7-1 Ratios and Proportions

1. **PETS** Out of a survey of 1000 households, 460 had at least one dog or cat as a pet. What is the ratio of pet owners to households?

   SOLUTION:

   $$\begin{aligned}\frac{\text{number of pet owners}}{\text{number of households}} &= \frac{460}{1000} \\ &= \frac{23}{50}\end{aligned}$$

   The ratio of per owners to households is 23:50.

   ANSWER:

   23:50

2. **SPORTS** Thirty girls tried out for 15 spots on the basketball team. What is the ratio of open spots to the number of girls competing?

   SOLUTION:

   $$\begin{aligned}\frac{\text{number of spots}}{\text{number of girls}} &= \frac{15}{30} \\ &= \frac{1}{2}\end{aligned}$$

   The ratio of open spots to the number of girls competing is 1:2.

   ANSWER:

   1:2

3. The ratio of the measures of three sides of a triangle is 2:5:4, and its perimeter is 165 units. Find the measure of each side of the triangle.

   SOLUTION:

   Just as the ratio $\frac{2}{5}$ or 2:5 is equivalent to $\frac{2x}{5x}$ or $2x{:}5x$, the extended ratio can be written as $2x{:}5x{:}4x$.

   The perimeter is 165 units, so the sum of the lengths of the sides is 165. Solve for x.

   $$\begin{aligned}2x + 5x + 4x &= 165 \\ 11x &= 165 \\ x &= 15\end{aligned}$$

   So the measures of the three sides are 2(15) or 30, 5(15) or 75, and 4(15) or 60.

   ANSWER:

   30, 75, 60

4. The ratios of the measures of three angles of a triangle are 4:6:8. Find the measure of each angle of the triangle.

SOLUTION:

Just as the ratio $\frac{4}{6}$ or 4:6 is equivalent to $\frac{4x}{6x}$ or $4x$:$6x$, the extended ratio can be written as $4x$:$6x$:$8x$. We know that sum of the measures of all interior angles in a triangle is 180.

$$\begin{aligned} 4x+6x+8x&=180 \\ 18x&=180 \\ x&=10 \end{aligned}$$

So the measures of the three angles are 4(10) or 40, 6(10) or 60, and 8(10) or 80.

ANSWER:

40, 60, 80

Solve each proportion.

5. $\frac{2}{3}=\frac{x}{24}$

SOLUTION:

$$\frac{2}{3}=\frac{x}{24}$$

Cross multiply.

$$2(24)=x(3)$$

Solve for x.

$48 = 3x$
$x = 16$

ANSWER:

16

6. $\frac{x}{5} = \frac{28}{100}$

SOLUTION:

$$\frac{x}{5} = \frac{28}{100}$$

Cross multiply.

$$x(100) = 28(5)$$

Solve for *x*.

$$\begin{aligned} 100x &= 140 \\ x &= 1.4 \end{aligned}$$

ANSWER:

1.4

7. $\frac{2.2}{x} = \frac{26.4}{96}$

SOLUTION:

$$\frac{2.2}{x} = \frac{26.4}{96}$$

Cross multiply.

$$2.2(96) = 26.4(x)$$

Solve for *x*.

$$\begin{aligned} 211.2 &= 26.4x \\ x &= 8 \end{aligned}$$

ANSWER:

8

8. $\frac{x-3}{3}=\frac{5}{8}$

SOLUTION:

$$\frac{x-3}{3}=\frac{5}{8}$$

Cross multiply.

$$(x-3)(8)=3(5)$$

Solve for *x*.

$$\begin{aligned}8x-24&=15\\8x&=39\\x&=4.875\end{aligned}$$

ANSWER:

4.875

9. **CCSS MODELING** Ella is baking apple muffins for the Student Council bake sale. The recipe that she is using calls for 2 eggs per dozen muffins, and she needs to make 108 muffins. How many eggs will she need?

SOLUTION:

Let the unknown number be *x*.

Form a proportion for the given information.

$$\frac{2\text{ eggs}}{\text{dozen}}=\frac{x\text{ eggs}}{108}$$

$$\frac{2}{12}=\frac{x}{108}$$

Cross multiply.

$$2(108)=x(12)$$

Solve for *x*.

$$\begin{aligned}216&=12x\\x&=18\end{aligned}$$

ANSWER:

18

MOVIES For Exercises 10 and 11, refer to the graphic below.

10. Of the films listed, which had the greatest ratio of Academy Awards to number of nominations?

SOLUTION:

$$\text{Movie A: } \frac{\text{number of Academy Awards}}{\text{number of nominations}}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$
$$= 1:2$$
$$\text{Movie B: } \frac{\text{number of Academy Awards}}{\text{number of nominations}}$$
$$= \frac{11}{11}$$
$$= 1:1$$
$$\text{Movie C: } \frac{\text{number of Academy Awards}}{\text{number of nominations}}$$
$$= \frac{11}{14}$$
$$= 11:14$$

To compare all the ratios, we can write them as decimals and then compare their values on the same scale. Therefore,

$A: 1:2 = \frac{1}{2} = 0.50$

$B: 1:1 = \frac{1}{1} = 1.00$

$C: 11:14 = \frac{11}{14} \approx 0.79$

Movie B has the greatest ratio, that is, 1:1.

ANSWER:

Movie B;1:1

11. Which film listed had the lowest ratio of awards to nominations?

SOLUTION:

$$\text{Movie A: } \frac{\text{number of Academy Awards}}{\text{number of nominations}}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$
$$= 1:2$$
$$\text{Movie B: } \frac{\text{number of Academy Awards}}{\text{number of nominations}}$$
$$= \frac{11}{11}$$
$$= 1:1$$
$$\text{Movie C: } \frac{\text{number of Academy Awards}}{\text{number of nominations}}$$
$$= \frac{11}{14}$$
$$= 11:14$$
$$\text{Movie D: } \frac{\text{number of Academy Awards}}{\text{number of nominations}}$$
$$= \frac{8}{12}$$
$$= 3:4$$

We can write them in decimal form and then compare their values on the same scale:

$$A:1:2 = \frac{1}{2} = 0.50$$
$$B:1:1 = \frac{1}{1} = 1.00$$
$$C:11:14 = \frac{11}{14} \approx 0.79$$

Movie A has the lowest ratio, that is, 1:2.

ANSWER:

Movie A; 1:2

7-1 Ratios and Proportions

12. **GAMES** A video game store has 60 games to choose from, including 40 sports games. What is the ratio of sports games to video games?

