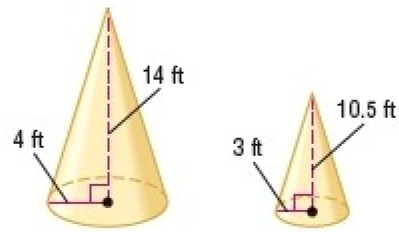
12-8 Congruent and Similar Solids

Determine whether each pair of solids is *similar*, *congruent*, or *neither*. If the solids are similar, state the scale factor.

1. 

SOLUTION:

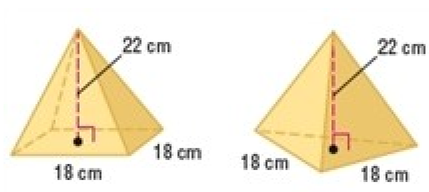
Ratio of radii: $\frac{4}{3}$

Ratio of heights: $\frac{14}{10.5} = \frac{4}{3}$

The ratios of the corresponding measures are equal, so the solids are similar. The scale factor is 4:3. Since the scale factor is not 1:1, the solids are not congruent.

ANSWER:

similar; 4:3

2. 

SOLUTION:

The base of one pyramid is a quadrilateral, while the base of the other is a triangle, so they are neither congruent nor similar.

ANSWER:

neither

3. Two similar cylinders have radii of 15 inches and 6 inches. What is the ratio of the surface area of the small cylinder to the surface area of the large cylinder?

SOLUTION:

Find the scale factor.

$$\frac{\text{radius(small)}}{\text{radius(large)}} = \frac{6}{15} = \frac{2}{5}$$

The scale factor is $\frac{2}{5}$.

If the scale factor is $\frac{a}{b}$, then the ratio of surface areas is $\frac{a^2}{b^2}$.

$$\frac{2}{5} = \frac{2^2}{5^2} = \frac{4}{25}$$

The ratio of the surface areas is 4:25.

ANSWER:

4:25

4. Two spheres have volumes of 36π cubic centimeters and 288π cubic centimeters. What is the ratio of the radius of the small sphere to the radius of the large sphere?

SOLUTION:

If two similar solids have a scale factor of *a*:*b*, then the volumes have a ratio of $a^3:b^3$.

$$\begin{aligned}\frac{\text{V(small)}}{\text{V(large)}} &= \frac{36\pi}{288\pi} \\ &= \frac{36}{288} \\ &= \frac{1}{8} \\ &= \frac{1^3}{2^3}\end{aligned}$$

Therefore, the scale factor is 1:2.

ANSWER:

1:2

5. **EXERCISE BALLS** A company sells two different sizes of exercise balls. The ratio of the diameters is 15:11. If the diameter of the smaller ball is 55 centimeters, what is the volume of the larger ball? Round to the nearest tenth.

SOLUTION:

$$\frac{\text{diameter (large)}}{\text{diameter (small)}} = \frac{15}{11}$$
$$\frac{\text{diameter (large)}}{55} = \frac{15}{11}$$
$$\text{diameter (large)} = \frac{15 \times 55}{11}$$
$$\text{diameter (large)} = 75$$

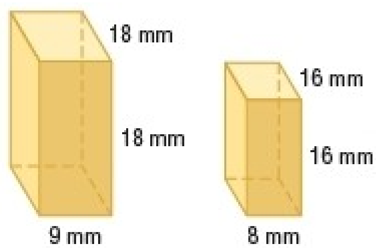
Since the diameter of the large ball is 75 cm, the radius is 37.5 cm.
Use the formula for the volume of a sphere to find the volume of the large ball.

$$\text{Volume of large ball} = \frac{4}{3}\pi(37.5)^3$$
$$\approx 220,893.2 \text{ cm}^3$$

ANSWER:

220,893.2 cm^3

CCSS REGULARITY Determine whether each pair of solids is *similar*, *congruent*, or *neither*. If the solids are similar, state the scale factor.



6.

SOLUTION:

Ratio of lengths: $\frac{18}{16} = \frac{9}{8}$

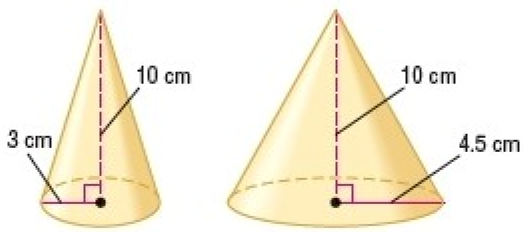
Ratio of widths: $\frac{18}{16} = \frac{9}{8}$

Ratio of heights: $\frac{9}{8}$

The ratios of the corresponding measures are equal, so the solids are similar. The scale factor is 9:8. Since the scale factor is not 1:1, the solids are not congruent.

ANSWER:

similar; 9:8



7.

SOLUTION:

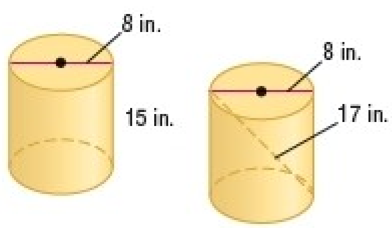
Ratio of radii: $\frac{3}{4.5}=\frac{2}{3}$

Ratio of heights: $\frac{10}{10}=\frac{1}{1}$

Since the ratios of corresponding measures are not equal, the prisms are neither congruent nor similar.

