R;$$
$$\frac{JM}{TQ} = \frac{ML}{QR} = \frac{KL}{SR} = \frac{JK}{TS}$$

ANSWER:

$$\angle J \cong \angle T, \angle K \cong \angle S, \angle M \cong \angle Q; \angle L \cong \angle R;$$
$$\frac{JM}{TQ} = \frac{ML}{QR} = \frac{KL}{SR} = \frac{JK}{TS}$$

7-2 Similar Polygons

Determine whether each pair of figures is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning.

3. 

SOLUTION:

Step 1: Compare corresponding angles:

Since all of the angles in the polygons are right angles, they are all congruent to each other. Therefore, corresponding angles are congruent.

Step 2: Compare corresponding sides:

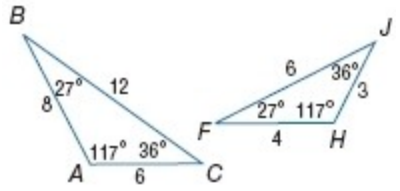
$$\frac{QR}{WX} = \frac{9}{4}$$
$$\frac{NQ}{WZ} = \frac{18}{10} = \frac{9}{5}$$

Since $\frac{NQ}{WZ} \neq \frac{QR}{WX}$, the figures are <u>not</u> similar.

ANSWER:

no; $\frac{NQ}{WZ} \neq \frac{QR}{WX}$

7-2 Similar Polygons

4. 

SOLUTION:

Step 1: Compare corresponding angles:

$$m\angle A = 117 = m\angle H$$
$$m\angle B = 27 = m\angle F$$
$$m\angle C = 36 = m\angle J$$

Therefore, $\angle A \cong \angle H$, $\angle B \cong \angle F$, *and* $\angle C \cong \angle J$.

Step 2: Compare corresponding sides:

$$\frac{AB}{HF} = \frac{8}{4} = \frac{2}{1}$$
$$\frac{BC}{FJ} = \frac{12}{6} = \frac{2}{1}$$
$$\frac{CA}{JH} = \frac{6}{3} = \frac{2}{1}$$

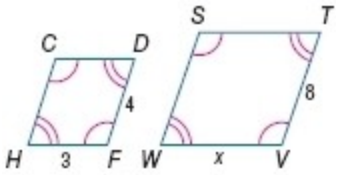
Therefore $\frac{AB}{HF} = \frac{BC}{FJ} = \frac{CA}{JH}$ and the corresponding sides are proportional.

Yes; $\Delta ABC \sim \Delta HFJ$ since $\angle A \cong \angle H, \angle B \cong \angle F, \angle C \cong \angle J$ and $\frac{AB}{HF} = \frac{BC}{FJ} = \frac{CA}{JH}$; scale factor: $\frac{2}{1}$.

ANSWER:

Yes; $\Delta ABC \sim \Delta HFJ$ since $\angle A \cong \angle H, \angle B \cong \angle F, \angle C \cong \angle J$ and $\frac{AB}{HF} = \frac{BC}{FJ} = \frac{CA}{JH}$; scale factor: $\frac{2}{1}$.

Each pair of polygons is similar. Find the value of *x*.

5. 

SOLUTION:

Use the corresponding side lengths to write a proportion.

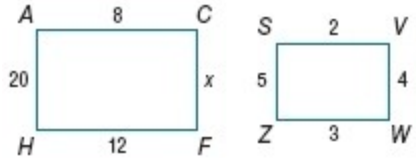
$$\frac{x}{3}=\frac{8}{4}$$

Solve for *x*.

$$4x=24$$
$$x=6$$

ANSWER:

6

6. 

SOLUTION:

Use the corresponding side lengths to write a proportion.

$$\frac{x}{12}=\frac{4}{3}$$

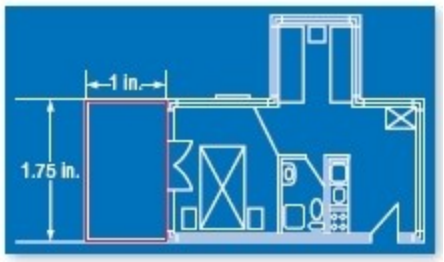
Solve for *x*.

$$3x=48$$
$$x=16$$

ANSWER:

16

7. **DESIGN** On the blueprint of the apartment shown, the balcony measures 1 inch wide by 1.75 inches long. If the actual length of the balcony is 7 feet, what is the perimeter of the balcony?



SOLUTION:

Write a proportion using the given information. Let x be the actual width of balcony. 1 foot = 12 inches. So, 7 feet = 84 inches.

$$\frac{x \text{ width of the balcony}}{84 \text{ total inches}} = \frac{1 \text{ scale inch}}{1.75 \text{ real inches}}$$

$$\frac{x}{84} = \frac{1}{1.75}$$
$$1.75x = 84$$
$$x = 48$$

So, width = 48 inches or 4 feet.

$$\begin{aligned} \text{Perimeter} &= 2(l + b) \\ &= 2(7 + 4) \\ &= 2(11) \\ &= 22 \end{aligned}$$

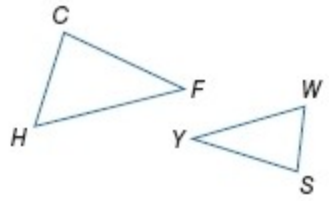
Therefore, the perimeter of the balcony is 22 ft.

ANSWER:

22 ft

List all pairs of congruent angles, and write a proportion that relates the corresponding sides for each pair of similar polygons.

8. $\Delta CHF \sim \Delta YWS$



SOLUTION:

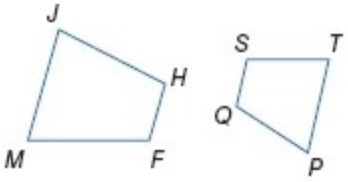
The order of vertices in a similarity statement identifies the corresponding angles and sides. Since we know that $\Delta CHF \sim \Delta YWS$, we can take the corresponding angles of this statement and set them congruent to each other. Then, since the corresponding sides of similar triangles are proportional to each other, we can write a proportion that relates the corresponding sides to each other.

$$\angle C \cong \angle Y, \angle H \cong \angle W, \angle F \cong \angle S;$$
$$\frac{CH}{YW} = \frac{HF}{WS} = \frac{FC}{SY}$$

ANSWER:

$$\angle C \cong \angle Y, \angle H \cong \angle W, \angle F \cong \angle S; \frac{CH}{YW} = \frac{HF}{WS} = \frac{FC}{SY}$$

9. $JHFM \sim PQST$



SOLUTION:

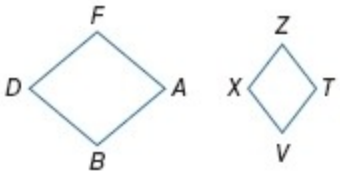
The order of vertices in a similarity statement identifies the corresponding angles and sides. Since we know that $JHFM \sim PQST$, we can take the corresponding angles of this statement and set them congruent to each other. Then, since the corresponding sides of similar polygons are proportional to each other, we can write a proportion that relates the corresponding sides to each other.

$$\angle J \cong \angle P, \angle H \cong \angle Q, \angle F \cong \angle S, \angle M \cong \angle T;$$
$$\frac{PQ}{JH} = \frac{TS}{MF} = \frac{SQ}{FH} = \frac{TP}{MJ}$$

ANSWER:

$$\angle J \cong \angle P, \angle F \cong \angle S, \angle M \cong \angle T; \angle H \cong \angle Q, \frac{PQ}{JH} = \frac{TS}{MF} = \frac{SQ}{FH} = \frac{TP}{MJ}$$

10. $ABDF \sim VXZT$



SOLUTION:

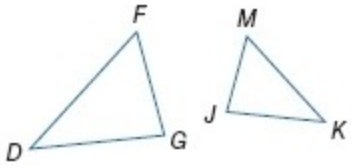
The order of vertices in a similarity statement identifies the corresponding angles and sides. Since we know that $ABDF \sim VXZT$, we can take the corresponding angles of this statement and set them congruent to each other. Then, since the corresponding sides of similar polygons are proportional to each other, we can write a proportion that relates the corresponding sides to each other.

$$\angle A \cong \angle V, \angle B \cong \angle X, \angle D \cong \angle Z, \angle F \cong \angle T;$$
$$\frac{AB}{VX} = \frac{BD}{XZ} = \frac{DF}{ZT} = \frac{FA}{TV}$$

ANSWER:

$\angle A \cong \angle V$, $\angle B \cong \angle X$, $\angle D \cong \angle Z$, $\angle F \cong \angle T$; $\frac{AB}{VX} = \frac{BD}{XZ} = \frac{DF}{ZT} = \frac{FA}{TV}$

11. $\Delta DFG \sim \Delta KMJ$



SOLUTION:

The order of vertices in a similarity statement identifies the corresponding angles and sides. Since we know that $\Delta DEF \sim \Delta KMJ$, we can take the corresponding angles of this statement and set them congruent to each other. Then, since the corresponding sides of similar triangles are proportional to each other, we can write a proportion that relates the corresponding sides to each other.

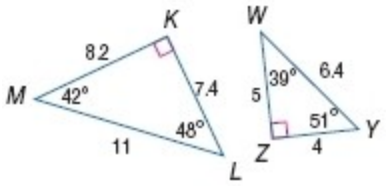
$$\angle D \cong \angle K, \angle F \cong \angle M, \angle G \cong \angle J;$$
$$\frac{DF}{KM} = \frac{FG}{MJ} = \frac{GD}{JK}$$

ANSWER:

$\angle D \cong \angle K$, $\angle F \cong \angle M$, $\angle G \cong \angle J$; $\frac{DF}{KM} = \frac{FG}{MJ} = \frac{GD}{JK}$

CCSS ARGUMENTS Determine whether each pair of figures is similar. If so, write the similarity statement and scale factor. If not, explain your reasoning.

12.