SOLUTION:

$$\frac{\text{number of sports games}}{\text{number of video games}} = \frac{40}{60}$$
$$= \frac{2}{3}$$

The ratio of sports games to video games is 2:3.

ANSWER:

2:3

13. The ratio of the measures of the three sides of a triangle is 9: 7: 5. Its perimeter is 191.1 inches. Find the measure of each side

SOLUTION:

Just as the ratio $\frac{9}{7}$ or 9:7 is equivalent to $\frac{9x}{7x}$ or $9x$:$7x$, the extended ratio can be written as $9x$:$7x$:$5x$.

The perimeter is 191.1 inches so the sum of the lengths of the sides is 191.1. Solve for x.

$$9x + 7x + 5x = 191.1$$
$$21x = 191.1$$
$$x = 9.1$$

So the measures of the three sides are 9(9.1) or 81.9 in., 7(9.1) or 63.7 in., and 5(9.1) or 45.5 in..

ANSWER:

81.9 in., 63.7 in., 45.5 in.

14. The ratio of the measures of the three sides of a triangle is 3: 7: 5, and its perimeter is 156.8 meters. Find the measure of each side.

SOLUTION:

Just as the ratio $\frac{3}{7}$ or 3:7 is equivalent to $\frac{3x}{7x}$ or $3x$:$7x$, the extended ratio can be written as $3x$:$7x$:$5x$.

The perimeter is156.8 meters, so the sum of the lengths of the sides is 156.8. Solve for x.

$$3x + 7x + 5x = 156.8$$
$$15x = 156.8$$
$$x \approx 10.45$$

So the measures of the three sides are 3(10.45) or about 31.4 m, 7(10.45) or about 73.2 m, and 5(10.45) or about 52.3 m.

ANSWER:

31.4 m, 73.2 m, 52.3 m

15. The ratio of the measures of the three sides of a triangle is $\frac{1}{4}:\frac{1}{8}:\frac{1}{6}$. Its perimeter is 4.75 feet. Find the length of the longest side.

SOLUTION:

The given ratio $\frac{1}{4}:\frac{1}{8}:\frac{1}{6}$ is equivalent to $\frac{1}{4}x:\frac{1}{8}x:\frac{1}{6}x$.

The perimeter is 4.75 feet, so the sum of the lengths of the sides is 4.75. Solve for *x*.

$$\begin{aligned}\tfrac{1}{4}x+\tfrac{1}{8}x+\tfrac{1}{6}x&=4.75\\24\left(\tfrac{1}{4}x+\tfrac{1}{8}x+\tfrac{1}{6}x\right)&=24(4.75)\\6x+3x+4x&=114\\13x&=114\\x&=\tfrac{114}{13}\approx 8.8\end{aligned}$$

So the measures of the three sides are $\frac{8.8}{4}$ or 2.2 ft, $\frac{8.8}{8}$ or 1.1 ft, and $\frac{8.8}{6}$ or 1.47 ft. The length of the longest side is 2.2 ft.

ANSWER:

2.2 ft

16. The ratio of the measures of the three sides of a triangle is $\frac{1}{4}:\frac{1}{3}:\frac{1}{6}$, and its perimeter is 31.5 centimeters. Find the length of the shortest side.

SOLUTION:

The given ratio $\frac{1}{4}:\frac{1}{3}:\frac{1}{6}$ is equivalent to $\frac{1}{4}x:\frac{1}{3}x:\frac{1}{6}x$.

The perimeter is 31.5 cm, so the sum of the lengths of the sides is 31.5. Solve for *x*.

$$\begin{aligned}\tfrac{1}{4}x+\tfrac{1}{3}x+\tfrac{1}{6}x&=31.5\\36\left(\tfrac{1}{4}x+\tfrac{1}{3}x+\tfrac{1}{6}x\right)&=36(31.5)\\9x+12x+6x&=1134\\27x&=1134\\x&=42\end{aligned}$$

So the measures of the three sides are $\frac{42}{4}$ or 10.5 cm, $\frac{42}{3}$ or 14 cm, and $\frac{42}{6}$ or 1.7 cm. The length of the shortest side is 7 cm.

ANSWER:

7 cm

Find the measures of the angles of each triangle.

17. The ratio of the measures of the three angles is 3:6:1.

SOLUTION:

Just as the ratio $\frac{3}{6}$ or 3:6 is equivalent to $\frac{3x}{6x}$ or $3x$:$6x$, the extended ratio can be written as $3x$:$6x$:$1x$.

We know that sum of the measures of all interior angles in a triangle is 180 degrees. Set the sum of the extended ratios equal to 180 and solve for x.

$$\begin{aligned} 3x + 6x + 1x &= 180 \\ 10x &= 180 \\ x &= 18 \end{aligned}$$

So the measures of the three angles are 3(18) or 54, 6(18) or 108, and 1(18) or 18.

ANSWER:

54, 108, 18

18. The ratio of the measures of the three angles is 7:5:8.

SOLUTION:

Just as the ratio $\frac{7}{5}$ or 7:5 is equivalent to $\frac{7x}{5x}$ or $7x$:$5x$, the extended ratio can be written as $7x$:$5x$:$8x$.

We know that sum of the measures of all interior angles in a triangle is 180. Set the sum of the extended ratios equal to 180 and solve for x.

$$\begin{aligned} 7x + 5x + 8x &= 180 \\ 20x &= 180 \\ x &= 9 \end{aligned}$$

So the measures of the three angles are 9(7) or 63, 5(9) or 45, and 8(9) or 72.

ANSWER:

63, 45, 72

19. The ratio of the measures of the three angles is 10:8:6.

SOLUTION:

Just as the ratio $\frac{10}{8}$ or 10:8 is equivalent to $\frac{10x}{8x}$ or $10x{:}8x$, the extended ratio can be written as $10x{:}8x{:}6x$.

We know that sum of the measures of all interior angles in a triangle is 180. Set the sum of the extended ratio equal to 180 and solve for x.

$$\begin{aligned} 10x + 8x + 6x &= 180 \\ 24x &= 180 \\ x &= 7.5 \end{aligned}$$

So the measures of the three angles are 10(7.5) or 75, 8(7.5) or 60, and 6(7.5) or 45.

ANSWER:

75, 60, 45

20. The ratio of the measures of the three angles is 5:4:7.

SOLUTION:

Just as the ratio $\frac{5}{4}$ or 5:4 is equivalent to $\frac{5x}{4x}$ or $5x{:}4x$, the extended ratio can be written as $5x{:}4x{:}7x$.