ANSWER:

neither



8.

SOLUTION:

Ratio of radii: $\frac{8}{8}$

Find the height of the second cylinder using the Pythagorean Theorem.

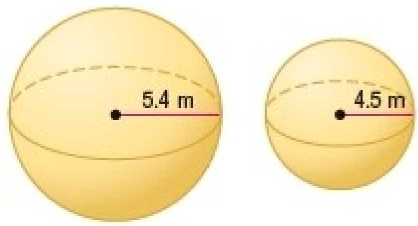
$$\begin{aligned} h^2+8^2&=17^2 \\ h^2&=17^2-8^2 \\ h&=\sqrt{17^2-8^2} \\ h&=\sqrt{225} \\ h&=15 \end{aligned}$$

Ratio of heights: $\frac{15}{15}=1$

The ratios of the corresponding measures are equal, so the solids are similar. The scale factor is 1:1. Since the scale factor is 1:1, the solids are congruent.

ANSWER:

similar and congruent; 1:1

9. 

SOLUTION:

All spheres are similar.
Find the scale factor.

Ratio of radii: $\frac{5.4}{4.5} = \frac{6}{5}$

The scale factor is 6:5. Since the scale factor is not 1:1, the solids are not congruent.

ANSWER:

similar; 6:5

10. Two similar pyramids have slant heights of 6 inches and 12 inches. What is the ratio of the surface area of the small pyramid to the surface area of the large pyramid?

SOLUTION:

Find the scale factor.

$$\frac{\text{slant height(small)}}{\text{slant height(large)}} = \frac{6}{12} = \frac{1}{2}$$

The scale factor is $\frac{1}{2}$.

If the scale factor is $\frac{a}{b}$, then the ratio of surface areas is $\frac{a^2}{b^2}$.

$$\frac{1}{2} = \frac{1^2}{2^2} = \frac{1}{4}$$

ANSWER:

1:4

11. Two similar cylinders have heights of 35 meters and 25 meters. What is the ratio of the volume of the large cylinder to the volume of the small cylinder?

SOLUTION:

Find the scale factor.

$$\frac{\text{height(large)}}{\text{height(small)}} = \frac{35}{25} = \frac{7}{5}$$

The scale factor is $\frac{7}{5}$.

If the scale factor is $\frac{a}{b}$, then the ratio of volumes is $\frac{a^3}{b^3}$.

$$\frac{7}{5} = \frac{7^3}{5^3} = \frac{343}{125}$$

The ratio of the volumes is 343:125.

ANSWER:

343:125

12. Two spheres have surface areas of 100π square centimeters and 16π square centimeters. What is the ratio of the volume of the large sphere to the volume of the small sphere?

SOLUTION:

$$\frac{S(\text{large})}{S(\text{small})} = \frac{100\pi}{16\pi} = \frac{25}{4} = \frac{5^2}{2^2}$$

Therefore, the scale factor is 5:2.

If the scale factor is $\frac{a}{b}$, then the ratio of volumes is $\frac{a^3}{b^3}$.

$$\frac{5}{2} = \frac{5^3}{2^3} = \frac{125}{8}$$

The ratio of the volumes is 125:8.

ANSWER:

125:8

13. Two similar hexagonal prisms have volumes of 250 cubic feet and 2 cubic feet. What is the ratio of the height of the large cylinder to the height of the small cylinder?

SOLUTION:

$$\frac{V(\text{large})}{V(\text{small})} = \frac{250}{2}$$
$$= \frac{125}{1}$$
$$= \frac{5^3}{1^3}$$

Therefore, the scale factor is 5:1.
The ratio of the height of the large cylinder to the height of the small cylinder is 5:1.

ANSWER:

5:1

14. **DIMENSIONAL ANALYSIS** Two rectangular prisms are similar. The height of the first prism is 6 yards and the height of the other prism is 9 feet. If the volume of the first prism is 810 cubic yards, what is the volume of the other prism?

SOLUTION:

Convert feet to yards.

$$\frac{\text{height(first)}}{\text{height(second)}} = \frac{6}{3} = \frac{2}{1}$$

If the scale factor is $\frac{a}{b}$, then the ratio of volumes is $\frac{a^3}{b^3}$.

$$\frac{2}{1} = \frac{2^3}{1^3} = \frac{8}{1}$$

Now find the volume of the second prism.

$$\frac{V(\text{first})}{V(\text{second})} = \frac{8}{1}$$
$$\frac{810}{V(\text{second})} = \frac{8}{1}$$
$$V(\text{second}) = \frac{810}{8}$$
$$V(\text{second}) = 101.25\ \text{yd}^3$$

ANSWER:

101.25 yd^3

15. **FOOD** A small cylindrical can of tuna has a radius of 4 centimeters and a height of 3.8 centimeters. A larger and similar can of tuna has a radius of 5.2 centimeters.

a. What is the scale factor of the cylinders?

b. What is the volume of the larger can? Round to the nearest tenth.