SOLUTION:

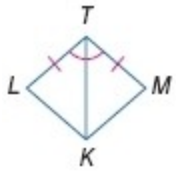
Step 1: Compare corresponding angles:

$$m\angle K = 90 = m\angle Z$$
$$m\angle M = 42 \neq m\angle Y$$
$$m\angle L = 48 \neq m\angle W$$

Since, $\angle M \ncong \angle Y$ *and* $\angle L \ncong \angle W$, then the triangles are <u>not</u> similar.

ANSWER:

no; $\angle L \ncong \angle W$

13. 

SOLUTION:

Since the $\Delta LTK \cong \Delta MTK$ (by SAS triangle congruence theorem), then their corresponding parts are congruent

Step 1: Compare corresponding angles:

$\angle L \cong \angle M$, $\angle LTK \cong \angle MTK$, *and* $\angle TKL \cong \angle TKM$

Step 2: Compare corresponding sides:

$\frac{TL}{TM} = \frac{1}{1}$
$\frac{LK}{MK} = \frac{1}{1}$
$\frac{TK}{TK} = \frac{1}{1}$

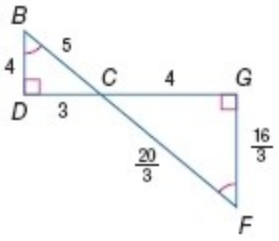
Therefore $\frac{TL}{TM} = \frac{LK}{MK} = \frac{TK}{TK}$ and the corresponding sides are proportional, with a scale factor of $\frac{1}{1}$.

Yes; $\Delta LTK \sim \Delta MTK$ because $\Delta LTK \cong \Delta MTK$; scale factor: 1.

ANSWER:

Yes; $\Delta LTK \sim \Delta MTK$ because $\Delta LTK \cong \Delta MTK$; scale factor: 1.

7-2 Similar Polygons

14. 

SOLUTION:

Step 1: Compare corresponding angles:

$\angle B \cong \angle F$ Given

$\angle D \cong \angle G$ All right angles are congruent.

$\angle BCD \cong \angle FCG$ Vertical angles are congruent.

Therefore, $\angle B \cong \angle F$, $\angle D \cong \angle G$, *and* $\angle BCD \cong \angle FCG$.

Step 2: Compare corresponding sides:

$$\frac{BD}{FG} = \frac{4}{\frac{16}{3}} = 4 \cdot \frac{3}{16} = \frac{12}{16} = \frac{3}{4}$$

$$\frac{DC}{GC} = \frac{3}{4}$$

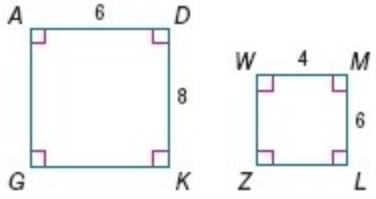
$$\frac{CB}{CF} = \frac{5}{\frac{20}{3}} = 5 \cdot \frac{3}{20} = \frac{15}{20} = \frac{3}{4}$$

Therefore $\frac{BD}{FG} = \frac{DC}{GC} = \frac{CA}{CF}$ and the corresponding sides are proportional with a scale factor of $\frac{3}{4}$.

Yes; $\Delta BDC \sim \Delta FGC$ because $\angle B \cong \angle F, \angle D \cong \angle G, \angle BCD \cong \angle FCG$, $\frac{BD}{FG} = \frac{DC}{GC} = \frac{CB}{CF}$; scale factor: $\frac{3}{4}$.

ANSWER:

Yes; $\Delta BDC \sim \Delta FGC$ because $\angle B \cong \angle F, \angle D \cong \angle G, \angle BCD \cong \angle FCG$, $\frac{BD}{FG} = \frac{DC}{GC} = \frac{CB}{CF}$; scale factor: $\frac{3}{4}$.

15. 

SOLUTION:

Step 1: Compare corresponding angles:

Since all of the angles in the polygons are right angles, they are all congruent to each other. Therefore, corresponding angles are congruent.

Step 2: Compare corresponding sides:

$\frac{AD}{WM} = \frac{6}{4} = \frac{3}{2}$

$\frac{DK}{ML} = \frac{8}{6} = \frac{4}{3}$

Since $\frac{AD}{WM} \neq \frac{DK}{ML}$, the figures are <u>not</u> similar.

ANSWER:

no; $\frac{AD}{WM} \neq \frac{DK}{ML}$

16. **GAMES** The dimensions of a hockey rink are 200 feet by 85 feet. Are the hockey rink and the air hockey table shown similar? Explain your reasoning.

SOLUTION:

No; sample answer: The ratio of the dimensions of the hockey rink and air hockey table are not the same.

$$\frac{\text{length of hockey rink}}{\text{length of hockey table}} = \frac{200}{98} \approx 2$$

$$\frac{\text{width of hockey rink}}{\text{width of hockey table}} = \frac{85}{49} \approx 1.7$$

The ratio of their lengths is about 2 and their widths is about 1.7.

ANSWER:

No; sample answer: The ratio of the lengths of the hockey rink and air hockey table is about 2 and the ratio of the widths is about 1.7.

17. **COMPUTERS** The dimensions of a 17-inch flat panel computer screen are approximately $13\frac{1}{4}$ by $10\frac{3}{4}$ inches. The dimensions of a 19-inch flat panel computer screen are approximately $14\frac{1}{2}$ by 12 inches. To the nearest tenth, are the computer screens similar? Explain your reasoning.

SOLUTION:

Yes; sample answer: In order to determine if two polygons are similar, you must compare the ratios of their corresponding sides.

$$\frac{\text{length of big screen}}{\text{length of small screen}} = \frac{14\frac{1}{2}}{13\frac{1}{4}} = \frac{\frac{29}{2}}{\frac{53}{4}} = \frac{29}{2} \cdot \frac{4}{53} = \frac{116}{106} \approx 1.09$$

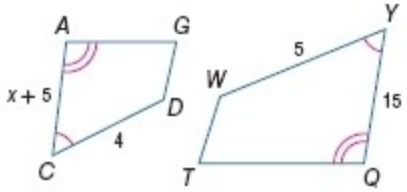
$$\frac{\text{width of big screen}}{\text{width of small screen}} = \frac{12}{10\frac{3}{4}} = \frac{12}{\frac{43}{4}} = \frac{12}{1} \cdot \frac{4}{43} = \frac{48}{43} \approx 1.12$$

The ratio of the longer dimensions of the screens is approximately 1.1 and the ratio of the shorter dimensions of the screens is approximately 1.1. Since the table and the rink are both rectangles, we know all of their angles are congruent to each other and, since the ratio of their corresponding sides are the same, the shapes are similar.

ANSWER:

Yes; sample answer: The ratio of the longer dimensions of the screens is approximately 1.1 and the ratio of the shorter dimensions of the screens is approximately 1.1.

CCSS REGULARITY Each pair of polygons is similar. Find the value of *x*.

18. 

SOLUTION:

Use the corresponding side lengths to write a proportion.

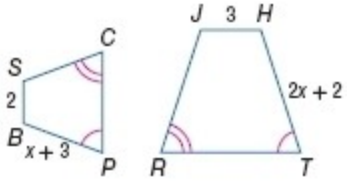
$$\frac{x+5}{15}=\frac{4}{5}$$

Solve for *x*.

$$\begin{aligned} 5(x+5) &= 60 \\ 5x+25 &= 60 \\ 5x &= 35 \\ x &= 7 \end{aligned}$$

ANSWER:

7

19. 

SOLUTION:

Use the corresponding side lengths to write a proportion.

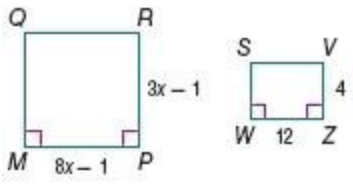
$$\frac{2x+2}{x+3}=\frac{3}{2}$$

Solve for *x*.

$$\begin{aligned} 2(2x+2) &= 3(x+3) \\ 4x+4 &= 3x+9 \\ x &= 5 \end{aligned}$$

ANSWER:

5



20.

SOLUTION:

Use the corresponding side lengths to write a proportion.

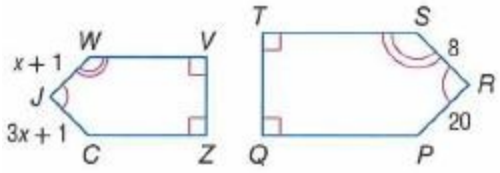
$$\frac{3x-1}{4}=\frac{8x-1}{12}$$

Solve for x.

$$\begin{aligned}12(3x-1)&=4(8x-1)\\36x-12&=32x-4\\4x&=8\\x&=2\end{aligned}$$

ANSWER:

2



21.

SOLUTION:

Use the corresponding side lengths to write a proportion.

$$\frac{x+1}{8}=\frac{3x+1}{20}$$

Solve for x.

$$\begin{aligned}20(x+1)&=8(3x+1)\\20x+20&=24x+8\\4x&=12\\x&=3\end{aligned}$$

ANSWER:

3

22. Rectangle *ABCD* has a width of 8 yards and a length of 20 yards. Rectangle *QRST*, which is similar to rectangle *ABCD*, has a length of 40 yards. Find the scale factor of rectangle *ABCD* to rectangle *QRST* and the perimeter of each rectangle.

SOLUTION:

Let x be the width of rectangle *QRST*.
Use the corresponding side lengths to write a proportion.

$$\frac{x}{40}=\frac{8}{20}$$

Solve for x.

$$20x = 320$$
$$x = 16$$

$$\text{Scale factor} = \frac{\text{Length of rectangle } ABCD}{\text{Length of rectangle } QRST}$$
$$= \frac{20}{40}$$
$$= \frac{1}{2}$$

Therefore, the scale factor is 1:2.

$$\text{Perimeter of rectangle } ABCD = 2(l+b)$$
$$= 2(20+8)$$
$$= 56$$

Therefore, the perimeter of rectangle *ABCD* is 56 yards.

$$\text{Perimeter of rectangle } QRST = 2(l+b)$$
$$= 2(40+16)$$
$$= 112$$

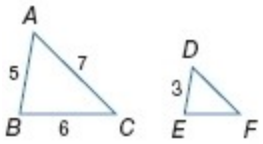
Therefore, the perimeter of rectangle *QRST* is 112 yards.