We know that sum of the measures of all interior angles in a triangle is 180. Set the sum of the extended ratio equal to 180 and solve for x.

$$\begin{aligned} 5x + 4x + 7x &= 180 \\ 16x &= 180 \\ x &= 11.25 \end{aligned}$$

So the measures of the three angles are 5(11.25) or 56.25, 4(11.25) or 45, and 7(11.25) or 78.75.

ANSWER:

56.25, 45, 78.75

Solve each proportion.

21. $\frac{5}{8} = \frac{y}{3}$

SOLUTION:

$\frac{5}{8} = \frac{y}{3}$

Cross multiply.

$5(3) = y(8)$

Solve for y.

$$15 = 8y$$
$$y = \frac{15}{8}$$

ANSWER:

$\frac{15}{8}$

22. $\frac{w}{6.4} = \frac{1}{2}$

SOLUTION:

$\frac{w}{6.4} = \frac{1}{2}$

Cross multiply.

$w(2) = 6.4$

Solve for w.

$$2w = 6.4$$
$$w = 3.2$$

ANSWER:

3.2

23. $\frac{4x}{24} = \frac{56}{112}$

SOLUTION:

$\frac{4x}{24} = \frac{56}{112}$

Cross multiply.

$4x(112) = 56(24)$

Solve for *x*.

$$\begin{aligned} 448x &= 1344 \\ x &= 3 \end{aligned}$$

ANSWER:

3

24. $\frac{11}{20} = \frac{55}{20x}$

SOLUTION:

$\frac{11}{20} = \frac{55}{20x}$

Cross multiply.

$11(20x) = 55(20)$

Solve for *x*.

$$\begin{aligned} 220x &= 1100 \\ x &= 5 \end{aligned}$$

ANSWER:

5

25. $\frac{2x+5}{10}=\frac{42}{20}$

SOLUTION:

$$\frac{2x+5}{10}=\frac{42}{20}$$

Cross multiply.

$$20(2x+5)=10(42)$$

Solve for x.

$$\begin{aligned} 40x+100&=420 \\ 40x&=320 \\ x&=8 \end{aligned}$$

ANSWER:

8

26. $\frac{a+2}{a-2}=\frac{3}{2}$

SOLUTION:

$$\frac{a+2}{a-2}=\frac{3}{2}$$

Cross multiply.

$$2(a+2)=3(a-2)$$

Solve for a.

$$\begin{aligned} 2a+4&=3a-6 \\ a&=10 \end{aligned}$$

ANSWER:

10

27. $\frac{3x-1}{4}=\frac{2x+4}{5}$

SOLUTION:

$$\frac{3x-1}{4}=\frac{2x+4}{5}$$

Cross multiply.

$$5(3x-1)=4(2x+4)$$

Solve for *x*.

$$\begin{aligned}15x-5&=8x+16\\7x&=21\\x&=3\end{aligned}$$

ANSWER:

3

28. $\frac{3x-6}{2}=\frac{4x-2}{4}$

SOLUTION:

$$\frac{3x-6}{2}=\frac{4x-2}{4}$$

Cross multiply.

$$4(3x-6)=2(4x-2)$$

Solve for *x*.

$$\begin{aligned}12x-24&=8x-4\\4x&=20\\x&=5\end{aligned}$$

ANSWER:

5

7-1 Ratios and Proportions

29. **NUTRITION** According to a recent study, 7 out of every 500 Americans aged 13 to 17 years are vegetarian. In a group of 350 13- to 17-year-olds, about how many would you expect to be vegetarian?

SOLUTION:

Let the unknown number be *x*.
Form a proportion for the given information.

$$\frac{7 \text{ Americans}}{500 \text{ total Americans}} = \frac{x \text{ vegetarians}}{350 \text{ total vegetarians}}$$
$$\frac{7}{500} = \frac{x}{350}$$

Cross multiply.

$$7(350) = 500x$$

Solve for *x*.

$$2450 = 500x$$
$$x \approx 5$$

ANSWER:

about 5

30. **CURRENCY** Your family is traveling to Mexico on vacation. You have saved \$500 to use for spending money. If 269 Mexican pesos is equivalent to 25 United States dollars, how much money will you get when you exchange your \$500 for pesos?

SOLUTION:

Let *x* be the unknown.
Form a proportion for the given information.

$$\frac{269 \text{ Pesos}}{25 \text{ US Dollars}} = \frac{x \text{ Pesos}}{500 \text{ US Dollars}}$$
$$\frac{269}{25} = \frac{x}{500}$$

Cross multiply.

$$269(500) = 25x$$

Solve for *x*.

$$134500 = 25x$$
$$x = 5380$$

ANSWER:

5380 pesos

ALGEBRA Solve each proportion. Round to the nearest tenth.

31. $\frac{2x+3}{3}=\frac{6}{x-1}$

SOLUTION:

$$
\begin{aligned}
\frac{2x+3}{3} &= \frac{6}{x-1} && \text{Original proportion} \\
(x-1)(2x+3) &= 18 && \text{Crossmultiply.} \\
2x^2+3x-2x-3 &= 18 && \text{Multiply.} \\
2x^2+3x-2x-3-18 &= 0 && -18 \text{ from each side.} \\
2x^2+x-21 &= 0 && \text{Simplify.} \\
2x^2+7x-6x-21 &= 0 && \text{Rewrite} x \text{ as } 7x - 6x. \\
x(2x+7)-3(2x+7) &= 0 && \text{Distributive Property} \\
(2x+7)(x-3) &= 0 && \text{Distributive Property} \\
(2x+7)=0 \text{ or } (x-3) &= 0 && \text{Zero Product Property} \\
2x=-7 \qquad x &= 3 && \text{Addition/Subtraction} \\
x=-3.5 \qquad x &= 3 && \text{Division.}
\end{aligned}
$$

ANSWER:

3, –3.5

32. $\frac{x^2+4x+4}{40}=\frac{x+2}{10}$

SOLUTION:

$$\frac{x^2+4x+4}{40}=\frac{x+2}{10} \quad \text{Original Proportion}$$
$$10(x^2+4x+4)=40(x+2) \quad \text{Cross Multiply.}$$
$$(x^2+4x+4)=4(x+2) \quad \div \text{ each side by 10.}$$
$$x^2+4x+4=4x+8 \quad \text{Distributive property}$$
$$x^2+4x+4-4x-8=0 \quad -(4x+8)\text{from each side.}$$
$$x^2-4=0 \quad \text{Simplify.}$$
$$(x+2)(x-2)=0 \quad \text{Factor.}$$
$$(x+2)=0 \text{ or } (x-2)=0 \quad \text{Zero Product Property}$$
$$x=-2 \qquad x=2 \quad \text{Subtraction/Addition.}$$