SOLUTION:

a. Find the scale factor.

$$\frac{\text{radius(small)}}{\text{radius(large)}} = \frac{4}{5.2} = \frac{10}{13}$$

b.

$$\frac{\text{height(small)}}{\text{height(large)}} = \frac{10}{13}$$

$$\frac{3.8}{\text{height(large)}} = \frac{10}{13}$$

$$\begin{aligned}\text{height(large)} &= \frac{13 \times 3.8}{10} \\ &= 4.94\end{aligned}$$

$$\begin{aligned}V(\text{large}) &= \pi r^2 h \\ &= \pi(5.2)^2(4.94) \\ &\approx 419.6 \text{ cm}^3\end{aligned}$$

ANSWER:

a. 10:13

b. 419.6 cm^3

16. **SUITCASES** Two suitcases are similar rectangular prisms. The smaller suitcase is 68 centimeters long, 47 centimeters wide, and 27 centimeters deep. The larger suitcase is 85 centimeters long.
a. What is the scale factor of the prisms?
b. What is the volume of the larger suitcase? Round to the nearest tenth.

SOLUTION:

a. Find the scale factor.

$$\frac{\text{length(small)}}{\text{length(large)}} = \frac{68}{85} = \frac{4}{5}$$

b.

$$\frac{\text{width(small)}}{\text{width(large)}} = \frac{4}{5}$$
$$\frac{47}{\text{width(large)}} = \frac{4}{5}$$
$$\text{width(large)} = \frac{5 \times 47}{4}$$
$$\text{width(large)} = 58.75$$

$$\frac{\text{height(small)}}{\text{height(large)}} = \frac{4}{5}$$
$$\frac{27}{\text{height(large)}} = \frac{4}{5}$$
$$\text{height(large)} = \frac{5 \times 27}{4}$$
$$\text{height(large)} = 33.75$$

$$\begin{aligned} V(\text{large}) &= Bh \\ &= (85 \times 58.75)33.75 \\ &\approx 168{,}539.1\,\text{cm}^3 \end{aligned}$$

ANSWER:

a. 4:5

b. 168,539.1 cm^3

17. **SCULPTURE** The sculpture shown is a scale model of a cornet. If the sculpture is 26 feet long and a standard cornet is 14 inches long, what is the scale factor of the sculpture to a standard cornet?
Refer to the image on Page 884.

SOLUTION:

26 feet = 312 inches

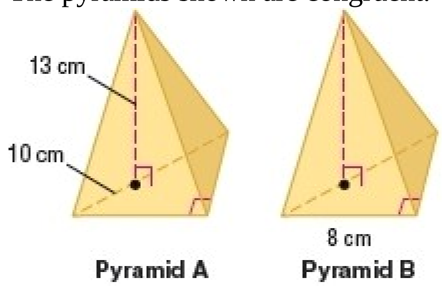
$$\frac{\text{length of the sculpture}}{\text{length of the standard cornet}} = \frac{312}{14} = \frac{156}{7}$$

The scale factor is 156:7.

ANSWER:

156:7

18. The pyramids shown are congruent.



a. What is the perimeter of the base of Pyramid A?

b. What is the area of the base of Pyramid B?

c. What is the volume of Pyramid B?

SOLUTION:

a. Since the pyramids are congruent, the scale factor is 1:1. The base of the pyramids is in the shape of a right triangle.

The base of the base of pyramid B is 8, so the same is true for pyramid A.
Find the height of the base of pyramid A.

$$\begin{aligned} h^2 + 8^2 &= 10^2 \\ h^2 &= 10^2 - 8^2 \\ h &= \sqrt{10^2 - 8^2} \\ &= \sqrt{100 - 64} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

The perimeter of the base of Pyramid A is 10 + 8 + 6 or 24 cm.

b. From part **a**, we know that the bases of the pyramids are right triangles with base 8 cm and height 6 cm.

The area of the base of pyramid B is ½ (8)(6) or 24 cm^2.

c. The area of the base of pyramid B is 24 cm^2. The height of pyramid B is the same as the height of pyramid A: 13 cm.

$$\begin{aligned} \text{Volume of pyramid B} &= \frac{1}{3}(Bh) \\ &= \frac{1}{3}(24)(13) \\ &= 104 \text{ cm}^3 \end{aligned}$$

ANSWER:

a. 24 cm

b. 24 cm^2

c. 104 cm^3

19. **TECHNOLOGY** Jalissa and Mateo each have the same type of MP3 player, but in different colors. The players are congruent rectangular prisms. The volume of Jalissa's player is 4.92 cubic inches, the width is 2.4 inches, and the depth is 0.5 inch. What is the height of Mateo's player?

