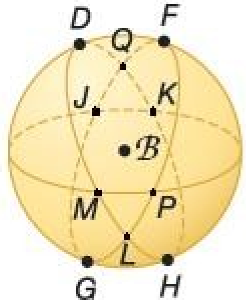
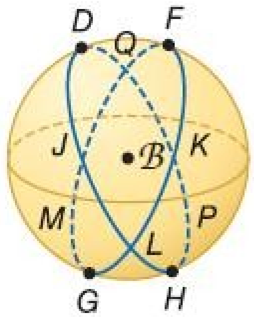
Name each of the following on sphere B.



1. two lines containing point Q

SOLUTION:

$\overleftrightarrow{DH}$ and $\overleftrightarrow{FJ}$ are lines on sphere *B* that contain point *Q*.



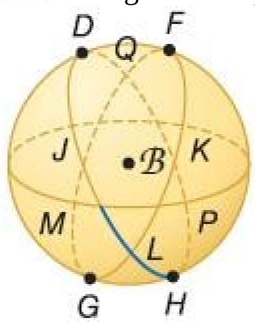
ANSWER:

$\overleftrightarrow{DH}$ and $\overleftrightarrow{FJ}$

2. a segment containing point *L*

SOLUTION:

$\overline{HM}$ is a segment on sphere *B* that contains point *L*.



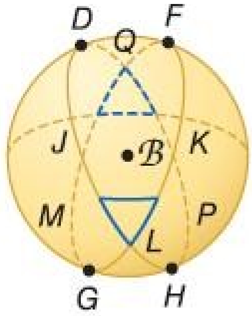
ANSWER:

$\overline{HM}$

3. a triangle

SOLUTION:

$\triangle JKQ$, $\triangle LMP$ are examples of triangles on sphere *B*.



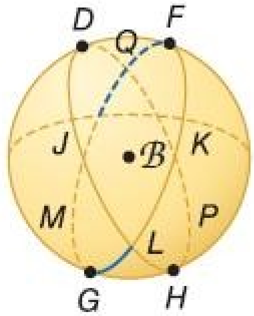
ANSWER:

$\triangle JKQ$, $\triangle LMP$

4. two segments on the same great circle

SOLUTION:

$\overline{LG}$ and $\overline{FJ}$ are segments on the same great circle.



ANSWER:

$\overline{LG}$ and $\overline{FJ}$

SPORTS Determine whether figure *X* on each of the spheres shown is a line in spherical geometry.

5. Refer to the image on Page 875.

SOLUTION:

Notice that figure *X* does not go through the pole of the sphere. Therefore, figure *X* is not a great circle and so not a line in spherical geometry.

ANSWER:

no

6. Refer to the image on Page 875.

SOLUTION:

Notice that the figure *X* passes through the center of the ball and is a great circle, so it is a line in spherical geometry.

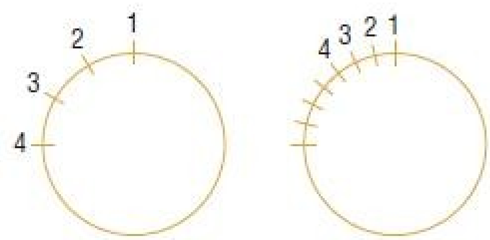
ANSWER:

yes

CCSS REASONING Tell whether the following postulate or property of plane Euclidean geometry has a corresponding statement in spherical geometry. If so, write the corresponding statement. If not, explain your reasoning.

7. The points on any line or line segment can be put into one-to-one correspondence with real numbers.

SOLUTION:



The points on any great circle or arc of a great circle can be put into one-to-one correspondence with real numbers.

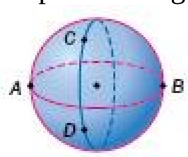
ANSWER:

The points on any great circle or arc of a great circle can be put into one-to-one correspondence with real numbers.

8. Perpendicular lines intersect at one point.

SOLUTION:

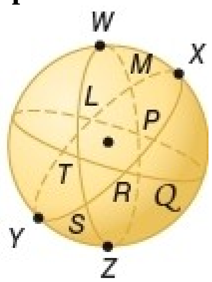
Perpendicular great circles intersect at two points.



ANSWER:

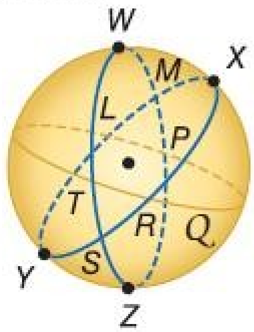
Perpendicular great circles intersect at two points.

Name two lines containing point M, a segment containing point S, and a triangle in each of the following spheres.

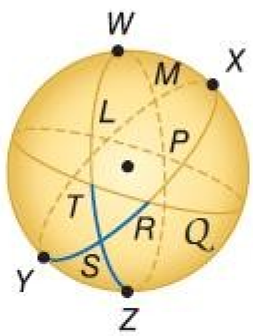
9. 

SOLUTION:

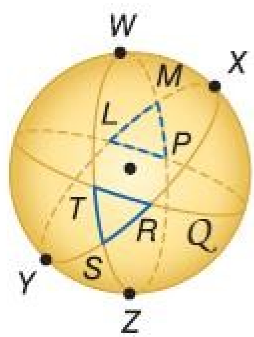
$\overleftrightarrow{WZ}$ and $\overleftrightarrow{XY}$ are two lines on sphere that contain point M.



$\overline{RY}$ or $\overline{TZ}$ is a segment on sphere that contains point S.