ANSWER:

2, –2

33. $\frac{9x+6}{18}=\frac{20x+4}{3x}$

SOLUTION:

$$\frac{9x+6}{18}=\frac{20x+4}{3x} \quad \text{Original proportion}$$
$$3x(9x+6)=18(20x+4) \quad \text{Cross multiply.}$$
$$x(9x+6)=6(20x+4) \quad \text{Divide each side by 3.}$$
$$9x^2+6x=120x+24 \quad \text{Distributive Property}$$
$$9x^2+6x-120x-24=0 \quad -(120x+24) \text{ from each side.}$$
$$9x^2-114x-24=0 \quad \text{Simplify.}$$

Use the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

a = 9, b = –114, c = –24

$$x=\frac{114\pm\sqrt{12996+864}}{18}$$
$$x=\frac{114\pm\sqrt{13860}}{18}$$
$$x=\frac{114\pm117.7}{18}$$
$$x=12.9 \quad \text{or} \quad -0.2$$

ANSWER:

12.9, –0.2

34. The perimeter of a rectangle is 98 feet. The ratio of its length to its width is 5: 2. Find the area of the rectangle.

SOLUTION:

The ratio $\frac{5}{2}$ or 5:2 is equivalent to $\frac{5x}{2x}$ or $5x{:}2x$.

The perimeter is 98 feet, so the sum of the lengths of the sides is 98. Solve for x.

$$\begin{aligned} 2(l+w) &= 98 \\ 2(5x+2x) &= 98 \\ 2(7x) &= 98 \\ 14x &= 98 \\ x &= 7 \end{aligned}$$

So the length and width of the rectangle are 5(7) or 35 feet and 2(7) or 14 feet respectively.

$$\begin{aligned} \text{Area} &= lw \\ &= 35 \times 14 \\ &= 490 \end{aligned}$$

Thus the area of the rectangle is 490 square feet.

ANSWER:

490 ft^2

35. The perimeter of a rectangle is 220 inches. The ratio of its length to its width is 7: 3. Find the area of the rectangle.

SOLUTION:

The ratio $\frac{7}{3}$ or 5:2 is equivalent to $\frac{7x}{3x}$ or $7x{:}3x$.

The perimeter is 220 feet, so the sum of the lengths of the sides is 220. Solve for x.

$$\begin{aligned} 2(l+w) &= 220 \\ 2(7x+3x) &= 220 \\ 2(10x) &= 220 \\ 20x &= 220 \\ x &= 11 \end{aligned}$$

So the length and width of the rectangle are 7(11) or 77 in. and 3(11) or 33 in. respectively.

$$\begin{aligned} \text{Area} &= lw \\ &= 77 \times 33 \\ &= 2541 \end{aligned}$$

Thus the area of the rectangle is 2541 square inches.

ANSWER:

2541 in^2

36. The ratio of the measures of the side lengths of a quadrilateral is 2:3:5:4. Its perimeter is 154 feet. Find the length of the shortest side.

SOLUTION:

The ratio $\frac{2}{3}$ or 2:3 is equivalent to $\frac{2x}{3x}$ or $2x{:}3x$, the extended ratio can be written as $2x{:}3x{:}5x{:}4x$.

The perimeter is 154 feet, so the sum of the lengths of the sides is 154. Solve for x.

$$\begin{aligned} 2x + 3x + 5x + 4x &= 154 \\ 14x &= 154 \\ x &= 11 \end{aligned}$$

So the measures of the four sides are 2(11) or 22 ft, 3(11) or 33 ft, 5(11) or 55 ft, and 4(11) or 44 ft. Therefore the length of the shortest side is 22 ft.

ANSWER:

22 ft

37. The ratio of the measures of the angles of a quadrilateral is 2:4:6:3. Find the measures of the angles of the quadrilateral.

SOLUTION:

Just as the ratio $\frac{2}{4}$ or 2:4 is equivalent to $\frac{2x}{4x}$ or $2x{:}4x$, the extended ratio can be written as $2x{:}4x{:}6x{:}3x$.

The sum of the measures of all interior angles in a quadrilateral is 360.Set the sum of the extended ratio equal to 360 and solve for x.

$$\begin{aligned} 2x + 4x + 6x + 3x &= 360 \\ 15x &= 360 \\ x &= 24 \end{aligned}$$

So the measures of the four angles are 2(24) or 48, 4(24) or 96, 6(24) or 144, and 3(24) or 72.

ANSWER:

48, 96, 144, 72

38. **SUMMER JOBS** In June of 2000, 60.2% of American teens 16 to 19 years old had summer jobs. By June of 2006,
51.6% of teens in that age group were a part of the summer work force.
a. Has the number of 16- to 19-year-olds with summer jobs increased or decreased since 2000? Explain your reasoning.
b. In June 2006, how many 16- to 19-year-olds would you expect to have jobs out of 700 in that age group? Explain your reasoning.

SOLUTION:

a. Decreased; 60.2% of teens had jobs in 2000 and 51.6% had jobs in 2006.

b. Let *x* be the number of teens with jobs in 2006.

Write a proportion comparing the number of American teens with jobs to the total number of American teens.

$$\frac{51.6 \text{ American teens with jobs}}{\text{out of } 100 \text{ American teens}} = \frac{x \text{ American teens with jobs}}{\text{out of } 700 \text{ American teens}}$$

$\frac{51.6}{100} = \frac{x}{700}$ Cross multiply.

$36120 = 100x$ Divide both sides by 100.

$x \approx 361$

There are about 361 American teens, out of 700, that have jobs.

ANSWER:

a. Decreased; 60.2% of teens had jobs in 2000 and 51.6% had jobs in 2006.
b. About 361 teens or 51.6% of 700.

39. **CCSS MODELING** Many artists have used golden rectangles in their work. In a golden rectangle, the ratio of the length to the width is about 1.618. This is known as the *golden ratio*.

a. Recall from page 461 that a standard television screen has an aspect ratio of 4: 3, while a high definition television screen has an aspect ratio of 16: 9. Is either type of screen a golden rectangle? Explain.

b. The golden ratio can also be used to determine column layouts for Web pages. Consider a site with two columns, the left for content and the right as a sidebar. The ratio of the left to right column widths is the golden ratio. Determine the width of each column if the page is 960 pixels wide.