SOLUTION:

Since the rectangular prisms are congruent, the scale factor is 1:1. Since the scale factor is 1:1, both rectangular prisms have the same length, width, and height.

$$\begin{aligned}\text{Volume} &= Bh \\ 4.92 &= (2.4 \times 0.5)h \\ 4.92 &= 1.2h \\ h &= 4.1\end{aligned}$$

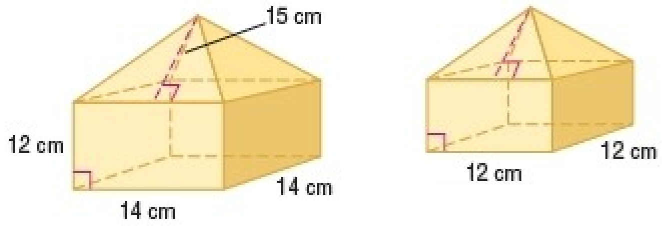
Since the height of Jalissa's player is 4.1 inches, the height of Mateo's player is 4.1 inches.

ANSWER:

4.1 in.

CCSS SENSE-MAKING Each pair of solids below is similar.

20. What is the surface area of the smaller solid shown below?



SOLUTION:

The ratio of the surface areas of the larger and smaller solids equals the square of the ratio of the lengths of any of their corresponding edges.
The lengths of the corresponding edges of the base are 14 cm and 12 cm. Find the surface area of the larger solid.

The larger solid is a composite solid composed of a square-based prism with sides of 14 cm and a height of 12 cm and a square-based pyramid with sides of 14 cm and a slant height of 15 cm.
Surface area of composite = lateral area of the prism + area of its bottom base + lateral area of the pyramid

$$\begin{aligned} SA_{\text{large solid}} &= Ph + B + \tfrac{1}{2}P\ell \\ &= \left(4 \times 14\right)\left(12\right) + (14)^2 + \tfrac{1}{2}(4 \times 14)(15) \\ &= 868 + 196 + 420 \\ &= 1288 \end{aligned}$$

So, the surface area of the large solid is 1288 cm^2.

Use the equal ratios to find the surface area of the smaller solid.

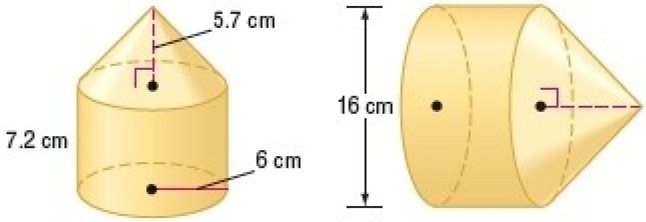
$$\frac{SA_l}{SA_s} = \left(\frac{7}{6}\right)^2$$
$$\frac{1288}{SA_s} = \frac{49}{36}$$
$$SA_s = \frac{1288 \times 36}{49}$$
$$SA_s \approx 946.3$$

Therefore, the surface area of the smaller solid is about 946.3 cm^2.

ANSWER:

946.3 cm^2

21. What is the volume of the larger solid shown below?



SOLUTION:

Since the solids are similar, the ratio of the volumes of the smaller and larger solids will equal the cube of the ratio of the diameters of their circular bases.

The diameter of the base of the smaller solid is 2 × 6 or 12 cm. The ratio of the diameters is $\frac{12}{16} = \frac{3}{4}$.

Each solid is a composite composed of a cone atop a cylinder. The volume of the smaller solid will equal the volume of the cone with a radius of 6 cm and a height of 5.7 cm and the volume of a cylinder with a radius of 6 cm and a height of 7.2 cm.

Volume of smaller solid = volume of the cone + volume of the cylinder

$$\begin{aligned} V_{\text{smaller solid}} &= V_{\text{cone}} + V_{\text{cylinder}} \\ &= \frac{1}{3}\pi(6)^2(5.7) + \pi(6)^2(7.2) \\ &= 68.4\pi + 259.2\pi \\ &= 327.6\pi \end{aligned}$$

So, the volume of the smaller solid is 327.6π cm^3.

Use the equal ratios to find the volume of the larger solid.

$$\begin{aligned} \frac{V_S}{V_L} &= \left(\frac{3}{4}\right)^3 \\ \frac{327.6\pi}{V_L} &= \frac{27}{64} \\ V_L &= \frac{327.6\pi \times 64}{27} \\ V_L &\approx 2439.6 \end{aligned}$$

Therefore, the volume of the larger solid is about 2439.6 cm^3.

ANSWER:

2439.6 cm^3

22. **DIMENSIONAL ANALYSIS** Two cylinders are similar. The height of the first cylinder is 23 cm and the height of the other cylinder is 8 in. If the volume of the first cylinder is 552π cm3, what is the volume of the other prism? Use 2.54 cm = 1 in.