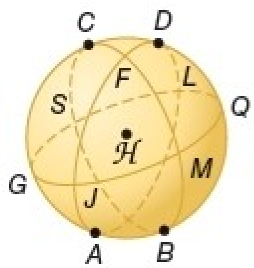


Examples of triangles include: ΔRST *or* ΔMPL



ANSWER:

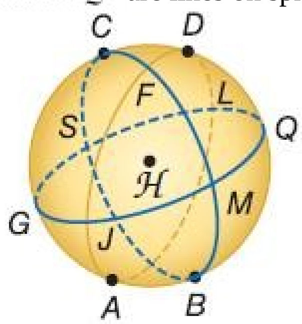
Sample answers: $\overleftrightarrow{WZ}$ and $\overleftrightarrow{XY}$, $\overline{RY}$ or $\overline{TZ}$, ΔRST or ΔMPL



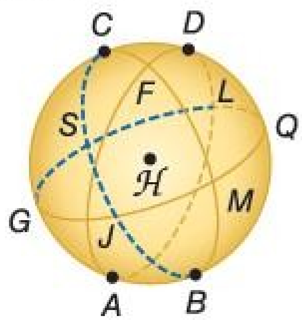
10.

SOLUTION:

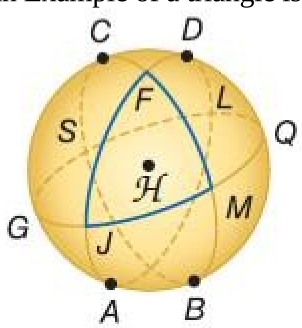
$\overleftrightarrow{FB}$ and $\overleftrightarrow{QJ}$ are lines on sphere that contain point M.



$\overline{CB}$ and $\overline{GL}$ is a segment on sphere that contains point S.



an Example of a triangle is ΔFJM



ANSWER:

Sample answers:

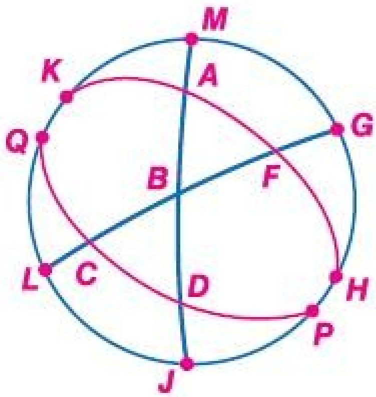
$\overleftrightarrow{FB}$ and $\overleftrightarrow{QJ}$, $\overline{CB}$ and $\overline{GL}$, ΔFJM

11. **SOCCER** Name each of the following on the soccer ball shown. Refer to the image in the text.

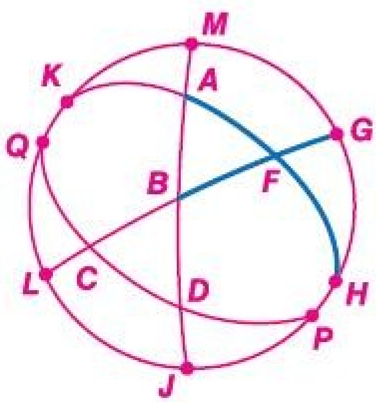
a. two lines containing point B
b. a segment containing point F
c. a triangle
d. a segment containing point C
e. a line
f. two lines containing point A

SOLUTION:

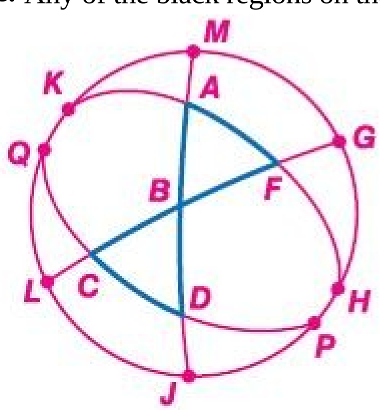
a. Point *B* lies on the intersection of two lines: $\overleftrightarrow{AD}$ and $\overleftrightarrow{FC}$



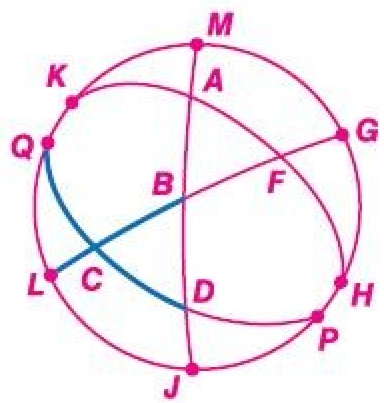
b. Point *F* lies on the intersection of two lines. Segments containing *F* include: $\overline{BG}$ and $\overline{AH}$



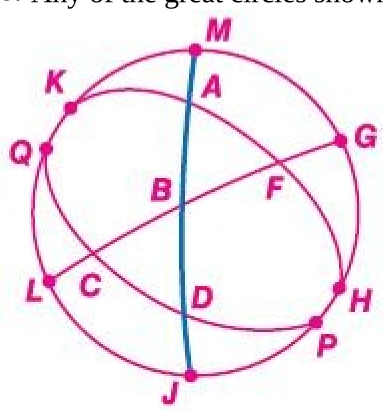
c. Any of the black regions on the ball are triangles. For example: ΔBCD and ΔABF



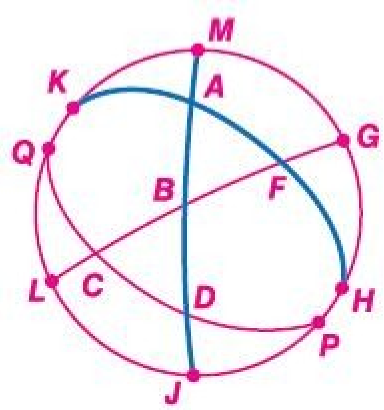
d. Point *C* lies on the intersection of two lines. Segments containing *C* include: $\overline{QD}$ and $\overline{BL}$



e. Any of the great circles shown are lines. For example: $\overleftrightarrow{MJ}$



f. *A* lies on the intersection of two lines: $\overleftrightarrow{MB}$ and $\overleftrightarrow{KF}$



ANSWER:

a. $\overleftrightarrow{AD}$ and $\overleftrightarrow{FC}$

b. Sample answers:
$\overline{BG}$ and $\overline{AH}$

c. Sample answers: ΔBCD and ΔABF

d. $\overline{QD}$ and $\overline{BL}$

e. $\overleftrightarrow{MJ}$

f. $\overleftrightarrow{MB}$ and $\overleftrightarrow{KF}$

ARCHITECTURE Determine whether figure w on each of the spheres shown is a line in spherical geometry.