SOLUTION:

a. No: the HDTV aspect ratio is $\frac{16}{9} \approx 1.7778$ and the standard aspect ratio is $\frac{4}{3} \approx 1.3333$. Neither television set is a golden rectangle since the ratios of the lengths to the widths are not the golden ratio (≈ 1.618).

b. The golden ratio is about 1.618. Let *x* be the right column and 960 – *x* be the left column.

$$\begin{aligned} \frac{\text{left column}}{\text{right column}} &= 1.618 \\ \frac{960-x}{x} &= 1.618 \\ 960-x &= 1.618x \\ 960 &= 2.618x \\ \frac{960}{2.618} &= x \\ 367 &\approx x \end{aligned}$$

593 pixels and 367 pixels

ANSWER:

a. No: the HDTV aspect ratio is 1.77778 and the standard aspect ratio is 1.33333. Neither television set is a golden rectangle since the ratios of the lengths to the widths are not the golden ratio.

b. 593 pixels and 367 pixels

40. **SCHOOL ACTIVITIES** A survey of club involvement showed that, of the 36 students surveyed, the ratio of French Club members to Spanish Club members to Drama Club members was 2:3:7. How many of those surveyed participate in Spanish Club? Assume that each student is active in only one club.

SOLUTION:

Just as the ratio $\frac{2}{3}$ or 2:3 is equivalent to $\frac{2x}{3x}$ or $2x{:}3x$, the extended ratio can be written as $2x{:}3x{:}7x$.

The sum of all the students surveyed is 36, so the sum of the extended ratio is 36. Solve for x.

$$\begin{aligned} 2x+3x+7x &= 36 \\ 12x &= 36 \\ x &= 3 \end{aligned}$$

So the number of students participated in Spanish club is 3(3) or 9.

ANSWER:
9 students

41. **PROOF** Write an algebraic proof of the Cross Products Property.

SOLUTION:

This proof uses algebraic strategies to simplify the equation $\frac{a}{b}=\frac{c}{d}, b\neq 0, d\neq 0$ by multiplying both sides of the equation by the common denominator of each side. Then, simplify your result to arrive at the conclusion of the proof.

Given: $\frac{a}{b}=\frac{c}{d}, b\neq 0, d\neq 0$
Prove: $ad = bc$

$\frac{a}{b}=\frac{c}{d}$	$b\neq 0, d\neq 0$
$(bd)\frac{a}{b}=(bd)\frac{c}{d}$	Multiply each side by the common denominator, bd.
$da = bc$	Simplify.
$ad = bc$	Commutative Property

ANSWER:

Given: $\frac{a}{b}=\frac{c}{d}, b\neq 0, d\neq 0$
Prove: $ad = bc$

$\frac{a}{b}=\frac{c}{d}$	$b\neq 0, d\neq 0$
$(bd)\frac{a}{b}=(bd)\frac{c}{d}$	Multiply each side by the common denominator, bd.
$da = bc$	Simplify
$ad = bc$	Commutative Property

42. **SPORTS** Jane jogs the same path every day in the winter to stay in shape for track season. She runs at a constant rate, and she spends a total of 39 minutes jogging. If the ratio of the times of the four legs of the jog is 3:5:1:4, how long does the second leg of the jog take her?

SOLUTION:

Just as the ratio $\frac{3}{5}$ or 3:5 is equivalent to $\frac{3x}{5x}$ or $3x$:$5x$, the extended ratio can be written as $3x$:$5x$:$1x$:$4x$.

Since she jogged a total of 39 minutes, we can set the sum of the extended ratio equal to 39. Solve for x.

$$\begin{aligned} 3x + 5x + 1x + 4x &= 39 \\ 13x &= 39 \\ x &= 3 \end{aligned}$$

She takes 5(3) or 15 minutes for second leg of the jog.

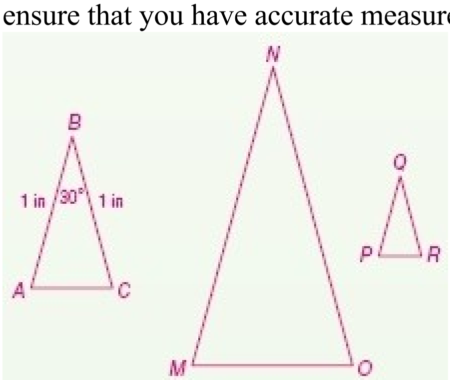
ANSWER:
15 min

43. **MULTIPLE REPRESENTATIONS** In this problem, you will explore proportional relationships in triangles.
a. GEOMETRIC Draw an isosceles triangle ABC. Measure and label the legs and the vertex angle. Draw a second triangle MNO with a congruent vertex angle and legs twice as long as ABC. Draw a third triangle PQR with a congruent vertex angle and legs half as long as ABC.
b. TABULAR Complete the table below using the appropriate measures.

Triangle	ABC	MNO	PQR
Leg length			
Perimeter			

c. VERBAL Make a conjecture about the change in the perimeter of an isosceles triangle if the vertex angle is held constant and the leg length is increased or decreased by a factor.

SOLUTION:
a. It is important that you measure the dimensions of your isosceles triangles carefully. Use a ruler and protractor to ensure that you have accurate measurements. You can use inches or centimeters for the units of your sides.



b. The legs of an isosceles triangle are the two congruent sides. Perimeter is the sum of all three sides of the triangle. Measure these dimensions of your triangles and record them in the table provided.

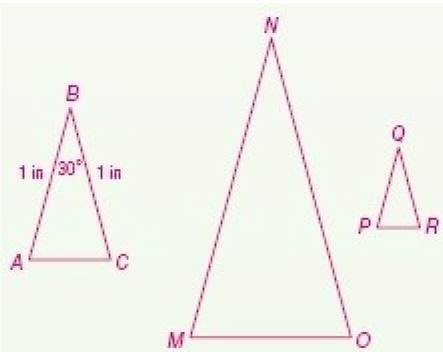
Triangle	ABC	MNO	PQR
Leg length	1 in.	2 in.	0.5 in.
Perimeter	2.5 in.	5 in.	1.25 in.

c. Look at your table and see if you notice a pattern between the relationship of the change in leg length and total perimeter of the triangle. Feel free to make more isosceles triangles to better see the pattern, if necessary.

Sample answer: When the vertex angle of an isosceles triangle is held constant and the leg length is increased or decreased by a factor, the perimeter of the triangle increases or decreases by the same factor.

ANSWER:

a.



b.

Triangle	*ABC*	*MNO*	*PQR*
Leg length	1 in.	2 in.	0.5 in.
Perimeter	2.5 in.	5 in.	1.25 in.

c. Sample answer: When the vertex angle of an isosceles triangle is held constant and the leg length is increased or decreased by a factor, the perimeter of the triangle increases or decreases by the same factor.