SOLUTION:

Find the scale factor.

$$\begin{aligned}\frac{\text{height(first)}}{\text{height(second)}} &= \frac{23}{8(2.54)}\\ &= \frac{23}{20.32}\\ &= \frac{2300}{2032}\\ &= \frac{575}{508}\end{aligned}$$

Find the radius of the first cylinder.

$$\begin{aligned}\text{Volume(first)} &= \pi r^2 h\\ 552\pi &= \pi r^2 h\\ 552 &= r^2(23)\\ 24 &= r^2\\ r &= 2\sqrt{6}\end{aligned}$$

Now, find the radius of the second.

$$\begin{aligned}\frac{\text{radius(first)}}{\text{radius(second)}} &= \frac{575}{508}\\ \frac{2\sqrt{6}}{\text{radius(second)}} &= \frac{575}{508}\\ \text{radius(second)} &= \frac{1016\sqrt{6}}{575}\\ &\approx 4.328\end{aligned}$$

$$\begin{aligned}\text{Volume(second)} &= \pi r^2 h\\ &= \pi\left(\frac{1016\sqrt{6}}{575}\right)^2(20.32)\\ &\approx 380.6\pi\end{aligned}$$

ANSWER:

380.65π cm^3

23. **DIMENSIONAL ANALYSIS** Two spheres are similar. The radius of the first sphere is 10 feet. The volume of the other sphere is 0.9 cubic meters. Use 2.54 cm = 1 in. to determine the scale factor from the first sphere to the second.

SOLUTION:

$$\frac{10\text{ft}}{1} \cdot \frac{12\text{in}}{1\text{ft}} \cdot \frac{2.54\text{cm}}{1\text{in}} \cdot \frac{1\text{m}}{100\text{cm}} = 3.048\text{m}$$

$$\begin{aligned} \text{V (first)} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(3.048)^3 \\ &\approx 118.6\ \text{m}^3 \end{aligned}$$

$$\begin{aligned} \frac{\text{V (first)}}{\text{V (second)}} &= \frac{118.6}{0.9} \\ &= \frac{1186}{9} \\ &\approx \frac{131.7}{1} \\ &\approx \frac{(5.08)^3}{1^3} \end{aligned}$$

So, the scale factor is about 5.08 to 1.

ANSWER:

about 5.08 to 1

24. **ALGEBRA** Two similar cones have volumes of 343π cubic centimeters and 512π cubic centimeters. The height of each cone is equal to 3 times its radius. Find the radius and height of both cones.

SOLUTION:

Let r represent the radius of the smaller cone and $3r$ represent its height. Use the volume to find the value of r.

$$\begin{aligned} V_{\text{cone}} &= \frac{1}{3}\pi r^2 h \\ 343\pi &= \frac{1}{3}\pi r^2(3r) \\ 343\pi &= \pi r^3 \\ 343 &= r^3 \\ \sqrt[3]{343} &= r \\ 7 &= r \end{aligned}$$

So, the radius of the smaller cone is 7 cm and its height is 3 × 7 or 21 cm.

The ratio of the volumes equals the cube of the ratio of the radii for the two cones.

$$\begin{aligned} \frac{V_S}{V_L} &= \left(\frac{r_S}{r_L}\right)^3 \\ \frac{343\pi}{512\pi} &= \left(\frac{7}{r_L}\right)^3 \\ \frac{343}{512} &= \frac{343}{{r_L}^3} \\ 512 &= {r_L}^3 \\ 8 &= r_L \end{aligned}$$

Therefore, the radius of the larger cone is 8 cm and its height is 3 × 8 or 24 cm.

ANSWER:

smaller cone: r = 7 cm, h = 21 cm; larger cone: r = 8 cm, h = 24 cm

25. **TENTS** Two tents are in the shape of hemispheres, with circular floors. The ratio of their floor areas is 9:12.25. If the diameter of the smaller tent is 6 feet, what is the volume of the larger tent? Round to the nearest tenth.

SOLUTION:

$$\frac{\text{Surface area of the small tent}}{\text{Surface area of the large tent}} = \frac{9}{12.25}$$
$$= \frac{3^2}{3.5^2}$$

Therefore, the ratio of the scale factor is 3:3.5.
Find the radius of the large tent.

$$\frac{\text{radius of the small tent}}{\text{radius of the large tent}} = \frac{3}{3.5}$$
$$\frac{3}{\text{radius of the large tent}} = \frac{3}{3.5}$$
$$\text{radius of the large tent} = 3$$
$$\text{volume of the large tent} = \frac{2}{3}\pi r^3$$
$$= \frac{2}{3}\pi(3.5)^3$$
$$\approx 89.8 \text{ ft}^3$$

ANSWER:

89.8 ft^3

26. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate similarity. The heights of two similar cylinders are in the ratio 2 to 3. The lateral area of the larger cylinder is 162π square centimeters, and the diameter of the smaller cylinder is 8 centimeters.
a. VERBAL What is the height of the larger cylinder? Explain your method.
b. GEOMETRIC Sketch and label the two cylinders.
c. ANALYTICAL How many times as great is the volume of the larger cylinder as the volume of the smaller cylinder?

SOLUTION:

a. Find the diameter of the larger cylinder.