12. Refer to the image on Page 876.

SOLUTION:

Notice the figure *W* passes through two poles of the sphere and is a great circle. It is therefore a line in spherical geometry.

ANSWER:

yes

13. Refer to the image on Page 876.

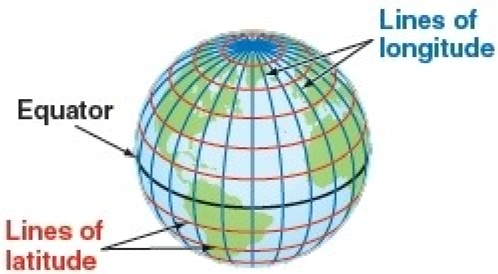
SOLUTION:

Notice that figure *W* does not go through the pole of the sphere. Therefore figure *W* is not a great circle and so not a line in spherical geometry.

ANSWER:

no

14. **CCSS MODELING** Lines of latitude and longitude are used to describe positions on the Earth's surface. By convention, lines of longitude divide Earth vertically, while lines of latitude divide it horizontally.



a. Are lines of longitude great circles? Explain.
b. Are lines of latitude great circles? Explain.

SOLUTION:

a. A great circle is a circle formed when a plane intersects a sphere with its center at the center of the sphere. This occurs when the plane intersects the sphere through the poles of the sphere. Therefore, since lines of longitude pass through poles of the sphere, they form great circles.

b. No; sample answer: Of the lines of latitude, only the Equator is a great circle. It passes through opposite poles of the sphere. Other latitude lines do not pass through opposite poles of the sphere, so they form smaller circles on the sphere and, thus, cannot be great circles.

ANSWER:

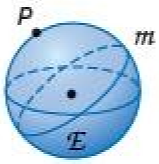
a. Yes; sample answer: Since lines of longitude pass through poles of the sphere, they form great circles.
b. No; sample answer: Of the lines of latitude, only the Equator is a great circle. It passes through opposite poles of the sphere. Other latitude lines do not pass through opposite poles of the sphere, so they cannot be great circles.

Tell whether the following postulate or property of plane Euclidean geometry has a corresponding statement in spherical geometry. If so, write the corresponding statement. If not, explain your reasoning.

15. A line goes on infinitely in two directions.

SOLUTION:

No; a great circle is finite and returns to its original starting point.



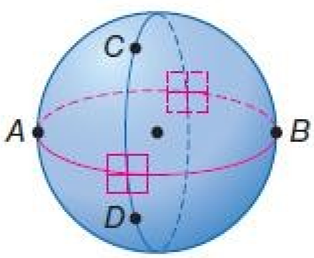
ANSWER:

No; a great circle is finite and returns to its original starting point.

16. Perpendicular lines form four 90° angles.

SOLUTION:

Yes; perpendicular great circles form eight 90° angles.



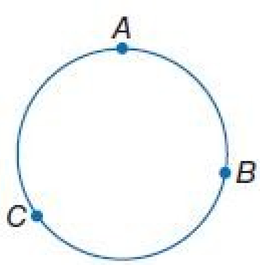
ANSWER:

Yes; perpendicular great circles form eight 90° angles.

17. If three points are collinear, exactly one is between the other two.

SOLUTION:

Yes; if three points are collinear, any one of the three points is between the other two.



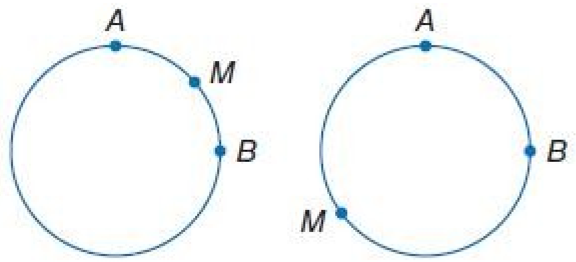
ANSWER:

Yes; if three points are collinear, any one of the three points is between the other two.

18. If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$

SOLUTION:

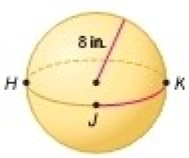
Yes; if M is the midpoint of $\overline{AB}$ on a great circle, then $\overline{AM} \cong \overline{MB}$



ANSWER:

Yes; if M is the midpoint of $\overline{AB}$ on a great circle, then $\overline{AM} \cong \overline{MB}$

On a sphere, there are two distances that can be measured between two points. Use each figure and the information given to determine the distance between points J and K on each sphere. Round to the nearest tenth. Justify your answer.



19. $m\widehat{JK} = 100$

SOLUTION:

The arc is 100°. Find the ratio of the arc to the circumference of the great circle.

$$\frac{100}{360} = \frac{5}{18}$$

100° is $\frac{5}{18}$ of 360°.

Find the circumference of the great circle.