44. **ERROR ANALYSIS** Mollie and Eva have solved the proportion $\frac{x-3}{4}=\frac{1}{2}$. Is either of them correct? Explain your reasoning.

SOLUTION:
Neither of these students are correct; In Mollie's case, she needed to cross multiply in her first step. She just multiplied the numerators by each other. Eva, on the other hand, did cross multiply correctly. However, she did not complete the distribution step correctly. A correct method of solving this problem is shown below:

$$\frac{x-3}{4}=\frac{1}{2}$$

$$(x-3)(2)=(4)(1)$$

$$2x-6=4$$

$$2x=10$$

$$x=5$$

ANSWER:
Neither; Mollie did not cross multiply and Eva did not distribute the 2.

45. **CHALLENGE** The dimensions of a rectangle are y and $y^2 + 1$ and the perimeter of the rectangle is 14 units. Find the ratio of the longer side of the rectangle to the shorter side of the rectangle.

SOLUTION:

The perimeter is 14 units, so we can set the formula for perimeter equal to 14 and substitute the corresponding values for the length and width.

$$\begin{aligned} 2(l + w) &= 14 \\ 2(y + y^2 + 1) &= 14 \\ y + y^2 + 1 &= 7 \\ y^2 + y - 6 &= 0 \\ (y + 3)(y - 2) &= 0 \end{aligned}$$

Use the Zero Product Property, set each product equal to zero and solve for y.

$y + 3 = 0$ or $y - 2 = 0$

$y = -3$ or $y = 2$

Since the length must be positive, $y = 2$. So the dimensions of the rectangle are $2^2 + 1$ or 5 and 2.

The ratio of the longer side of the rectangle to the shorter side of the rectangle is 5:2.

ANSWER:

5:2

46. **CCSS REASONING** The ratio of the lengths of the diagonals of a quadrilateral is 1: 1. The ratio of the lengths of the consecutive sides of the quadrilateral is 3:4:3:5. Classify the quadrilateral. Explain.

SOLUTION:

Since the ratios of the lengths of the diagonals are 1:1, we know that diagonals of a quadrilateral are congruent. The diagonals are congruent in rectangles, squares, and isosceles trapezoids.If the ratio of the lengths of the consecutive sides of the quadrilateral are 3:4:3:5, then we know that one pair of opposite sides are congruent but the other pair is not. Therefore, the shape cannot be a rectangle or a square. Hence, it must be an isosceles trapezoid.

Isosceles trapezoid; sample answer: Both pairs of opposite sides cannot be congruent since the ratios are not congruent, so the quadrilateral is not a parallelogram. The diagonals of an isosceles trapezoid are also congruent, and two of the sides are congruent based on the ratio of the sides. Therefore, the quadrilateral is an isosceles trapezoid.

ANSWER:

Isosceles trapezoid; sample answer: Both pairs of opposite sides cannot be congruent since the ratios are not congruent, so the quadrilateral is not a parallelogram. The diagonals of an isosceles trapezoid are also congruent, and two of the sides are congruent based on the ratio of the sides. Therefore, the quadrilateral is an isosceles trapezoid.

47. **WHICH ONE DOESN'T BELONG?** Identify the proportion that does not belong with the other three. Explain your reasoning.

$\frac{3}{8} = \frac{8.4}{22.4}$ $\frac{2}{3} = \frac{5}{7.5}$ $\frac{5}{6} = \frac{14}{16.8}$ $\frac{7}{9} = \frac{19.6}{25.2}$

SOLUTION:

$\frac{2}{3} = \frac{5}{7.5}$; If you divide each numerator of the second fraction in each proportion by the numerator of the first fraction, and repeat for the denominators, you get the scale factor of the proportion. You need to multiply $\frac{2}{3}$ by a factor of 2.5 to get $\frac{5}{7.5}$. The factor of the other three proportions is 2.8.

ANSWER:

$\frac{2}{3} = \frac{5}{7.5}$; you need to multiply $\frac{2}{3}$ by a factor of 2.5 to get $\frac{5}{7.5}$. The factor of the other three proportions is 2.8.

48. **OPEN ENDED** Write four ratios that are equivalent to the ratio 2: 5. Explain why all of the ratios are equivalent.

SOLUTION:

One strategy to find ratios equivalent to 2:5 is to multiply both parts of the ratio by the same value. For example, 2:5 multiplied by 3 would result is $2 \cdot 3 : 5 \cdot 3$ = 6:15. Sample answer: 6: 15, 8: 20, 10: 25, 12: 30; All of the ratios simplify to 2: 5.

ANSWER:

Sample answer: 6: 15, 8: 20, 10: 25, 12: 30; All of the ratios simplify to 2: 5.

49. **WRITING IN MATH** Compare and contrast a ratio and a proportion. Explain how you use both to solve a problem.

SOLUTION:

It helps to make a list of the properties and applications of both ratios and proportions in order to compare and contrast them. Both are written with fractions. A ratio compares two quantities with division, while a proportion equates two ratios. First, ratios are used to write proportions. Then the cross product is used to solve the proportion.

ANSWER:

Both are written with fractions. A ratio compares two quantities with division, while a proportion equates two ratios. First, ratios are used to write proportions. Then the cross product is used to solve the proportion.

50. Solve the following proportion.

$$\frac{x}{-8} = \frac{12}{6}$$

A –12
B –14
C –16
D –18

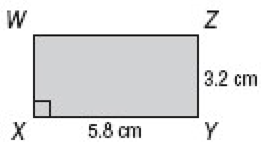
SOLUTION:

$$\frac{x}{-8} = \frac{12}{6}$$
$$6x = -96$$
$$x = -16$$

So, the correct choice is C.

ANSWER:
C

51. What is the area of rectangle *WXYZ*?



F 18.6 cm^2
G 20.4 cm^2
H 21.2 cm^2
J 22.8 cm^2

SOLUTION:

$$\begin{aligned}\text{Area} &= lw\\ &= 5.8 \times 3.2\\ &= 18.56\end{aligned}$$

Thus the area of the rectangle is about 18.6 square centimeters.
The correct choice is F.

ANSWER:
F

52. **GRIDDED RESPONSE** Mrs. Sullivan's rectangular bedroom measures 12 feet by 10 feet. She wants to purchase carpet for the bedroom that costs \$2.56 per square foot, including tax. How much will it cost in dollars to carpet her bedroom?

SOLUTION:

$$\begin{aligned}\text{Area of the bedroom} &= lw \\ &= 12 \times 10 \\ &= 120\end{aligned}$$

The area of the bedroom is 120 square feet.
So, the cost of the bedroom carpet is 120(\$2.56) or \$307.20.
Therefore, the answer is \$307.20.