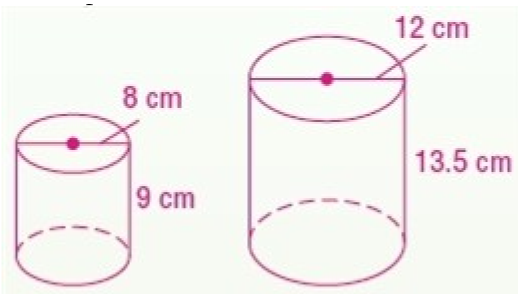
$$\frac{2}{3} = \frac{8}{d}$$
$$2d = 8 \cdot 3$$
$$2d = 24$$
$$d = 12$$

The diameter is 12, so the radius is 6. Use this and the lateral area to find the height of the larger cylinder.

$$L = 2\pi rh$$
$$162\pi = 2\pi(6)h$$
$$162\pi = 12\pi h$$
$$13.5 = h$$

.

b. Sample answer:



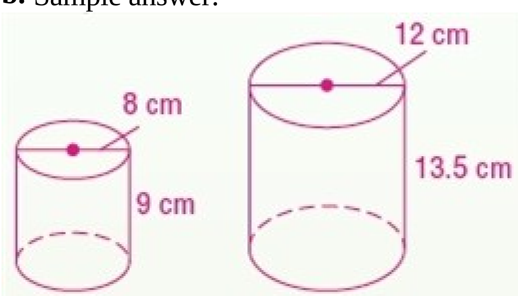
c. Simplify the ratio of the volume of the larger cylinder to the volume of the smaller cylinder.

$$\begin{aligned}\frac{V(\text{large})}{V(\text{small})} &= \frac{\pi(6)^2(13.5)}{\pi(4)^2(9)}\\ &= \frac{486}{144}\\ &= 3.375\\ V(\text{large}) &= 3.375 \cdot V(\text{small})\end{aligned}$$

ANSWER:

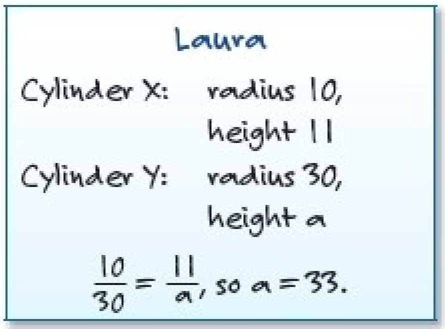
a. 13.5 cm; The heights are in the ratio 2 to 3, so the scale factor is 2:3. Since $\frac{8}{12}=\frac{2}{3}$, the diameter of the larger cylinder is 12 cm. The lateral area of the larger cylinder is 162π cm^2 , so the height is 162π ÷ 12π or 13.5 cm.

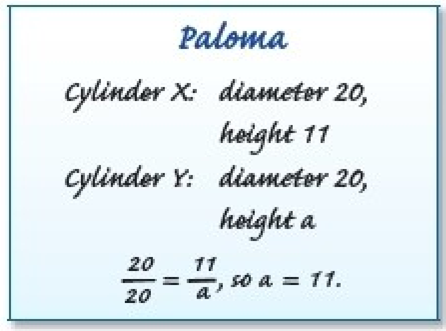
b. Sample answer:



c. 3.375 times as great

27. **ERROR ANALYSIS** Cylinder X has a diameter of 20 centimeters and a height of 11 centimeters. Cylinder Y has a radius of 30 centimeters and is similar to Cylinder X. Did Laura or Paloma correctly find the height of Cylinder Y? Explain your reasoning.





SOLUTION:

When similar figures are compared, we need to compare corresponding parts, like this:

$$\frac{diameter\ X}{diameter\ Y} = \frac{height\ X}{height\ Y}$$

Paloma incorrectly compared the diameter of X to the radius of Y.

ANSWER:

Laura; because she compared corresponding parts of the similar figures. Paloma incorrectly compared the diameter of X to the radius of Y.

28. **CHALLENGE** The ratio of the volume of Cylinder A to the volume of Cylinder B is 1:5. Cylinder A is similar to Cylinder C with a scale factor of 1:2 and Cylinder B is similar to Cylinder D with a scale factor of 1:3. What is the ratio of the volume of Cylinder C to the volume of Cylinder D? Explain your reasoning.

SOLUTION:

Convert all of the given ratios to volumes.

$$\begin{aligned}\frac{V(A)}{V(B)} &= \frac{1}{5}\\ \frac{V(A)}{V(C)} &= \frac{1^3}{2^3}\\ &= \frac{1}{8}\\ \frac{V(B)}{V(D)} &= \frac{1^3}{3^3}\\ &= \frac{1}{27}\end{aligned}$$

Now use these ratios to get ratio for *C* to *D*. Set one of the values equal to one.

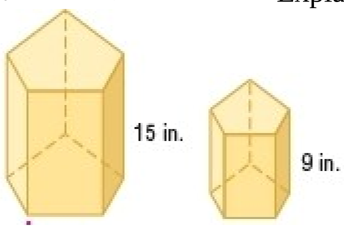
If $\frac{V(A)}{V(B)} = \frac{1}{5}$ and $\frac{V(A)}{V(C)} = \frac{1}{8}$, then $\frac{V(B)}{V(C)} = \frac{5}{8} = \frac{1}{\frac{8}{5}}$.

If $\frac{V(B)}{V(C)} = \frac{1}{\frac{8}{5}}$ and $\frac{V(B)}{V(D)} = \frac{1}{27}$, then $\frac{V(C)}{V(D)} = \frac{\frac{8}{5}}{27} = \frac{8}{135}$.