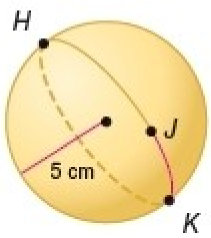
$$C = 2\pi \cdot 8$$
$$= 16\pi$$

The distance between *J* and *K* is $\frac{5}{18}$ of the circumference.

$$\frac{5}{18} \times 16\pi \approx 14$$

ANSWER:

14.0 in.; since 100 degrees is $\frac{5}{18}$ of 360 degrees, $\frac{5}{18} \times$ circumference of great circle ≈ 14.0



20. $m\widehat{JK} = 60$

SOLUTION:

The arc is 60°. Find the ratio of the arc to the circumference of the great circle.

$$\frac{60}{360} = \frac{1}{6}$$

Find the circumference of the great circle.

$$C = 2\pi \cdot 5$$
$$= 10\pi$$

The distance between *J* and *K* is $\frac{1}{6}$ of the circumference.

$$\frac{1}{6} \times 10\pi \approx 5.2$$

ANSWER:

5.2 cm.; since 60 degrees is $\frac{1}{6}$ of 360 degrees, $\frac{1}{6}$ × circumference of great circle ≈ 5.2

21. **GEOGRAPHY** The location of Phoenix, Arizona, is 112° W longitude, 33.4° N latitude, and the location of Helena, Montana, is 112° W longitude, 46.6° N latitude. West indicates the location in terms of the prime meridian, and north indicates the location in terms of the equator. The mean radius of Earth is about 3960 miles.
a. Estimate the distance between Phoenix and Helena. Explain your reasoning.
b. Is there another way to express the distance between these two cities? Explain.
c. Can the distance between Washington, D.C., and London, England, which lie on approximately the same lines of latitude, be calculated in the same way? Explain your reasoning.
d. How many other locations are there that are the same distance from Phoenix, Arizona as Helena, Montana is? Explain.

SOLUTION:

a. The cities are on the same longitude, so they are on the same great circle. They are 46.6° – 33.4° = 13.2° apart on the latitude, and there are 360° in the great circle, so their distance apart is $\frac{13.2}{360}$ of the circumference of the Earth.

$$\frac{13.2}{360} \times 2\pi \times 3960 \approx 912$$

b. Yes; sample answer: Since the cities lie on a great circle, the distance between the cities can be expressed as the major arc or the minor arc. The sum of the two values is the circumference of Earth.

In this case, the major arc is much longer than the minor arc.

c. No; sample answer: Since lines of latitude do not go through opposite poles of the sphere, they are not great circles. Therefore, the distance cannot be calculated in the same way. The great circles are needed because we need to know the circumference. If we knew the circumference of the small circle that represented the latitude and included the two cities, then we would be able to calculate the distance this way.

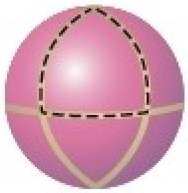
d. Sample answer: Infinite locations. If Phoenix were a point on the sphere, then there are infinite points that are equidistant from that point.

ANSWER:

a. about 912 mi; the cities are 13.2° apart on the same great circle, so $\frac{13.2}{360} \times 2\pi \times 3960$ gives the distance between them.

b.Yes; sample answer: Since the cities lie on a great circle, the distance between the cities can be expressed as the major arc or the minor arc. The sum of the two values is the circumference of Earth.
c. No; sample answer: Since lines of latitude do not go through opposite poles of the sphere, they are not great circles. Therefore, the distance cannot be calculated in the same way.
d. Sample answer: Infinite locations. If Phoenix were a point on the sphere, then there are infinite points that are equidistant from that point.

22. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate triangles in spherical geometry.



a. CONCRETE Use masking tape on a ball to mark three great circles. One of the three great circles should go through different poles than the other two. The great circles will form a triangle. Use a protractor to estimate the measure of each angle of the triangle.
b. TABULAR Tabulate the measure of each angle of the triangle formed. Remove the tape and repeat the process two times so that you have tabulated the measure of three different triangles. Record the sum of the measures of each triangle.
c. VERBAL Make a conjecture about the sum of the measures of a triangle in spherical geometry.

SOLUTION:

a. See students' work.
b. Sample answer:

Triangle	$m\angle 1$	$m\angle 2$	$m\angle 3$	Sum
1	90	90	90	270
2	100	106	114	320
3	75	80	30	185

c. Sample answer: The sum of the measures of the angles of a triangle in spherical geometry is greater than 180°.

The sums are also inconsistent.

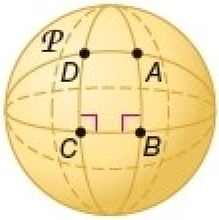
ANSWER:

a. See students' work.
b.

Triangle	$m\angle 1$	$m\angle 2$	$m\angle 3$	Sum
1	90	90	90	270
2	100	106	114	320
3	75	80	30	185

c. Sample answer: The sum of the measures of the angles of a triangle in spherical geometry is greater than 180°.

23. **QUADRILATERALS** Consider quadrilateral *ABCD* on sphere *P*. Note that it has four sides with $\overline{DC} \perp \overline{CB}$, $\overline{AB} \perp \overline{CB}$, and $\overline{DC} \cong \overline{AB}$.



a. Is $\overline{CD} \perp \overline{DA}$? Explain your reasoning.
b. How does *DA* compare to *CB*?
c. Can a rectangle, as defined in Euclidean geometry, exist in non-Euclidean geometry? Explain your reasoning.

SOLUTION:

a. No; if $\overline{CD}$ was perpendicular to $\overline{DA}$, then $\overline{DA}$ would be parallel to $\overline{CB}$ (lines perpendicular to the same line must be parallel). This is not possible, because there are no parallel lines in spherical geometry.

b. *DA* < *CB* because $\overline{CB}$ appears to lie on a great circle.

c. No; since there are no parallel lines in spherical geometry, the sides of a figure cannot be parallel. So, a rectangle, as defined in Euclidean geometry, cannot exist in non-Euclidean geometry.