ANSWER:

1: 2; perimeter of *ABCD* = 56 yd, perimeter of *QRST* = 112 yd

Find the perimeter of the given triangle.

23. ΔDEF, if $\Delta ABC \sim \Delta DEF$, $AB = 5$, $BC = 6$, $AC = 7$, and $DE = 3$



SOLUTION:

Use the corresponding side lengths to write a proportion.

$$\frac{EF}{6} = \frac{3}{5} \qquad \frac{DF}{7} = \frac{3}{5}$$

Solve for EF.

$$5EF = 18$$
$$EF = 3.6$$

Solve for DF.

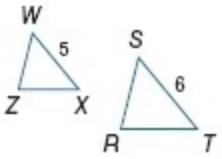
$$5DF = 21$$
$$EF = 4.2$$

$$\begin{aligned}\text{Perimeter of triangle } DEF &= DE + EF + DF \\ &= 3 + 3.6 + 4.2 \\ &= 10.8\end{aligned}$$

ANSWER:

10.8

24. ΔWZX, if $\Delta WZX \sim \Delta SRT$, $ST = 6$, $WX = 5$, and the perimeter of $\Delta SRT = 15$



SOLUTION:

The scale factor of triangle *SRT* to triangle *WZX* is $\frac{ST}{WX}$ or $\frac{6}{5}$.

Use the perimeter of triangle *SRT* and the scale factor to write a proportion and then substitute in the value of the perimeter of triangle SRT and solve for the perimeter of triangle WZX.

$$\frac{6}{5} = \frac{\text{Perimeter of triangle } SRT}{\text{Perimeter of triangle } WZX}$$

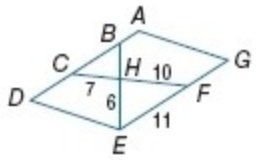
$$\frac{6}{5} = \frac{15}{\text{Perimeter of triangle } WZX}$$

$$\text{Perimeter of triangle } WZX = \frac{75}{6}$$
$$= 12.5$$

ANSWER:

12.5

25. ΔCBH, if $\Delta CBH \sim \Delta FEH$, $ADEG$ is a parallelogram, $CH = 7$, $FH = 10$, $FE = 11$, and $EH = 6$



SOLUTION:

Use the corresponding side lengths to write a proportion.

$$\frac{7}{10} = \frac{BH}{6} \qquad \frac{7}{10} = \frac{BC}{11}$$

Solve for BH.

$$\frac{7}{10} = \frac{BH}{6}$$
$$10BH = 42$$
$$BH = 4.2$$

Solve for BC.

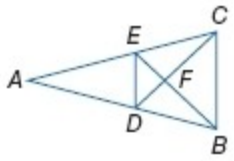
$$\frac{7}{10} = \frac{BC}{11}$$
$$10BC = 77$$
$$BC = 7.7$$

$$\begin{aligned}\text{Perimeter of triangle } CBH &= CB + BH + HC \\ &= 7.7 + 4.2 + 7 \\ &= 18.9\end{aligned}$$

ANSWER:

18.9

26. ΔDEF, if $\Delta DEF \sim \Delta CBF$, perimeter of $\Delta CBF = 27$, $DF = 6$, $FC = 8$



SOLUTION:

The scale factor of triangle *CBF* to triangle *DEF is* $\frac{FC}{DF}$ or $\frac{8}{6} = \frac{4}{3}$.

Use the perimeter of triangle *CBF* and the scale factor to write a proportion.Then, substitute the given value of the perimeter of triangle CBF and solve for the perimeter of triangle DEF.

$$\frac{4}{3} = \frac{\text{Perimeter of } \Delta CBF}{\text{Perimeter of } \Delta DEF}$$

$$\frac{4}{3} = \frac{27}{\text{Perimeter of } \Delta DEF}$$

$$4(\text{Perimeter of } \Delta DEF) = 3(27)$$

$$4(\text{Perimeter of } \Delta DEF) = 81$$

$$\text{Perimeter of } \Delta DEF = \frac{81}{4}$$

$$\text{Perimeter of } \Delta DEF = 20.25$$

$$\text{Perimeter of triangle } DEF = \frac{81}{4}$$
$$= 20.25$$

ANSWER:

20.25

27. Two similar rectangles have a scale factor of 2: 4. The perimeter of the large rectangle is 80 meters. Find the perimeter of the small rectangle.

SOLUTION:

Use the perimeter of the large rectangle and the scale factor to write a proportion. Then, substitute in the given value of the large rectangle and solve for the perimeter of the small rectangle.

$$\frac{4}{2} = \frac{\text{Perimeter of the large rectangle}}{\text{Perimeter of the small rectangle}}$$

$$\frac{4}{2} = \frac{80}{\text{Perimeter of the small rectangle}}$$

$$4(\text{Perimeter of small rectangle}) = 2(80)$$

$$4(\text{Perimeter of small rectangle}) = 160$$

$$\text{Perimeter of small rectangle} = \frac{160}{4}$$

$$\text{Perimeter of the small rectangle} = \frac{160}{4}$$

$$= 40 \text{ meters}$$

ANSWER:

40 m

28. Two similar squares have a scale factor of 3: 2. The perimeter of the small rectangle is 50 feet. Find the perimeter of the large rectangle.

SOLUTION:

Use the perimeter of the large rectangle and the scale factor to write a proportion.Then, substitute the given value of the perimeter of the small rectangle into the proportion. Solve for the perimeter of the large rectangle.

$$\frac{3}{2} = \frac{\text{Perimeter of the large rectangle}}{\text{Perimeter of the small rectangle}}$$

$$\frac{3}{2} = \frac{\text{Perimeter of the large rectangle}}{50}$$

$$3(50) = 2(\text{Perimeter of the large rectangle})$$

$$150 = 2(\text{Perimeter of the large rectangle})$$

$$\frac{150}{2} = \text{Perimeter of the large rectangle}$$

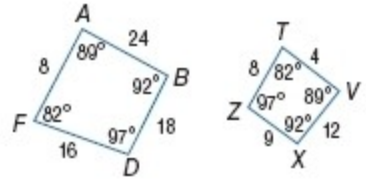
$$75 = \text{Perimeter of the large rectangle}$$

Thus, the perimeter of the large rectangle is 75 ft.

ANSWER:

75 ft

List all pairs of congruent angles, and write a proportion that relates the corresponding sides.

29. 

SOLUTION:

The order of vertices in a similarity statement identifies the corresponding angles and sides.

$m\angle A = 89 = m\angle V$
$m\angle B = 92 = m\angle X$
$m\angle D = 97 = m\angle Z$
$m\angle F = 82 = m\angle T$

Therefore, $\angle A \cong \angle V$; $\angle B \cong \angle X$; $\angle D \cong \angle Z$; $\angle F \cong \angle T$.

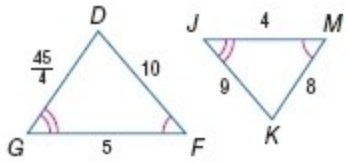
$\frac{AB}{VX} = \frac{24}{12} = \frac{2}{1}$
$\frac{BD}{XZ} = \frac{18}{9} = \frac{2}{1}$
$\frac{DF}{ZT} = \frac{16}{8} = \frac{2}{1}$
$\frac{FA}{TV} = \frac{8}{4} = \frac{2}{1}$

Hence, $\frac{AB}{VX} = \frac{BD}{XZ} = \frac{DF}{ZT} = \frac{FA}{TV} = 2$

ANSWER:

$\angle A \cong \angle V$, $\angle B \cong \angle X$, $\angle D \cong \angle Z$, $\angle F \cong \angle T$; $\frac{AB}{VX} = \frac{BD}{XZ} = \frac{DF}{ZT} = \frac{FA}{TV} = 2$

7-2 Similar Polygons

30. 

SOLUTION:

The order of vertices in a similarity statement identifies the corresponding angles and sides.

$\angle G \cong \angle J$ *Given*
$\angle F \cong \angle M$ *Given*
$\angle D \cong \angle K$ *Third Angle Theorem*

Therefore, $\angle D \cong \angle K, \angle F \cong \angle M, \angle G \cong \angle J;$

$\frac{DF}{KM} = \frac{10}{8} = \frac{5}{4}$

$\frac{FG}{MJ} = \frac{5}{4}$

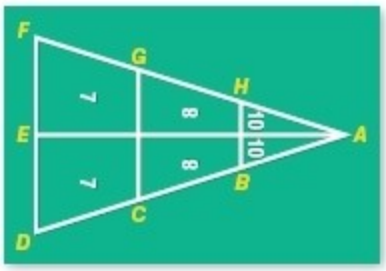
$\frac{GD}{JK} = \frac{\frac{45}{4}}{9} = \frac{45}{4} \cdot \frac{1}{9} = \frac{5}{4}$

Hence, $\frac{DF}{KM} = \frac{FG}{MJ} = \frac{GD}{JK} = \frac{5}{4} = 1.25$.

ANSWER:

$\angle D \cong \angle K, \angle F \cong \angle M, \angle G \cong \angle J; \frac{DF}{KM} = \frac{FG}{MJ} = \frac{GD}{JK} = 1.25$

SHUFFLEBOARD A shuffleboard court forms three similar triangles in which $\angle AHB \cong \angle AGC \cong \angle AFD$. Find the side(s) that correspond to the given side or angles that are congruent to the given angle.



31. $\overline{AB}$

SOLUTION:

Since we know that $\Delta ABC \sim \Delta ACG \sim \Delta ADF$, then we know that the order of vertices in a similarity statement identifies the corresponding angles and sides. Therefore, $\overline{AB}$ would correspond to $\overline{AC}$ and $\overline{AD}$.