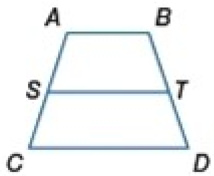
ANSWER:
\$307.20

53. **SAT/ACT** Kamilah has 5 more than 4 times the number of DVDs that Mercedes has. If Mercedes has x DVDs, then in terms of x, how many DVDs does Kamilah have?
A $4(x + 5)$
B $4(x + 3)$
C $9x$
D $4x + 5$
E $5x + 4$

SOLUTION:
In order to determine this answer, start by thinking of how many DVD's Mercedes has. According to the problem, she has x DVD's. Since Kamilah has *5 more* than *4 times* what Mercedes has, we can take rephrase this expression as *4 times Mercedes plus 5 more*, or $4x+5$. Therefore, the answer is D.

ANSWER:
D

For trapezoid *ABCD*, *S* and *T* are midpoints of the legs.



54. If $CD = 14$, $ST = 10$, and $AB = 2x$, find x.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{AB}$ and $\overline{CD}$ are the bases and $\overline{ST}$ is the midsegment. So, $ST = \frac{AB + CD}{2}$.

Substitute and solve for x.

$$10 = \frac{2x + 14}{2}$$
$$20 = 2x + 14$$
$$2x = 6$$
$$x = 3$$

ANSWER:

3

55. If $AB = 3x$, $ST = 15$, and $CD = 9x$, find x.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{AB}$ and $\overline{CD}$ are the bases and $\overline{ST}$ is the midsegment. So, $ST = \frac{AB + CD}{2}$.

Substitute and solve for x.

$$15 = \frac{3x + 9x}{2}$$
$$30 = 12x$$
$$x = 2.5$$

ANSWER:

2.5

56. If $AB = x + 4$, $CD = 3x + 2$, and $ST = 9$, find AB.

SOLUTION:

By the Trapezoid Midsegment Theorem, the midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

$\overline{AB}$ and $\overline{CD}$ are the bases and $\overline{ST}$ is the midsegment. So, $ST = \frac{AB + CD}{2}$.

Substitute and solve for x.

$$9 = \frac{x + 4 + 3x + 2}{2}$$
$$18 = 4x + 6$$
$$4x = 12$$
$$x = 3$$

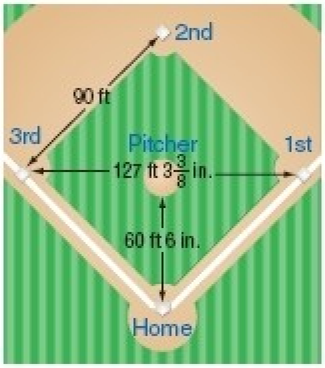
Substitute $x = 3$ in AB.

AB = x + 4
AB = 4 + 4
AB = 7

ANSWER:

7

57. **SPORTS** The infield of a baseball diamond is a square. Is the pitcher's mound located in the center of the infield? Explain.



SOLUTION:

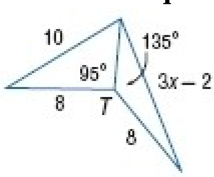
In order to solve this problem, you need to think about the properties of the geometric shape of the baseball diamond. Since it is a square, it takes on the properties of a every parallelogram (rhombus, rectangle, and parallelogram), as well as some of its own properties.

Since a square is a parallelogram, the diagonals bisect each other. Since a square is a rhombus, the diagonals are congruent. Therefore, the distance from first base to third base is equal to the distance between home plate and second base. Thus, the distance from home plate to the center of the infield is 127 ft $3\frac{3}{8}$ in. divided by 2 or 63 ft $7\frac{11}{16}$ in. This distance is longer than the distance from home plate to the pitcher's mound so the pitcher's mound is not located in the center of the field. It is about 3 ft closer to home.

ANSWER:

Since a square is a parallelogram, the diagonals bisect each other. Since a square is a rhombus, the diagonals are congruent. Therefore, the distance from first base to third base is equal to the distance between home plate and second base. Thus, the distance from home plate to the center of the infield is 127 ft $3\frac{3}{8}$ in. divided by 2 or 63 ft $7\frac{11}{16}$ in. This distance is longer than the distance from home plate to the pitcher's mound so the pitcher's mound is not located in the center of the field. It is about 3 ft closer to home.

Write an inequality for the range of values for *x*.

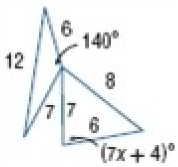
58. 

SOLUTION:

By the Hinge Theorem, $3x - 2 > 10$.
Solve for x.
$3x > 12$
$x > 4$

ANSWER:

$x > 4$

59. 

SOLUTION:

12 > 7

By the Converse of the Hinge Theorem, $140 > 7x + 4$.

$140 > 7x + 4$

$136 > 7x$

$\frac{136}{7} > x$

Use the fact that the measure of any angle is greater than 0 to write a second inequality.

$7x + 7 > 0$

$7x > -4$

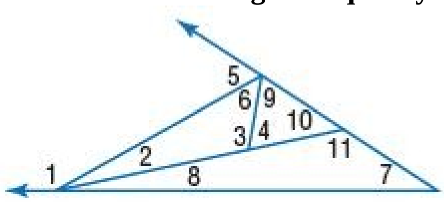
$x > -\frac{4}{7}$

Write $x > -\frac{4}{7}$ and $x < \frac{136}{7}$ as the compound inequality $-\frac{4}{7} < x < \frac{136}{7}$.

ANSWER:

$-\frac{4}{7} < x < \frac{136}{7}$

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.



60. measures less than $m\angle 5$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 5$) is larger than either remote interior angle ($\angle 10$ and $\angle 2$ from one triangle and $\angle 7$ and $\angle 2$ & $\angle 8$ from another triangle).

$\angle 2, \angle 7, \angle 8, \angle 10$

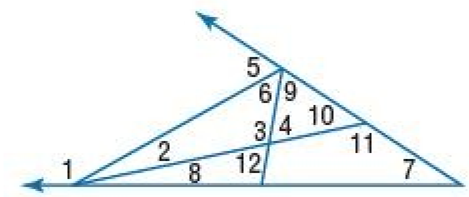
ANSWER:

$\angle 2$, $\angle 7$, $\angle 8$, $\angle 10$

61. measures greater than $m\angle 6$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle is larger than either remote interior angle. Since $\angle 6$ is a remote interior angle for the exterior angles of $\angle 4$ and $\angle 11$, it is less than either of these two angles. $\angle 1$ is greater than $\angle 6$ because, if the line containing $\angle 3$ and $\angle 6$ were extended down to form a triangle including $\angle 2$, $\angle 8$, $\angle 6$, and the new angle formed by the extension ($\angle 12$), then $\angle 6$ would be a remote interior angle for $\angle 1$. See the art below.