ANSWER:

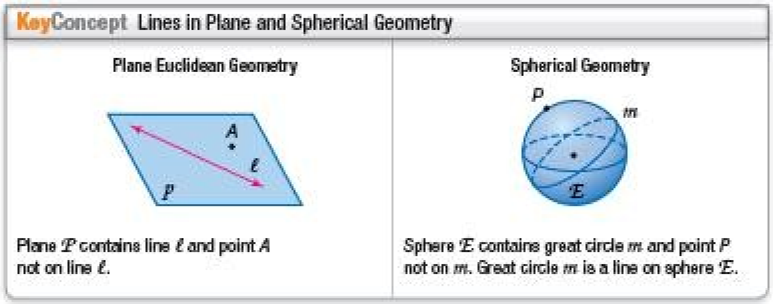
a. No; if $\overline{CD}$ was perpendicular to $\overline{DA}$, then $\overline{DA}$ would be parallel to $\overline{CB}$. This is not possible, because there are no parallel lines in spherical geometry.
b. $DA < CB$
c. No; the sides are not parallel.

12-7 Spherical Geometry

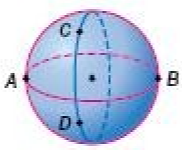
24. **WRITING IN MATH** Compare and contrast Euclidean and spherical geometries. Be sure to include a discussion of planes and lines in both geometries.

SOLUTION:

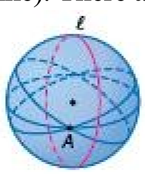
Visually, the two geometries are quite different.



In Euclidean geometry, there is exactly one line that passes through any two points. In spherical geometry there is more than one great circle that can pass through polar points, but only one great circle that can pass through non-polar points.



In Euclidean geometry, given a point and a line, there is exactly one line passing through the point that is parallel to the line. In spherical geometry, any great circle (line) that passes through a point will intersect any other great circle (line). There are no parallel lines.

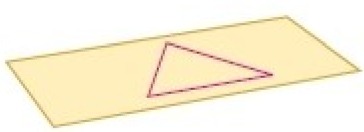


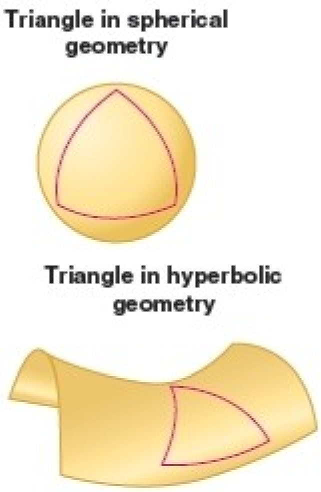
ANSWER:

Sample answer: Points, lines, and planes exist in both Euclidean and spherical geometries. In Euclidean geometry, lines continue indefinitely, and in spherical geometry, lines occur as great circles. In Euclidean geometry, planes extend indefinitely in two dimensions, and in spherical geometry, the plane is the surface of the sphere. Spherical geometry is non-Euclidean because the parallel postulate is invalid.

25. **CHALLENGE** Geometries can be defined on curved surfaces other than spheres. Another type of non-Euclidean geometry is hyperbolic geometry. This geometry is defined on a curved saddle-like surface. Compare the sum of the angle measures of a triangle in hyperbolic, spherical, and Euclidean geometries.

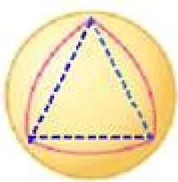
Triangle in plane geometry





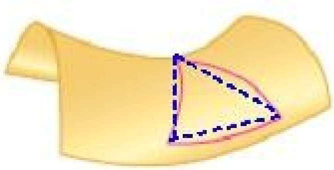
SOLUTION:

Sample answer: In plane geometry, the sum of the measures of the angles of a triangle is 180.
In spherical geometry, the sides of a triangle are curved segments. If the vertices of the spherical triangle are connected in a plane as shown by the dotted blue lines in the figure below, the triangle formed would lie inside the spherical triangle.



Thus, the sum of the measures of the angles of a spherical triangle is greater than 180.

In hyperbolic geometry, the sides of a triangle are segments that curve inward. If the vertices of the hyperbolic triangle are connected in a plane as shown by the dotted blue lines in the figure below, the triangle formed would generally lie outside the hyperbolic triangle.



Thus, the sum of the measures of the angles of a hyperbolic geometry is less than 180.

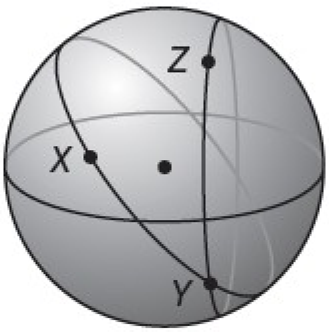
ANSWER:

Sample answer: In plane geometry, the sum of the measures of the angles of a triangle is 180. In spherical geometry, the sum of the measures of the angles of a triangle is greater than 180. In hyperbolic geometry, the sum of the measures of the angles of a triangle is less than 180.

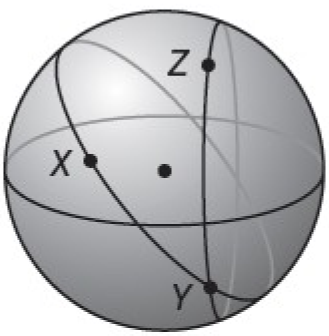
26. **OPEN ENDED** Sketch a sphere with three points so that two of the points lie on a great circle and two of the points do not lie on a great circle.