ANSWER:

$\overline{AC}, \overline{AD}$

32. $\overline{FD}$

SOLUTION:

Since we know that $\Delta ABC \sim \Delta ACG \sim \Delta ADF$, then we know that the order of vertices in a similarity statement identifies the corresponding angles and sides. Therefore, $\overline{FD}$ would correspond to $\overline{HB}$ and $\overline{GC}$.

ANSWER:

$\overline{HB}, \overline{GC}$

33. $\angle ACG$

SOLUTION:

Since we know that $\Delta ABC \sim \Delta ACG \sim \Delta ADF$, then we know that the order of vertices in a similarity statement identifies the corresponding angles and sides. Therefore, $\angle ACG$ would correspond to $\angle ABH$ and $\angle ADF$.

ANSWER:

$\angle ABH, \angle ADF$

34. $\angle A$

SOLUTION:

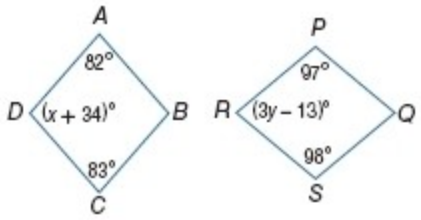
Angle A is included in all of the triangles. No other angles are congruent to A. In the similarity involving ΔHAE and ΔBAE, and with every other similarity, A corresponds with A.

ANSWER:

$\angle A$ is included in all of the triangles.

Find the value of each variable.

35. $ABCD \sim QSRP$



SOLUTION:

Two polygons are similar if and only if their corresponding angles are congruent and corresponding side lengths are proportional.

So, $\angle A \cong \angle Q, \angle B \cong \angle S, \angle C \cong \angle R, \angle D \cong \angle P.$

Therefore, $x + 34 = 97$ and $3y - 13 = 83$.

Solve for x.

$x + 34 = 97$

$x + 34 - 34 = 97 - 34$

$x = 63$

Solve for y.

$3y - 13 = 83$

$3y - 13 + 13 = 83 + 13$

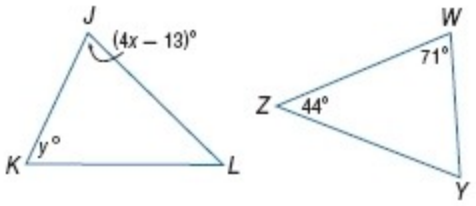
$3y = 96$

$y = 32$

ANSWER:

$x = 63, y = 32$

36. $\Delta JKL \sim \Delta WYZ$



SOLUTION:

Two polygons are similar if and only if their corresponding angles are congruent and corresponding side lengths are proportional.

So, $\angle J \cong \angle W, \angle K \cong \angle Y, \angle L \cong \angle Z.$

Therefore, $4x - 13 = 71$ and $m\angle L = 44$.

Solve for x.

$4x - 13 = 71$

$4x-13+13=71+13$

$4x=84$

$x=21$

So, $m\angle J = 4(21) - 13 = 71$.

We know that the sum of measures of all interior angles of a triangle is 180.

$$\begin{aligned} m\angle J + m\angle K + m\angle L &= 180 \\ 71 + y + 44 &= 180 \\ 115 + y &= 180 \\ y &= 65 \end{aligned}$$

ANSWER:

$x = 21, y = 65$

37. **SLIDE SHOW** You are using a digital projector for a slide show. The photos are 13 inches by $9\frac{1}{4}$ inches on the computer screen, and the scale factor of the computer image to the projected image is 1:4. What are the dimensions of the projected image?

SOLUTION:

Since the scale factor of the computer image to the projected image is 1:4, we can set up a proportion to find the projected dimensions.

Let the unknown width be w and the unknown length be l.

Form two proportions with the given information and solve for w and l.

$\frac{1}{4} = \frac{13}{w}$ *and* $\frac{1}{4} = \frac{9\frac{1}{4}}{l}$

$1 \cdot w = 4 \cdot 13$ $1 \cdot l = 9\frac{1}{4} \cdot 4$

$w = 52$ $l = \frac{37}{4} \cdot 4 = 37$

Therefore, the dimensions of the projected image would be 52 inches by 37 inches.

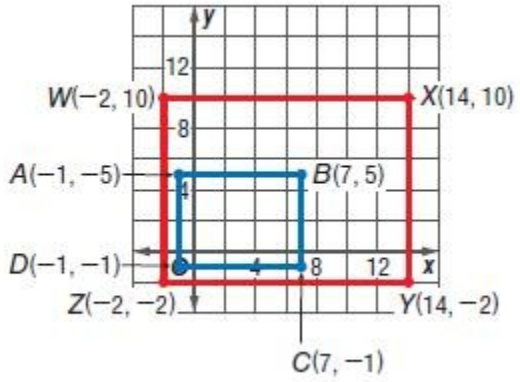
ANSWER:

52 in. by 37 in.

COORDINATE GEOMETRY For the given vertices, determine whether rectangle *ABCD* is similar to rectangle *WXYZ*. Justify your answer.

38. $A(-1, 5)$, $B(7, 5)$, $C(7, -1)$, $D(-1, -1)$;
$W(-2, 10)$, $X(14, 10)$, $Y(14, -2)$, $Z(-2, -2)$

SOLUTION:



Since it is given that these shapes are rectangles, we know that all the angles formed are right angles. Therefore, the right angles are all congruent, $\angle A \cong \angle W, \angle B \cong \angle X, \angle C \cong \angle Y,$ and $\angle D \cong \angle Z$.

Use the distance formula to find the length of each side.

$$AB = \sqrt{(7-(-1))^2 + (5-5)^2}$$
$$= \sqrt{64+0}$$
$$= 8$$
$$BC = \sqrt{(7-7)^2 + (-1-5)^2}$$
$$= \sqrt{0+36}$$
$$= 6$$
$$CD = \sqrt{(-1-7)^2 + (-1-(-1))^2}$$
$$= \sqrt{64}$$
$$= 8$$
$$DA = \sqrt{(-1-(-1))^2 + (-1-5)^2}$$
$$= \sqrt{0+36}$$
$$= 6$$
$$WX = \sqrt{(14-(-2))^2 + (10-10)^2}$$
$$= \sqrt{256+0}$$
$$= 16$$
$$XY = \sqrt{(14-14)^2 + (-2-10)^2}$$
$$= \sqrt{0+144}$$
$$= 12$$
$$YZ = \sqrt{(-2-14)^2 + (-2-(-2))^2}$$
$$= \sqrt{256+0}$$
$$= 16$$

Now, compare the scale factors of each pair of corresponding sides:

$$\frac{AB}{WX} = \frac{8}{16} = \frac{1}{2}$$
$$\frac{BC}{XY} = \frac{6}{12} = \frac{1}{2}$$
$$\frac{CD}{YZ} = \frac{8}{16} = \frac{1}{2}$$
$$\frac{DA}{ZW} = \frac{6}{12} = \frac{1}{2}$$

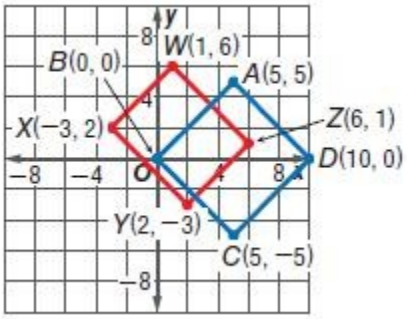
Therefore, $ABCD \sim WXYZ$ because $\angle A \cong \angle W, \angle B \cong \angle X, \angle C \cong \angle Y, \angle D \cong \angle Z$, and $\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW} = \frac{1}{2}$.

ANSWER:

$ABCD \sim WXYZ$ because $\angle A \cong \angle W$, $\angle B \cong \angle X$, $\angle C \cong \angle Y$, $\angle D \cong \angle Z$, $\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW} = \frac{1}{2}$

39. $A(5, 5)$, $B(0, 0)$, $C(5, -5)$, $D(10, 0)$;
$W(1, 6)$, $X(-3, 2)$, $Y(2, -3)$, $Z(6, 1)$

SOLUTION:



Since we know this is a rectangle, we know all the angles are right angles and are congruent to each other.

Use the distance formula to find the length of each side.

$$AB = \sqrt{(0-5)^2 + (0-5)^2}$$
$$= \sqrt{25+25}$$
$$= 5\sqrt{2}$$
$$BC = \sqrt{(5-0)^2 + (-5-0)^2}$$
$$= \sqrt{25+25}$$
$$= 5\sqrt{2}$$
$$WX = \sqrt{(-3-1)^2 + (2-6)^2}$$
$$= \sqrt{16+16}$$
$$= 4\sqrt{2}$$
$$XY = \sqrt{(2-(-3))^2 + (-3-2)^2}$$
$$= \sqrt{25+25}$$
$$= 5\sqrt{2}$$

$$\frac{BC}{XY} = \frac{5\sqrt{2}}{5\sqrt{2}} = \frac{1}{1}$$

$$\frac{AB}{WX} = \frac{5\sqrt{2}}{4\sqrt{2}} = \frac{5}{4}$$

Therefore, $\frac{BC}{XY} \neq \frac{AB}{WX}$.

So the given rectangles are not similar.