ANSWER:

8:135; The volume of Cylinder C is 8 times the volume of Cylinder A, and the volume of Cylinder D is 27 times the volume of Cylinder B. If the original ratio of volumes was 1*x* :5*x*, the new ratio is 8*x* :135*x*. So, the ratio of volumes is 8:135.

29. **WRITING IN MATH** Explain how the surface areas and volumes of the similar prisms are related.



SOLUTION:

$\text{scale factor}: \frac{15}{9} = \frac{5}{3}$

$\text{surface area}: \frac{5^2}{3^2} = \frac{25}{9}$

$\text{volume}: \frac{5^3}{3^3} = \frac{125}{27}$

ANSWER:

Since the scale factor is 15:9 or 5:3, the ratio of the surface areas is 25:9 and the ratio of the volumes is 125:27. So, the surface area of the larger prism is $\frac{25}{9}$ or about 2.8 times the surface area of the smaller prism. The volume of the larger prism is $\frac{125}{27}$ or about 4.6 times the volume of the smaller prism.

30. **OPEN ENDED** Describe two nonsimilar triangular pyramids with similar bases.

SOLUTION:

Select some random values for one of the bases: (3, 4, 5)

Multiply these by a constant (like 2) to get the values for the similar base: (6, 8, 10)

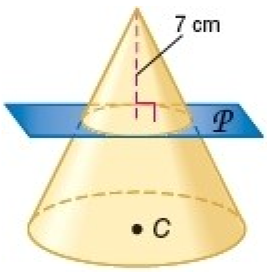
These bases are in the ratio of 1:2. Select a value for the height of the first pyramid: 6

The height of the second pyramid cannot be $6 \times 2 = 12$.

ANSWER:

Sample answer: a pyramid with a right triangle base of 3, 4, and 5 units and a height of 6 units; a pyramid with a right triangle base of 6, 8, and 10 units and a height of 6 units.

31. **CCSS SENSE-MAKING** Plane *P* is parallel to the base of cone *C*, and the volume of the cone above the plane is $\frac{1}{8}$ of the volume of cone *C*. Find the height of cone *C*.



SOLUTION:

Since plane *P* is parallel to the base of cone *C*, the cone above the plane is similar to cone *C*. The ratio of the volumes of the cones will equal the cube of the ratio of their heights.

$$\frac{V_a}{V_C} = \left(\frac{h_a}{h_C}\right)^3$$
$$\sqrt[3]{\frac{1}{8}} = \frac{7}{h_C}$$
$$\frac{1}{2} = \frac{7}{h_C}$$
$$h_C = 14$$

Therefore, the height of cone *C* is 14 cm.

ANSWER:

14 cm

32. **WRITING IN MATH** Explain why all spheres are similar.

SOLUTION:

Sample answer: All spheres are the same shape. The only parameter that can vary is the radius, so all spheres are similar.

ANSWER:

Sample answer: All spheres are the same shape. The only parameter that can vary is the radius, so all spheres are similar.

33. Two similar spheres have radii of 20π meters and 6π meters. What is the ratio of the surface area of the large sphere to the surface area of the small sphere?

A $\frac{100}{3}$

B $\frac{100}{9}$

C $\frac{10}{3}$

D $\frac{10}{9}$

SOLUTION:

Find the scale factor.

$$\frac{\text{radius of the large sphere}}{\text{radius of the small sphere}} = \frac{20\pi}{6\pi} = \frac{10}{3}$$

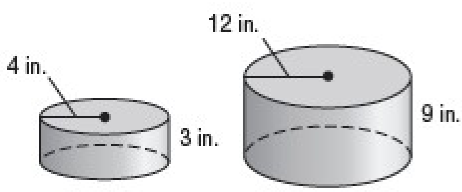
Therefore, the ratio of the surface area of the large sphere to the surface area of the small sphere is $\frac{10^2}{3^2}$ or $\frac{100}{9}$.

So, the correct choice is B.

ANSWER:

B

34. What is the scale factor of the similar figures?



F 0.25
G 0.33
H 0.5
J 0.75

SOLUTION:

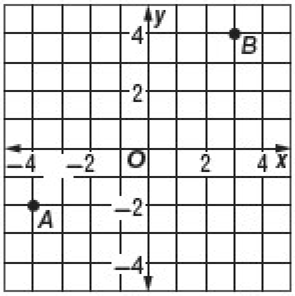
$$\frac{\text{radius of small cylinder}}{\text{radius of large cylinder}} = \frac{4}{12} = 0.33$$

The scale factor is 0.33.
So, the correct choice is G.

ANSWER:

G

35. **SHORT RESPONSE** Point A and point B represent the locations of Timothy's and Quincy's houses. If each unit on the map represents one kilometer, how far apart are the two houses?



SOLUTION:

The coordinates of points A and B are (–4, –2) and (3, 4) respectively.
Use the distance formula to find the distance.

$$d = \sqrt{(3-(-4))^2 + (4-(-2))^2}$$
$$= \sqrt{(7)^2 + (6)^2}$$
$$= \sqrt{85}$$
$$\approx 9.2$$

The distance between the locations is about 9.2 km.