SOLUTION:

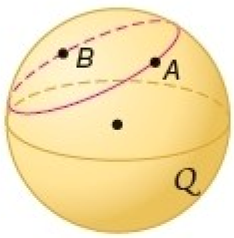
Draw a sphere. Place points *X* and *Y* on the sphere so that they lie on a circle that has the center of the sphere as its center. Place point *Z* on the sphere so that points *Z* and *Y* lie on a circle that does not contain the center of sphere in its interior.



ANSWER:

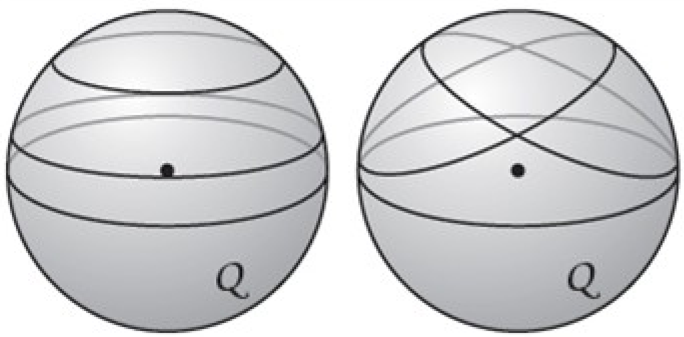


27. **CCSS ARGUMENTS** A small circle of a sphere intersects at least two points, but does not go through opposite poles. Points A and B lie on a small circle of sphere Q. Will two small circles sometimes, always, or never be parallel? Draw a sketch and explain your reasoning.



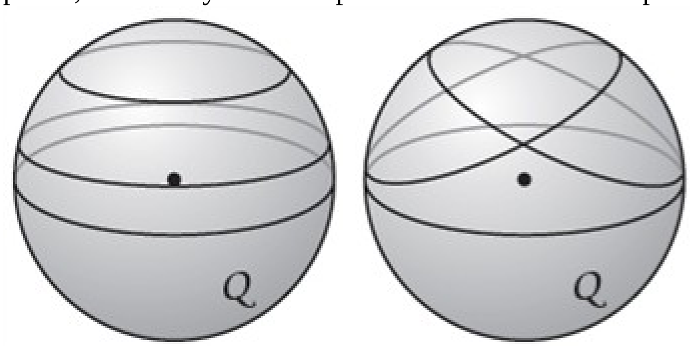
SOLUTION:

Sometimes; sample answer: Since small circles cannot go through opposite poles, it is possible for them to be parallel, such as lines of latitude. It is also possible for them to intersect when two small circles can be drawn through three points, where they have one point in common and two points that occur on one small circle and not the other.



ANSWER:

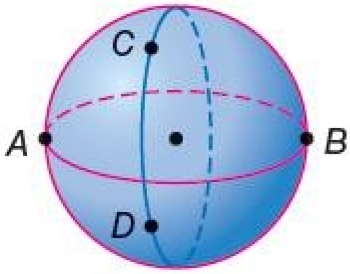
Sometimes; sample answer: Since small circles cannot go through opposite poles, it is possible for them to be parallel, such as lines of latitude. It is also possible for them to intersect when two small circles can be drawn through three points, where they have one point in common and two points that occur on one small circle and not the other.



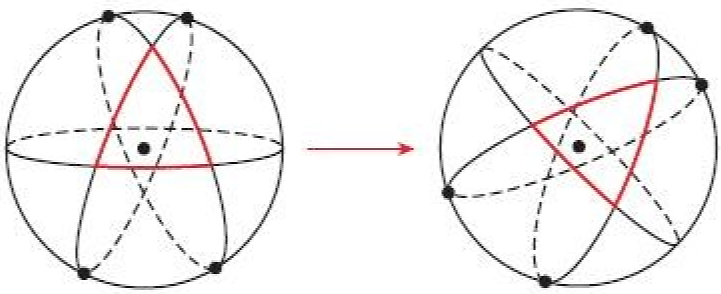
28. **WRITING IN MATH** Do similar or congruent triangles exist in spherical geometry? Explain your reasoning.

SOLUTION:

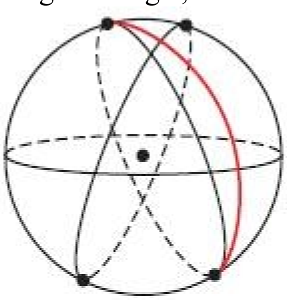
Consider different examples. If lines are drawn so that each angle is 90° then this will divide the sphere into 8 congruent triangles.



Also, any triangle can be rotated to form a congruent triangle elsewhere on the surface of the sphere.



Similar triangles should have the same angle measures and corresponding sides should be longer or shorter by a common scale factor. In Euclidean geometry this is the result of a dilation that preserves angles and maps lines to parallel lines. However, in spherical geometry we see that changing the length of the sides of a triangle will also change the angle, and there are no parallel lines.



ANSWER:

Sample answer: Congruent triangles exist, because three great circles that form a triangle will form identical triangles on opposite sides of the sphere. Similar triangles do not exist because the sum of the measures of the angles of a triangle is not constant. If two triangles in spherical geometry have the same angle measures, they are congruent.

29. **REASONING** Is the statement Spherical geometry is a subset of Euclidean geometry true or false? Explain your reasoning.

SOLUTION:

When a group is a subset of another group, then everything in the first group is included in the second group. For example, if Group A is *odd numbers* and Group B is *real numbers*, then every object in Group A is in Group B, so Group A is a subset of Group B.

Spherical geometry is non-Euclidean, so none of the objects in Spherical geometry are also in Euclidean geometry. Therefore, it cannot be a subset of Euclidean geometry.