ANSWER:

no; $\frac{BC}{XY} \neq \frac{AB}{WX}$

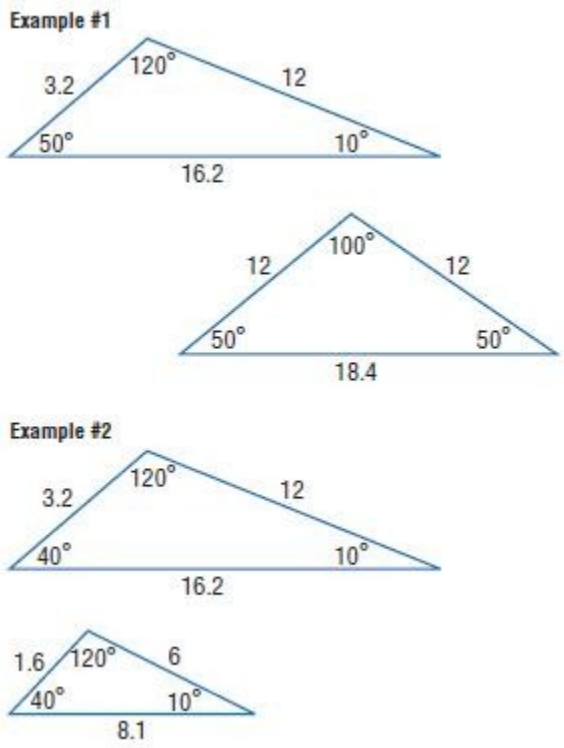
CCSS ARGUMENTS Determine whether the polygons are *always*, *sometimes*, or *never* similar. Explain your reasoning.

40. two obtuse triangles

SOLUTION:

Sometimes; sample answer: If corresponding angles are congruent and corresponding sides are proportional, two obtuse triangles are similar.

In Example #1 below, you can see that both triangles are obtuse, however, their corresponding angles are not congruent and their corresponding sides are not proportional. In Example #2, the triangles are obtuse and similar, since their corresponding angles are congruent and the corresponding sides each have a scale factor of 2:1.



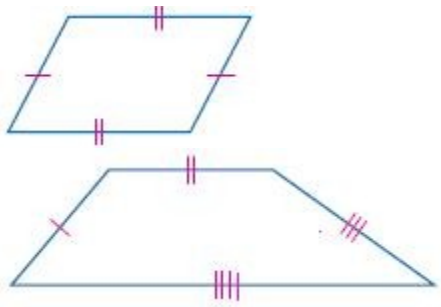
ANSWER:

Sometimes; sample answer: If corresponding angles are congruent and corresponding sides are proportional, two obtuse triangles are similar.

41. a trapezoid and a parallelogram

SOLUTION:

Never; sample answer: Parallelograms have both pairs of opposite sides congruent. Trapezoids can only have one pair of opposite sides congruent, as their one pair of parallel sides aren't congruent. Therefore, the two figures cannot be similar because they can never be the same type of figure.



ANSWER:

Never; sample answer: Parallelograms have both pairs of opposite sides parallel. Trapezoids have exactly one pair of parallel legs. Therefore, the two figures cannot be similar because they can never be the same type of figure.

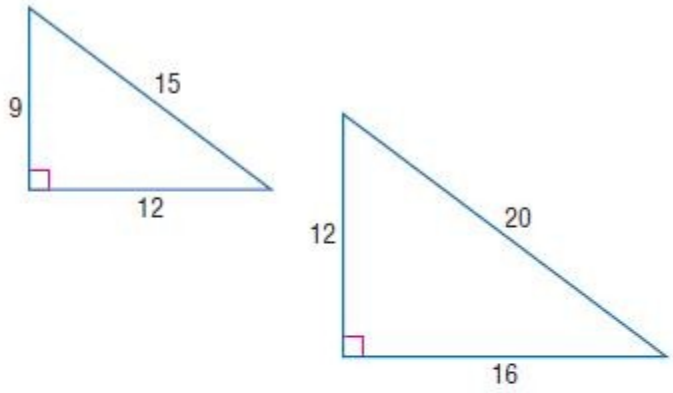
42. two right triangles

SOLUTION:

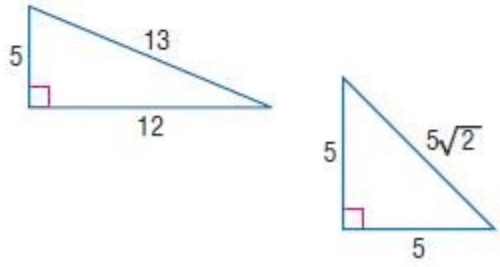
Sometimes; sample answer: If corresponding angles are congruent and corresponding sides are proportional, two right triangles are similar.

As seen in Example #1 below, the right triangles are similar, because their corresponding angles are congruent and their corresponding sides all have the same scale factor of 3:4. However, in Example #2, the right triangles are not similar becuase the corresponding sides do not have the same scale factor.

Example #1



Example #2



ANSWER:

Sometimes; sample answer: If corresponding angles are congruent and corresponding sides are proportional, two right triangles are similar.

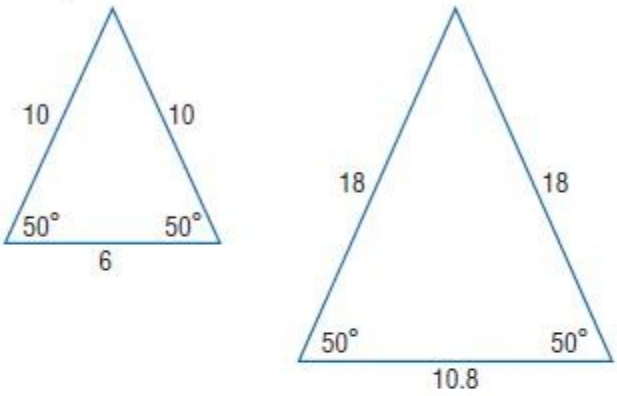
43. two isosceles triangles

SOLUTION:

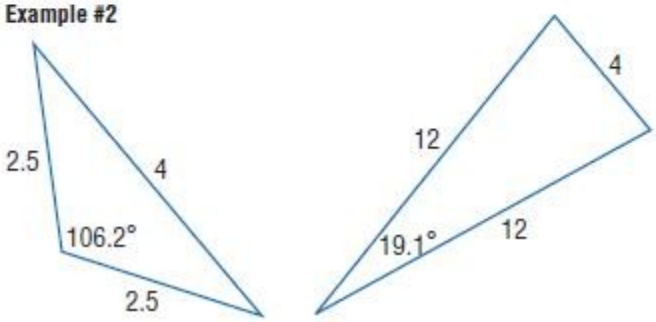
Sometimes; sample answer: If corresponding angles are congruent and corresponding sides are proportional, two isosceles triangles are similar.

As shown in Example #1 below, the two isosceles triangles are similar because they have congruent corresponding angles and the scale factor of the corresponding sides is 5:9. However, in Example #2, although still isosceles triangles, their corresponding parts are not related.

Example #1



Example #2



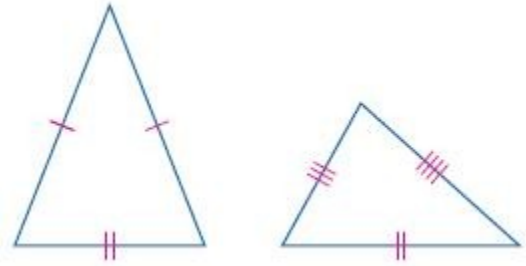
ANSWER:

Sometimes; sample answer: If corresponding angles are congruent and corresponding sides are proportional, two isosceles triangles are similar.

44. a scalene triangle and an isosceles triangle

SOLUTION:

Never; sample answer: Since an isosceles triangle has two congruent sides and a scalene triangle has three non-congruent sides, the ratios of the three pairs of sides can never be equal. Therefore, an isosceles triangle and a scalene triangle can never be similar.



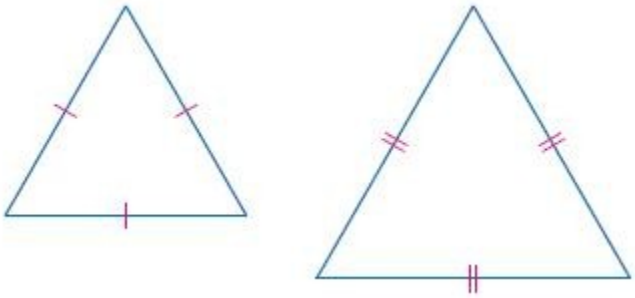
ANSWER:

Never; sample answer: Since an isosceles triangle has two congruent sides and a scalene triangle has three non-congruent sides, the ratios of the three pairs of sides can never be equal. Therefore, an isosceles triangle and a scalene triangle can never be similar.

45. two equilateral triangles

SOLUTION:

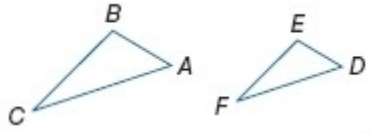
Always; sample answer: Equilateral triangles always have three 60° angles, so the angles of one equilateral triangle are always congruent to the angles of a second equilateral triangle. The three sides of an equilateral triangle are always congruent, so the ratio of each pair of legs of one triangle to a second triangle will always be the same. Therefore, a pair of equilateral triangles is always similar.



ANSWER:

Always; sample answer: Equilateral triangles always have three 60° angles, so the angles of one equilateral triangle are always congruent to the angles of a second equilateral triangle. The three sides of an equilateral triangle are always congruent, so the ratio of each pair of legs of one triangle to a second triangle will always be the same. Therefore, a pair of equilateral triangles is always similar.

46. **PROOF** Write a paragraph proof of Theorem 7.1.

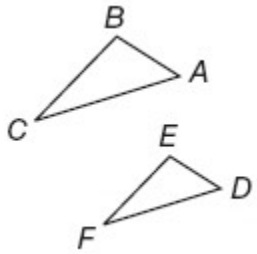


Given: $\Delta ABC \sim \Delta DEF$ and $\frac{AB}{DE} = \frac{m}{n}$

Prove: $\frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta DEF} = \frac{m}{n}$

SOLUTION:

For this proof, because we know that $\Delta ABC \sim \Delta DEF$, we can write a similarity statement relating the corresponding sides, such as $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$. Notice that we also know that $\frac{AB}{DE} = \frac{m}{n}$. Therefore, we can set each of these ratios equal to $\frac{m}{n}$, due to the Transitive property, and isolate each individual side length of ΔABC into separate equations. We are doing this because we want to get both perimeter formulas in terms of the same side lengths. Then, substitute each side relationship into the perimeter formula for each triangle and simplify.