ANSWER:

$\sqrt{85} \approx 9.2\text{km}$

36. **SAT/ACT** If $\frac{x+2}{3} = \frac{(x+2)^2}{15}$, what is one possible value of x?

A 0
B 1
C 2
D 3
E 4

SOLUTION:

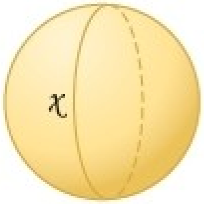
$$\frac{x+2}{3} = \frac{(x+2)^2}{15}$$
$$3(x+2)^2 = 15(x+2)$$
$$3(x+2) = 15$$
$$x+2 = 5$$
$$x = 3$$

So, the correct choice is D.

ANSWER:

D.

Determine whether figure *x* on each of the spheres shown is a line in spherical geometry.

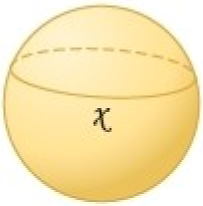
37. 

SOLUTION:

Great circles, circles containing the center of the sphere, are lines in spherical geometry. *X* is a great circle.

ANSWER:

Yes

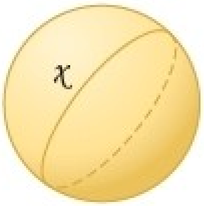
38. 

SOLUTION:

Notice that circle *x* does not go through the pole of the sphere. Therefore circle *x* is not a great circle and so not a line in spherical geometry.

ANSWER:

no

39. 

SOLUTION:

Great circles, circles containing the center of the sphere, are lines in spherical geometry. *X* is a great circle.

ANSWER:

Yes

40. **ENTERTAINMENT** Some people think that the Spaceship Earth geosphere at Epcot in Disney World in Orlando, Florida, resembles a golf ball. The building is a sphere measuring 165 feet in diameter. A typical golf ball has a diameter of approximately 1.5 inches.

a. Find the volume of Spaceship Earth to the nearest cubic foot.
b. Find the volume of a golf ball to the nearest tenth.
c. What is the scale factor that compares Spaceship Earth to a golf ball?
d. What is the ratio of the volumes of Spaceship Earth to a golf ball?

SOLUTION:

a.

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(82.5)^3$$
$$\approx 2,352,071$$

b.

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(0.75)^3$$
$$\approx 1.8$$

c.

165 feet = 1980 inches

$$\frac{\text{radius of Spaceship Earth}}{\text{radius of golf ball}} = \frac{990}{0.75}$$
$$\frac{\text{radius of Spaceship Earth}}{\text{radius of golf ball}} = \frac{1320}{1}$$

The scale factor is 1320:1.

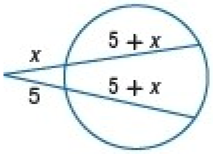
d.

The ratio of the volumes of Spaceship Earth to a golf ball is $1320^3 : 1^3$ or 2,299,968,000 to 1.

ANSWER:

a. 2,352,071 ft^3
b. 1.8 in^3
c. 1320 to 1
d. 1320^3 to 1 or 2,299,968,000 to 1

Find *x*. Assume that segments that appear to be tangent are tangent.

41. 

SOLUTION:

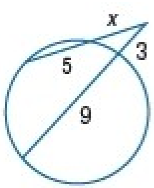
If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

Solve for *x*.

$$x(5+x) = 5(5+x)$$
$$x = 5$$

ANSWER:

5

42. 

SOLUTION:

By Theorem 10.16, $AD \cdot AB = AE \cdot AC$.

Solve for *x*.

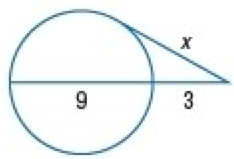
$$(5+x)x = (9+3)3$$
$$(5+x)x = (12)3$$
$$5x + x^2 = 36$$
$$x^2 + 5x - 36 = 0$$
$$(x-4)(x+9) = 0$$

$$x - 4 = 0 \text{ and } x + 9 = 0$$
$$x = 4 \text{ and } x = -9$$

Since the length should be positive, $x = 4$.

ANSWER:

4

43. 

SOLUTION:

By Theorem 10.17,

$x^2 = (9+3)3$

Solve for *x*.

$x^2 = 36$

$x = 6$

ANSWER:

6

Express each fraction as a decimal to the nearest hundredth.

44. $\frac{8}{13}$

SOLUTION:

$$\frac{8}{13} = 0.61538\ldots$$
$$\approx 0.62$$

ANSWER:

0.62

45. $\frac{17}{54}$

SOLUTION:

$$\frac{17}{54} = 0.3148\overline{148}$$
$$\approx 0.31$$

ANSWER:

0.31

46. $\frac{11}{78}$

SOLUTION:

$$\frac{11}{78} = 0.1410$$
$$\approx 0.14$$

ANSWER:

0.14

47. $\frac{43}{46}$

SOLUTION:

$$\begin{aligned}\frac{43}{46} &= 0.9347\ldots \\ &\approx 0.93\end{aligned}$$

ANSWER:

0.93