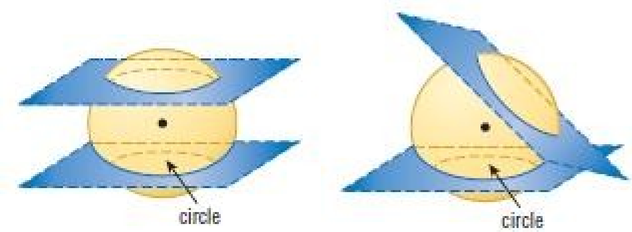
ANSWER:

False; sample answer: Spherical geometry is non-Euclidean, so it cannot be a subset of Euclidean geometry.

30. **REASONING** Two planes are equidistant from the center of a sphere and intersect the sphere. What is true of the circles? Are they lines in spherical geometry? Explain.

SOLUTION:

Consider a few examples:



Since the planes do not pass through the center of the sphere, the circles are not great circles and so are not lines in spherical geometry. However, they do form equivalent circles.

ANSWER:

The circles are congruent. They are not lines in spherical geometry because they are not great circles since they do not have their centers at the center of the sphere.

12-7 Spherical Geometry

31. Which of the following postulates or properties of spherical geometry is false?
A The shortest path between two points on a circle is an arc.
B If three points are collinear, any of the three points lies between the other two.
C A great circle is infinite and never returns to its original starting point.
D Perpendicular great circles intersect at two points.

SOLUTION:

The statement “A great circle is infinite and never returns to its original starting point.” is a false statement in spherical geometry. Therefore, the correct choice is C.

Perpendicular great circles will intersect at opposite points on the sphere.

If three points are collinear on a sphere, then the *line* in which they lie is continuous, so if *B* is between *A* and *C*, then *C* is between *A* and *B* and *A* is between *C* and *B*.

The shortest path between two points on a circle is an arc. Remember that the imaginary *chord* that connects the points is not a part of Spherical geometry.

ANSWER:

C

32. **SAT/ACT** A car travels 50 miles due north in 1 hour and 120 miles due west in 2 hours. What is the average speed of the car?
F 50 mph
G 55 mph
H 60 mph
J none of the above

SOLUTION:

Total distance travelled = 170 miles
Total time taken = 3 hours

$$\text{Average speed} = \frac{170}{3}\text{mph}$$

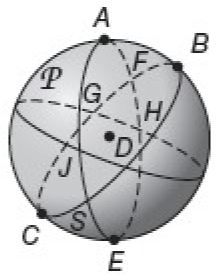
None of the choices is $\frac{170}{3}$ mph .

The correct choice is J.

ANSWER:

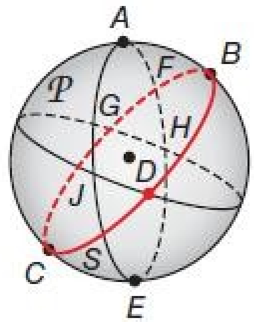
J

33. **SHORT RESPONSE** Name a line in sphere *P* that contains point *D*.



SOLUTION:

We are looking for a great circle that passes though the point *D*. Observing the arcs shown, we can see that point *D* lies on the line $\overleftrightarrow{BC}$.



ANSWER:

Sample answer: $\overleftrightarrow{BC}$

34. **ALGEBRA** The ratio of males to females in a classroom is 3:5. How many females are in the room if the total number of students is 32?

A 12
B 20
C 29
D 51
E 53

SOLUTION:

Let *x* be the number of females in a classroom.
Form a proportion.

$$\frac{x}{32} = \frac{5}{3+5}$$
$$\frac{x}{32} = \frac{5}{8}$$
$$x = 20$$

So, the correct choice is B.

ANSWER:

B

Find the volume of each sphere or hemisphere. Round to the nearest tenth.

35. sphere: area of great circle = 98.5 m^2

SOLUTION:

We know that the area of a great circle is πr^2.

$$\pi r^2 = 98.5$$
$$r^2 = \frac{98.5}{\pi}$$
$$r \approx 5.5994$$

The volume V of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius.

Use the formula.

$$V = \frac{4}{3}\pi(5.5994)^3$$
$$\approx 735.4 \text{ m}^3$$

ANSWER:

735.4 m^3

36. sphere: circumference of great circle ≈ 23.1 in.

SOLUTION:

We know that the circumference of a great circle is $2\pi r$.

$$2\pi r \approx 23.1$$
$$\pi r \approx 11.55$$
$$r \approx \frac{11.55}{\pi}$$

The volume V of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius.

Use the formula.

$$V \approx \frac{4}{3}\pi\left(\frac{11.55}{\pi}\right)^3$$
$$\approx 208.2 \text{ in}^3$$

ANSWER:

208.2 in^3

37. hemisphere: circumference of great circle 50.3 cm

SOLUTION:

We know that the circumference of a great circle is $2\pi r$.

$$\begin{aligned} 2\pi r &\approx 50.3 \\ \pi r &\approx 25.15 \\ r &\approx \frac{25.15}{\pi} \end{aligned}$$

The volume V of a hemisphere is $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ or $V = \frac{2}{3}\pi r^3$, where r is the radius.

Use the formula.

$$\begin{aligned} V &\approx \frac{2}{3}\pi\left(\frac{25.15}{\pi}\right)^3 \\ &\approx 1074.5 \text{ cm}^3 \end{aligned}$$

ANSWER:

1074.5 cm^3

38. hemisphere: area of great circle ≈ 3416 ft^2

SOLUTION:

We know that the area of a great circle is πr^2.

$$\begin{aligned} \pi r^2 &\approx 3416 \\ r &\approx 32.97494 \end{aligned}$$

The volume V of a hemisphere is $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ or $V = \frac{2}{3}\pi r^3$, where r is the radius. Here, the diameter is 16 cm.