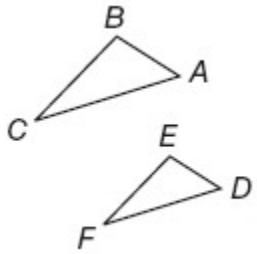


Given: $\Delta ABC \sim \Delta DEF$ and $\frac{AB}{DE} = \frac{m}{n}$

Prove: $\frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta DEF} = \frac{m}{n}$

Proof: Because $\Delta ABC \sim \Delta DEF$, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$. So $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{m}{n}$. Cross products yield $AB = DE\left(\frac{m}{n}\right)$, $BC = EF\left(\frac{m}{n}\right)$, and $AC = DF\left(\frac{m}{n}\right)$. Using substitution, the perimeter of $\Delta ABC = DE\left(\frac{m}{n}\right) + EF\left(\frac{m}{n}\right) + DF\left(\frac{m}{n}\right)$, or $\frac{m}{n}(DE + EF + DF)$. The ratio of the two perimeters = $\frac{\frac{m}{n}(DE + EF + DF)}{DE + EF + DF}$ or $\frac{m}{n}$.

ANSWER:



Given: $\Delta ABC \sim \Delta DEF$ and $\frac{AB}{DE} = \frac{m}{n}$

Prove: $\frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta DEF} = \frac{m}{n}$

Proof: Because $\Delta ABC \sim \Delta DEF$, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$. So $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{m}{n}$. Cross products yield $AB = DE\left(\frac{m}{n}\right)$, $BC = EF\left(\frac{m}{n}\right)$, and $AC = DF\left(\frac{m}{n}\right)$. Using substitution, the perimeter of $\Delta ABC = DE\left(\frac{m}{n}\right) + EF\left(\frac{m}{n}\right) + DF\left(\frac{m}{n}\right)$, or $\frac{m}{n}(DE + EF + DF)$. The ratio of the two perimeters = $\frac{\frac{m}{n}(DE + EF + DF)}{DE + EF + DF}$ *or* $\frac{m}{n}$.

47. **PHOTOS** You are enlarging the photo shown for your school yearbook. If the dimensions of the original photo are $2\frac{1}{3}$ inches by $1\frac{2}{3}$ inches and the scale factor of the old photo to the new photo is 2: 3, what are the dimensions of the new photo?

Refer to Page 472.

SOLUTION:

Let the unknown width be *w* and the unknown length be *l*.

Since the scale factor of the original photo to the new photo is 2:3, we can write proportions comparing their dimensions.

Form two proportions with the given information and solve for *w* and *l*.

$\frac{2}{3} = \frac{1\frac{2}{3}}{w}$ *and* $\frac{2}{3} = \frac{2\frac{1}{3}}{l}$

$2 \cdot w = 1\frac{2}{3} \cdot 3$ $\quad$ $2 \cdot l = 2\frac{1}{3} \cdot 3$

$2w = \frac{5}{3} \cdot 3$ $\quad$ $2l = \frac{7}{3} \cdot 3$

$2w = 5$ $\quad$ $2l = 7$

$w = \frac{5}{2}$ $\quad$ $l = \frac{7}{2}$

Therefore, the dimensions of the new photo are $\frac{7}{2}$ *or* $3\frac{1}{2}$ inches by $\frac{5}{2}$ *or* $2\frac{1}{2}$ inches.

ANSWER:

$3\frac{1}{2}$ in. by $2\frac{1}{2}$ in.

48. **CHANGING DIMENSIONS** Rectangle *QRST* is similar to rectangle *JKLM* with sides in a ratio of 4: 1.
a. What is the ratio of the areas of the two rectangles?
b. Suppose the dimension of each rectangle is tripled. What is the new ratio of the sides of the rectangles?
c. What is the ratio of the areas of these larger rectangles?

SOLUTION:

a. The area of rectangle involves squaring the relationship of the width to the length , therefore the ratio would be $4^2 : 1^2$ *or* $16 : 1$.

b. If the dimensions of each side were tripled, we would have a resulting ratio 4(3): 1(3) = 12: 3, which results in a ratio of 4:1. There is no change in the ratio through multiplying each dimension by the same number; the ratio of the sides remains the same.

c. The area of rectangle involves squaring the relationship of the width to the length , therefore its ratio would be $12^2 : 3^2$ *or* $144 : 9$, which reduces to 16:1.

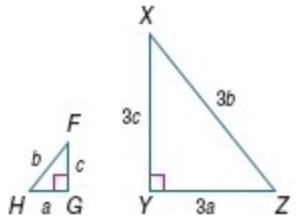
ANSWER:

a. 16:1
b. 4:1
c. 16:1

49. **CHANGING DIMENSIONS** In the figure shown, $\Delta FGH \sim \Delta XYZ$.

a. Show that the perimeters of ΔFGH and ΔXYZ have the same ratio as their corresponding sides.

b. If 6 units are added to the lengths of each side, are the new triangles similar? Explain.



SOLUTION:

a. The ratio of the corresponding sides of these two right triangles are $\frac{a}{3a} = \frac{b}{3b} = \frac{c}{3c}$ and the ratio of their perimeters is $\frac{a+b+c}{3a+3b+3c}$. When each ratio written in its most simplified form, you get $\frac{1}{3}$.

$$\frac{a}{3a} = \frac{b}{3b} = \frac{c}{3c} = \frac{a+b+c}{3(a+b+c)} = \frac{1}{3}$$

$$\frac{a+6}{3a+6} = \frac{b+6}{3b+6} = \frac{c+6}{3c+6}$$

b. $\frac{a+6}{3(a+2)} = \frac{b+6}{3(b+2)} = \frac{c+6}{3(c+2)}$, which does not equal a ratio of 1:3. Therefore, no, the sides are no longer proportional**.**

ANSWER:

a. $\frac{a}{3a} = \frac{b}{3b} = \frac{c}{3c} = \frac{a+b+c}{3(a+b+c)} = \frac{1}{3}$

b. No; the sides are no longer proportional.

50. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate similarity in squares.

a. GEOMETRIC Draw three different-sized squares. Label them *ABCD*, *PQRS*, and *WXYZ*. Measure and label each square with its side length.

b. TABULAR Calculate and record in a table the ratios of corresponding sides for each pair of squares: *ABCD* and *PQRS*, *PQRS* and *WXYZ*, and *WXYZ* and *ABCD*. Is each pair of squares similar?

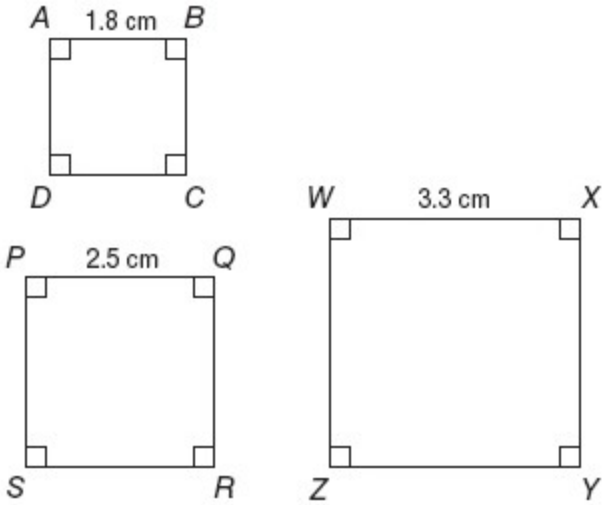
c. VERBAL Make a conjecture about the similarity of all squares.

SOLUTION:

a. Choose three different side lengths for the sketches of your three squares. It would be a good idea to use some decimals, as well as whole number side lengths as options. Label them as directed.

Sample answer:

7-2 Similar Polygons



b. When filling in this table, remember that all sides of a square are congruent to each other. What do you observe from your calculations?

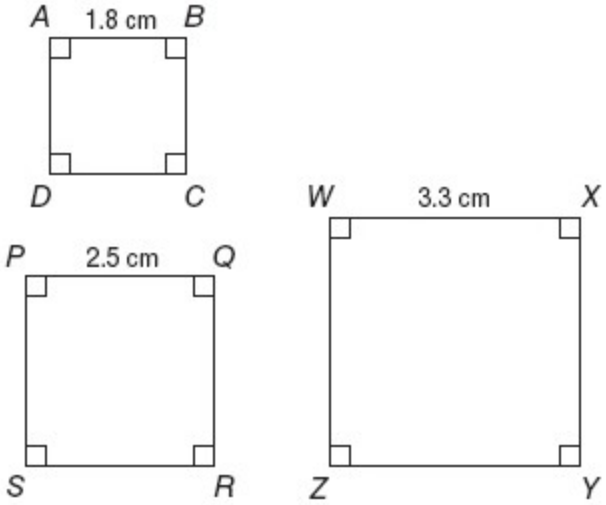
ABCD and *PQRS*		*PQRS* and *WXYZ*		*WXYZ* and *ABCD*	
AB:PQ	0.72	*PQ:WX*	0.76	*WX:AB*	1.8
BC:QR	0.72	*QR:XY*	0.76	*XY:BC*	1.8
CD:RS	0.72	*RS:YZ*	0.76	*YZ:CD*	1.8
AD:SP	0.72	*SP:ZW*	0.76	*ZW:DA*	1.8

ABCD is similar to *PQRS*; *PQRS* is similar to *WXYZ*; *WXYZ* is similar to *ABCD*.

c. Sample answer: Since all the ratios of corresponding sides of squares are the same, all squares are similar.

ANSWER:

a. Sample answer:



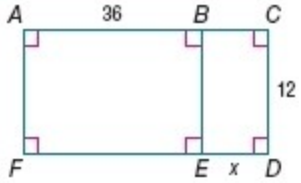
b.