$\angle 4$, $\angle 11$, $\angle 1$

ANSWER:

$\angle 4$, $\angle 11$, $\angle 1$

62. measures greater than $m\angle 10$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle is larger than either remote interior angle. Since $\angle 10$ is a remote interior angle for the exterior angles of $\angle 3$ and $\angle 5$, it is less than either of these two angles.

$\angle 3, \angle 5$

ANSWER:

$\angle 3$, $\angle 5$

63. measures less than $m\angle 11$

SOLUTION:

By the Exterior Angle Inequality Theorem, the exterior angle ($\angle 11$) is larger than either remote interior angle ($\angle 4$ and $\angle 9$ from one triangle and $\angle 2$ and $\angle 6$ & $\angle 9$ from another triangle).

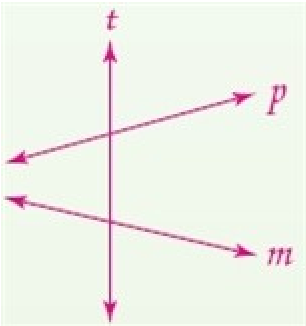
$\angle 2, \angle 6, \angle 9, \angle 4$

ANSWER:

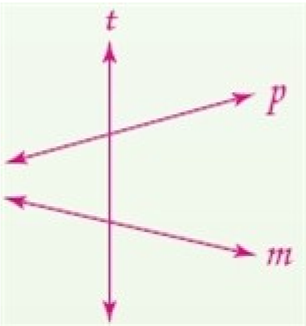
$\angle 2$, $\angle 6$, $\angle 9$, $\angle 4$

64. **REASONING** Find a counterexample for the following statement.
*If lines **p** and **m** are cut by transversal **t** so that consecutive interior angle are congruent, then lines **p** and **m** are parallel and **t** is perpendicular to both lines.*

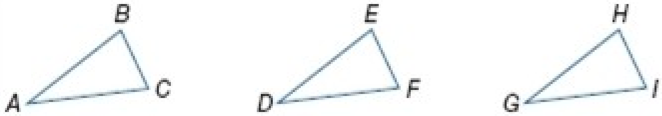
SOLUTION:
To find a counterexample for a conditional statement, one must keep the hypothesis true but falsify the conclusion. Therefore, we must think of an example in which the lines p and m are cut by a transversal so that the consecutive interior angles formed are congruent; however, lines p and m are not parallel to each other. In the example shown below, you can see that these conditions are met.



ANSWER:

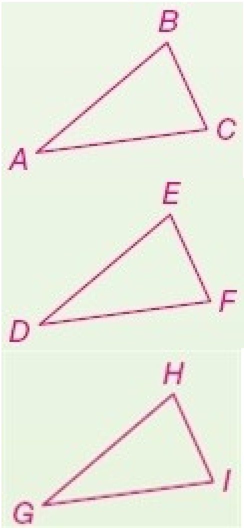


Write a paragraph proof.
65. **Given:** $\Delta ABC \cong \Delta DEF$; $\Delta DEF \cong \Delta GHI$
Prove: $\Delta ABC \cong \Delta GHI$



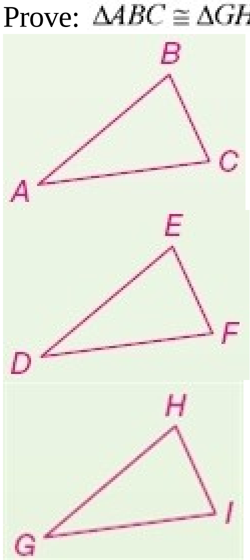
SOLUTION:
A good strategy for solving this proof is to think backwards - what corresponding parts of triangles ABC and GHI do you you need to have proven congruent to each other, in order to prove the triangles are congruent? If you want to use the definition of congruent triangles to prove that $\Delta ABC \cong \Delta GHI$, you would need to prove that all corresponding angles and sides are congruent to each other. For example, you can prove that $\angle A \cong \angle G$ because, since $\Delta ABC \cong \Delta DEF$, we know that $\angle A \cong \angle D$, and since $\Delta DEF \cong \Delta GHI$, we know that $\angle D \cong \angle G$. By transitive property of congruence, since $\angle A \cong \angle D$ and $\angle D \cong \angle G$, then we know that $\angle A \cong \angle G$. You can complete this proof by continuing this approach for all corresponding parts of the triangles ABC and GHI.

Given: $\Delta ABC \cong \Delta DEF$
$\Delta DEF \cong \Delta GHI$
Prove: $\Delta ABC \cong \Delta GHI$



Proof:
You are given that $\Delta ABC \cong \Delta DEF$. Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. You are also given that $\Delta DEF \cong \Delta GHI$. So $\angle D \cong \angle G$, $\angle E \cong \angle H$, $\angle F \cong \angle I$, $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HI}$, and $\overline{DF} \cong \overline{GI}$ by CPCTC. Therefore, $\angle A \cong \angle G$, $\angle B \cong \angle H$, $\angle C \cong \angle I$, $\overline{AB} \cong \overline{GH}$, $\overline{BC} \cong \overline{HI}$ and $\overline{AC} \cong \overline{GI}$ because congruence of angles and segments is transitive. Thus, $\Delta ABC \cong \Delta GHI$ by the definition of congruent triangles.

ANSWER:
Given: $\Delta ABC \cong \Delta DEF$
$\Delta DEF \cong \Delta GHI$
Prove: $\Delta ABC \cong \Delta GHI$



Proof:
You are given that $\Delta ABC \cong \Delta DEF$. Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$. You are also given that $\Delta DEF \cong \Delta GHI$. So $\angle D \cong \angle G$, $\angle E \cong \angle H$, $\angle F \cong \angle I$, $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HI}$, and $\overline{DF} \cong \overline{GI}$ by CPCTC. Therefore, $\angle A \cong \angle G$, $\angle B$

$\cong \angle H$, $\angle C \cong \angle I$, $\overline{AB} \cong \overline{GH}, \overline{BC} \cong \overline{HI}$ and $\overline{AC} \cong \overline{GI}$ because congruence of angles and segments is transitive. Thus, $\Delta ABC \cong \Delta GHI$ by the definition of congruent triangles.