ABCD and *PQRS*		*PQRS* and *WXYZ*		*WXYZ* and *ABCD*	
AB:*PQ*	0.72	*PQ*:*WX*	0.76	*WX*:*AB*	1.8
BC:*QR*	0.72	*QR*:*XY*	0.76	*XY*:*BC*	1.8
CD:*RS*	0.72	*RS*:*YZ*	0.76	*YZ*:*CD*	1.8
AD:*SP*	0.72	*SP*:*ZW*	0.76	*ZW*:*DA*	1.8

ABCD is similar to *PQRS*; *PQRS* is similar to *WXYZ*; *WXYZ* is similar to *ABCD*.

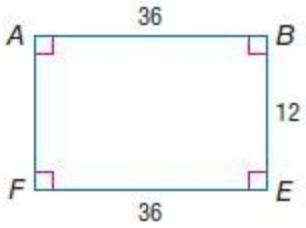
c. Sample answer: All squares are similar.

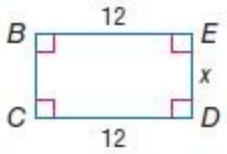
51. **CHALLENGE** For what value(s) of x is $BEFA \sim EDCB$?



SOLUTION:

Use the corresponding side lengths to write a proportion. Recall that the opposite sides in a rectangle are congruent.





$$\frac{x}{12} = \frac{12}{36}$$

Solve for x.

$$36x = 144$$

$$x = 4$$

ANSWER:

4

52. **REASONING** Recall that an *equivalence relation* is any relationship that satisfies the Reflexive, Symmetric, and Transitive Properties. Is similarity an equivalence relation? Explain.

SOLUTION:

Yes; sample answer:
Similarity is reflexive, then the ratio of lengths of two corresponding sides should equal each other. If $\Delta ABC \sim \Delta XYZ$, then $\frac{AB}{XY} = \frac{AB}{XY}$.

Recall that the symmetric property states *if a=b, then b=a*. Therefore, similarity is symmetric because, if $\frac{AB}{XY} = \frac{BC}{YZ}$, then $\frac{BC}{YZ} = \frac{AB}{XY}$.

Recall that the transitive property states *if a=b, and b=c, then a=c*. Therefore, similarity is transitive because if $\frac{AB}{XY} = \frac{BC}{YZ}$ and $\frac{BC}{YZ} = \frac{CA}{ZX}$, then $\frac{AB}{XY} = \frac{CA}{ZX}$.

ANSWER:

Yes; sample answer: Similarity is reflexive, symmetric, and transitive. If $\Delta ABC \sim \Delta XYZ$, then $\frac{AB}{XY} = \frac{AB}{XY}$. If $\frac{AB}{XY} = \frac{BC}{YZ}$, then $\frac{BC}{YZ} = \frac{AB}{XY}$. If $\frac{AB}{XY} = \frac{BC}{YZ}$ and $\frac{BC}{YZ} = \frac{CA}{ZX}$, then $\frac{AB}{XY} = \frac{CA}{ZX}$.

53. **OPEN ENDED** Find a counterexample for the following statement.
All rectangles are similar.

SOLUTION:

Sample answer:

Shown are two rectangles; however, the ratios of their corresponding sides are not the same. Therefore, they cannot be similar.

ANSWER:

Sample answer:

54. **CCSS REASONING** Draw two regular pentagons of different sizes. Are the pentagons similar? Will any two regular polygons with the same number of sides be similar? Explain.

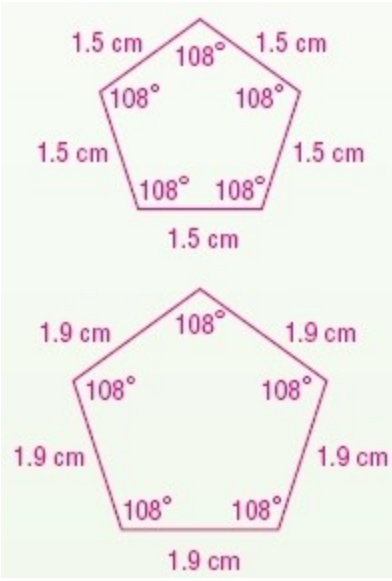
SOLUTION:

When you draw two regular pentagons (5-sided polygons) recall that all sides of the same pentagon are congruent to each other and all angles are congruent to each other as well. To determine one angle measure of any regular pentagon, you can use the sum of the interior angles of a polygon formula 180(*n*-2), where *n* represents the number of sides, and substitute in 5 for *n*

$$180(5-2) = 180(3) = 540$$

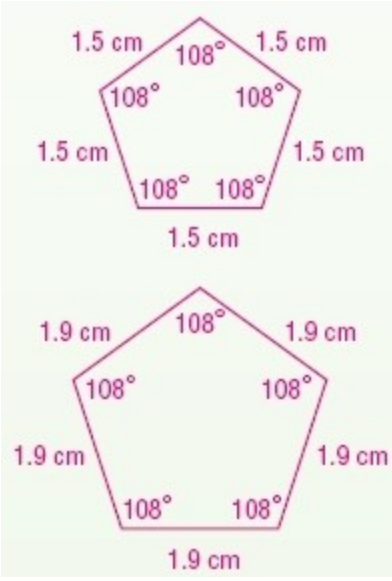
Divide 540 degrees by the 5 and determine the measure of one angle in a regular pentagon:

$\frac{540}{5} = 108$ degrees



Yes; yes; sample answer: The pentagons are similar because their corresponding angles are congruent and their corresponding sides are proportional. All of the angles and sides in a regular polygon are congruent. The angles will be congruent regardless of the size of the figure, and since all of the sides are congruent the ratios of the sides of one regular figure to a second regular figure with the same number of sides will all be the same. Therefore, all regular polygons with the same number of sides are congruent.

ANSWER:



Yes; yes; sample answer: The pentagons are similar because their corresponding angles are congruent and their corresponding sides are proportional. All of the angles and sides in a regular polygon are congruent. The angles will be congruent regardless of the size of the figure, and since all of the sides are congruent the ratios of the sides of one regular figure to a second regular figure with the same number of sides will all be the same. Therefore, all regular polygons with the same number of sides are congruent.

55. **WRITING IN MATH** How can you describe the relationship between two figures?

SOLUTION:

Sample answer: The figures could be described as congruent if they are the same size and shape, similar if their corresponding angles are congruent and their corresponding sides are proportional, and equal if they are the same exact figure.

ANSWER:

Sample answer: The figures could be described as congruent if they are the same size and shape, similar if their corresponding angles are congruent and their corresponding sides are proportional, and equal if they are the same exact figure.

56. **ALGEBRA** If the arithmetic mean of $4x$, $3x$, and 12 is 18, then what is the value of x?

A 6
B 5
C 4
D 3

SOLUTION:

$$\frac{4x+3x+12}{3}=18$$

Solve for x.

$$\frac{7x+12}{3}=18$$
$$7x+12=54$$
$$7x=42$$
$$x=6$$

So, the correct choice is A.

ANSWER:

A

57. Two similar rectangles have a scale factor of 3: 5. The perimeter of the large rectangle is 65 meters. What is the perimeter of the small rectangle?
F 29 m
G 39 m
H 49 m
J 59 m

SOLUTION:
Use the perimeter of the large rectangle and the scale factor to write a proportion.

$$\frac{5}{3} = \frac{\text{P(large)}}{\text{P(small)}}$$
$$\frac{5}{3} = \frac{65}{\text{P(small)}}$$
$$5P(small) = 3(65)$$
$$5P(small) = 195$$
$$P(small) = 39$$

So, the correct choice is G.

ANSWER:
G

58. **SHORT RESPONSE** If a jar contains 25 dimes and 7 quarters, what is the probability that a coin selected from the jar at random will be a dime?

SOLUTION:
Possible outcomes: { 25 dimes, 7 quarters}
Number of possible outcomes :32
Favorable outcomes: {25 dimes}
Number of favorable outcomes: 25

$$\text{P(dime)} = \frac{\text{favorable}}{\text{possible}}$$
$$= \frac{25}{32}$$
$$\approx 0.78$$

ANSWER:
0.78

59. **SAT/ACT** If the side of a square is $x + 3$, then what is the diagonal of the square?

A $x^2 + 3$
B $3x + 3$
C $x\sqrt{2} + 3\sqrt{2}$
D $x\sqrt{3} + 3\sqrt{3}$

SOLUTION:

Use the Pythagorean Theorem to find the diagonal of the square. Let d be the diagonal of the square.

$$d^2 = (x+3)^2 + (x+3)^2$$
$$= 2(x+3)^2$$
$$d = \pm\sqrt{2}(x+3)$$

Since the length must be positive, $d = \sqrt{2}(x+3)$ *or* $x\sqrt{2} + 3\sqrt{2}$
So, the correct choice is E.

ANSWER:

E

60. **COMPUTERS** In a survey of 5000 households, 4200 had at least one computer. What is the ratio of computers to households?

SOLUTION:

$$\frac{\text{number of computers}}{\text{number of households}} = \frac{4200}{5000}$$
$$= \frac{21}{25}$$

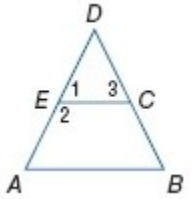
The ratio of computers to households is 21:25.

ANSWER:

21:25

61. **PROOF** Write a flow proof.

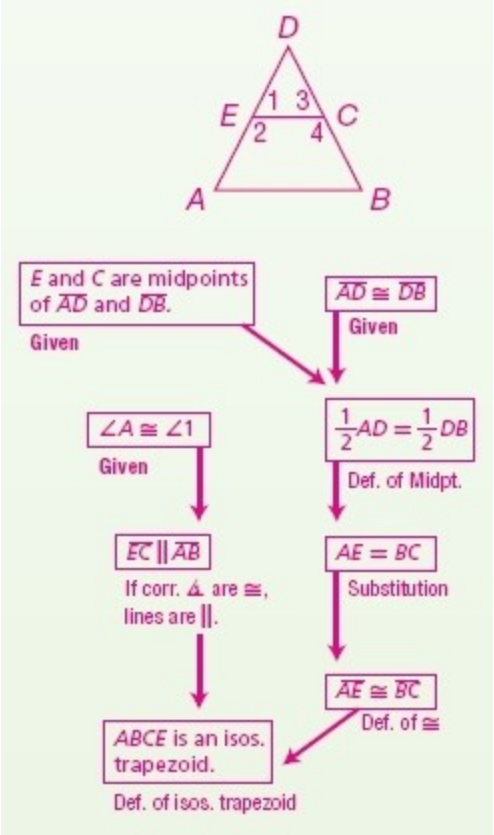
Given: E and C are midpoints of $\overline{AD}$ and $\overline{DB}$, $\overline{AD} \cong \overline{DB}$, $\angle A \cong \angle 1$.
Prove: $ABCE$ is an isosceles trapezoid.



SOLUTION:

A good approach to this proof is to think backwards. What do you need to do to prove that ABCE is an isosceles trapezoid? In order to do this, you will need to establish two facts: that the bases are parallel ($\overline{EC} \parallel \overline{AB}$) and the legs are congruent ($\overline{AE} \cong \overline{BC}$). To prove that lines are parallel to each other, you can use the given relationship that $\angle A \cong \angle 1$, which are congruent corresponding angles. Now, to prove that $\overline{AE} \cong \overline{BC}$, you can refer to the relationships that E and C are midpoints (meaning that they divide each segment into two congruent halves) and the given statement that $\overline{AD} \cong \overline{DB}$, to prove that half of each of these congruent segments must also be congruent.

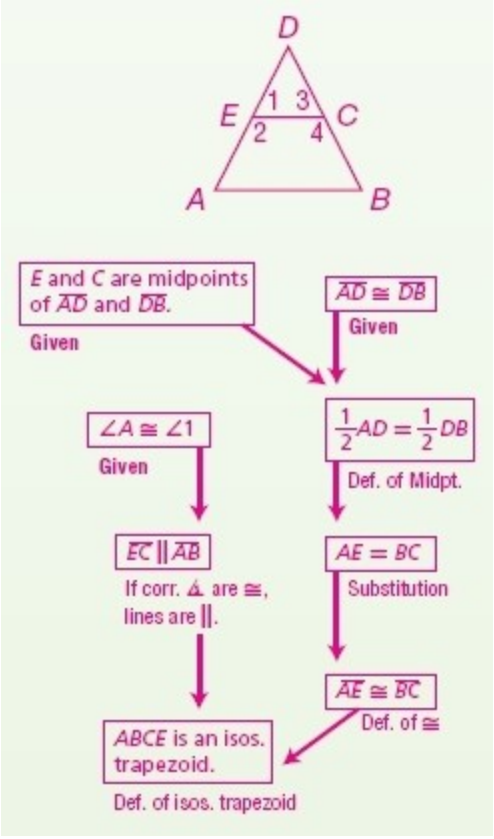
Given: E and C are midpoints of $\overline{AD}$ and $\overline{DB}$, $\overline{AD} \cong \overline{DB}$, $\angle A \cong \angle 1$.
Prove: $ABCE$ is an isosceles trapezoid.



ANSWER:

Given: E and C are midpoints of $\overline{AD}$ and $\overline{DB}$, $\overline{AD} \cong \overline{DB}$, $\angle A \cong \angle 1$.
Prove: $ABCE$ is an isosceles trapezoid.

7-2 Similar Polygons



62. **COORDINATE GEOMETRY** Determine the coordinates of the intersection of the diagonals of $\square JKLM$ with vertices $J(2, 5)$, $K(6, 6)$, $L(4, 0)$, and $M(0, -1)$.

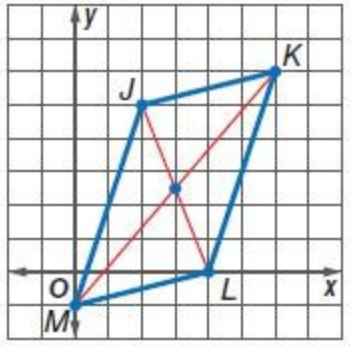
SOLUTION:

Since the diagonals of a parallelogram bisect each other, their intersection point is the midpoint of the diagonals, $\overline{JL}$ and $\overline{KM}$. Choose either diagonal and find its midpoint.

Find the midpoint of $\overline{JL}$ with endpoints (2, 5) and (4, 0), using the Midpoint Formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

$$\left(\frac{2+4}{2}, \frac{5+0}{2}\right) = (3, 2.5)$$

The coordinates of the intersection of the diagonals of parallelogram *JKLM* are (3, 2.5).



ANSWER:

(3, 2.5)

State the assumption you would make to start an indirect proof of each statement.

63. If $3x > 12$, then $x > 4$.

SOLUTION:

Identify the conclusion you wish to prove. The assumption is that this conclusion is false.

$x \leq 4$

ANSWER:

$x \leq 4$

64. $\overline{PQ} \cong \overline{ST}$

SOLUTION:

Identify the conclusion you wish to prove. The assumption is that this conclusion is false.

$\overline{PQ} \ncong \overline{ST}$

ANSWER:

$\overline{PQ} \ncong \overline{ST}$

65. The angle bisector of the vertex angle of an isosceles triangle is also an altitude of the triangle.

SOLUTION:

Identify the conclusion you wish to prove. The assumption is that this conclusion is false.

The angle bisector of the vertex angle of an isosceles triangle is not an altitude of the triangle.

ANSWER:

The angle bisector of the vertex angle of an isosceles triangle is not an altitude of the triangle.

66. If a rational number is any number that can be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$, then 6 is a rational number.

SOLUTION:

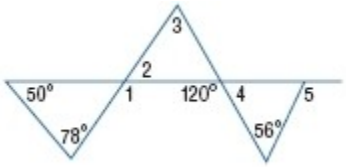
Identify the conclusion you wish to prove. The assumption is that this conclusion is false.

6 cannot be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

ANSWER:

6 cannot be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Find the measures of each numbered angle.



67. $m\angle 1$

SOLUTION:

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

By the Exterior Angle Theorem, $m\angle 1 = 50 + 78$.

Therefore, $m\angle 1 = 128$.

ANSWER:

128

68. $m\angle 2$

SOLUTION:

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

By the Exterior Angle Theorem, $m\angle 1 = 50 + 78$.

Therefore, $m\angle 1 = 128$.

$\angle 1$ and $\angle 2$ form a linear pair. So, $m\angle 1 + m\angle 2 = 180$.

Substitute $m\angle 1 = 128$ and solve for $m\angle 2$.

$$128 + m\angle 2 = 180$$
$$m\angle 2 = 52$$

ANSWER:

52

69. $m\angle 3$

SOLUTION:

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

By the Exterior Angle Theorem, $m\angle 1 = 50 + 78$.

Therefore, $m\angle 1 = 128$.

$\angle 1$ and $\angle 2$ form a linear pair. So, $m\angle 1 + m\angle 2 = 180$.

Substitute the $m\angle 1 = 128$ and solve for $m\angle 2$.

$$128 + m\angle 2 = 180$$
$$m\angle 2 = 52$$

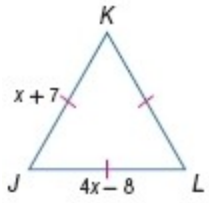
Since $120°$ is the exterior angle to the remote interior angles $\angle 3$ and $\angle 2$, we can state that, by the Exterior Angle Theorem:

$$m\angle 2 + m\angle 3 = 120$$
$$52 + m\angle 3 = 120$$
$$m\angle 3 = 68$$

ANSWER:

68

ALGEBRA Find *x* and the unknown side measures of each triangle.

70. 

SOLUTION:

In the figure, $\overline{JK} \cong \overline{KL} \cong \overline{JL}$. So, $4x - 8 = x + 7$.

Solve for *x*.

$$4x - 8 = x + 7$$
$$3x = 15$$
$$x = 5$$

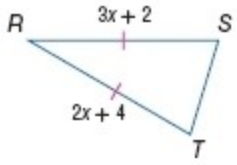
Substitute $x = 5$ in *JK*..

$$JK = x + 7$$
$$= 5 + 7$$
$$= 12$$

Since all the sides are congruent, *JK* = *KL* = *JL* = 12.

ANSWER:

x = 5, *JK* = *JL* = *KL* = 12

71. 

SOLUTION:

In the figure, $\overline{RS} \cong \overline{RT}$. So, $3x + 2 = 2x + 4$.

Solve for x.

$$3x + 2 = 2x + 4$$
$$x + 2 = 4$$
$$x = 2$$

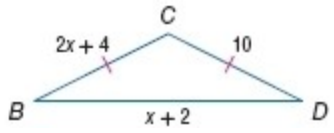
Substitute $x = 2$ in RT and RS.

$$RT = 3x + 2$$
$$= 3(2) + 2$$
$$= 6 + 2$$
$$= 8$$

$$RS = 2x + 4$$
$$= 2(2) + 4$$
$$= 4 + 4$$
$$= 8$$

ANSWER:

$x = 2$, $RT = 8$, $RS = 8$

72. 

SOLUTION:

In the figure, $\overline{BC} \cong \overline{CD}$. So, $2x + 4 = 10$.

Solve for x.

$$2x + 4 = 10$$
$$2x = 6$$
$$x = 3$$

Substitute $x = 3$ in BC and BD.

$$BC = 2x + 4$$
$$= 2(3) + 4$$
$$= 6 + 4$$
$$= 10$$

$$BD = x + 2$$
$$= 3 + 2$$
$$= 5$$

ANSWER:

$x = 3$, $BC = 10$, $CD = 10$, $BD = 5$