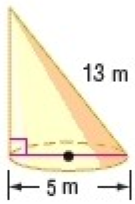
So, the radius is 8 cm.
Use the formula.

$$\begin{aligned} V &\approx \frac{2}{3}\pi(32.97494)^3 \\ &\approx 75094.9 \text{ ft}^3 \end{aligned}$$

ANSWER:

75,094.9 ft^3

Find the volume of each cone. Round to the nearest tenth.

39. 

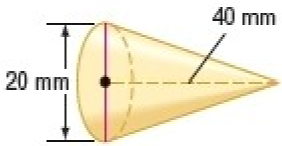
SOLUTION:
Use the Pythagorean theorem to find *h*.

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
5^2 + b^2 &= 13^2 \\
b^2 &= 13^2 - 5^2 \\
b^2 &= 169 - 25 \\
b &= \sqrt{144} \\
b &= 12
\end{aligned}
$$

Now find the volume.

$$
\begin{aligned}
V &= \tfrac{1}{3}\pi r^2 h \\
&= \tfrac{1}{3}\pi (2.5)^2 (12) \\
&\approx 78.5
\end{aligned}
$$

ANSWER:
78.5 m^3

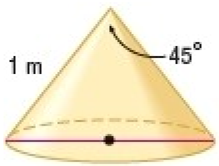
40. 

SOLUTION:

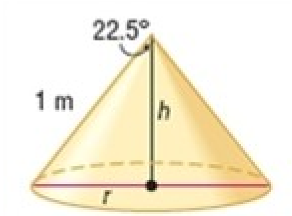
$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi (10)^2 (40) \\ &\approx 4188.8 \end{aligned}$$

ANSWER:

4188.8 mm^3

41. 

SOLUTION:



Use trigonometry to find the height and radius of the cone.

$$\sin 22.5 = \frac{opposite}{hypotenuse}$$
$$\sin 22.5 = \frac{r}{1}$$
$$\sin 22.5 = r$$

$$\cos 22.5 = \frac{adjacent}{hypotenuse}$$
$$\cos 22.5 = \frac{h}{1}$$
$$\cos 22.5 = h$$

Now find the volume.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(\sin 22.5)^2(\cos 22.5) \\ &\approx 0.1 \text{ in}^3 \end{aligned}$$

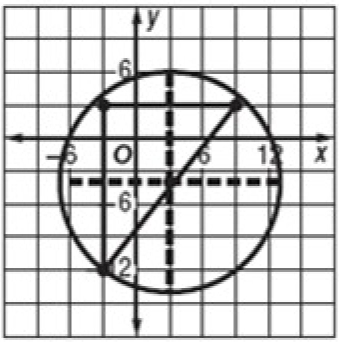
ANSWER:

0.1 in^3

42. **RADIOS** Three radio towers are modeled by the points A(–3, 4), B(9, 4), and C(–3, –12). Determine the location of another tower equidistant from all three towers, and write an equation for the circle which all three points lie on.

SOLUTION:

You are given three points that lie on a circle. Graph triangle ABC and construct the perpendicular bisectors of two sides to locate the center of the circle. Find the radius and then use the center and radius to write an equation. Construct the perpendicular bisectors of two sides. The center appears to be at (3, –4).



Find the distance between the points to determine the radius.

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3-3)^2 + (4-(-4))^2} \\ &= \sqrt{36+64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

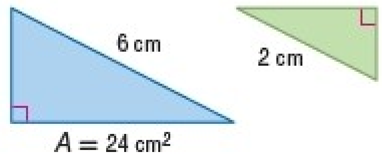
The standard form of the equation of a circle with center at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. Substitute.

$$(x-3)^2 + (y+4)^2 = 100$$

ANSWER:

(3, –4); $(x - 3)^2 + (y + 4)^2 = 100$

For each pair of similar figures, find the area of the green figure.

43. 

SOLUTION:

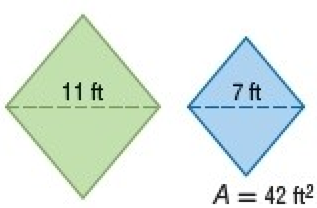
The scale factor between the blue triangle and the green triangle is 6 ÷ 2 = 3, so the ratio of their areas is 3^2.

$$\frac{\text{Area(blue)}}{\text{Area(green)}} = (3)^2$$
$$\frac{24}{\text{Area(green)}} = 9$$
$$\text{Area(green)} = \frac{24}{9}$$
$$\text{Area(green)} = 2.7$$

The area of the green triangle is 2.7 cm^2.

ANSWER:

2.7 cm^2

44. 

SOLUTION:

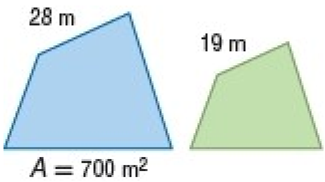
The scale factor between the blue quadrilateral and the green quadrilateral is $\frac{7}{11}$, so the ratio of their areas is $\left(\frac{7}{11}\right)^2$.

$$\frac{\text{Area(blue)}}{\text{Area(green)}} = \left(\frac{7}{11}\right)^2$$
$$\frac{42}{\text{Area(green)}} = \frac{49}{121}$$
$$\text{Area(green)} = \frac{42 \times 121}{49}$$
$$\text{Area(green)} \approx 103.7$$

The area of the green quadrilateral is about 103.7 ft^2.

ANSWER:

103.7 ft^2

45. 

SOLUTION:

The scale factor between the blue quadrilateral and the green quadrilateral is $\frac{28}{19}$, so the ratio of their areas is $\left(\frac{28}{19}\right)^2$.

$$\frac{\text{Area(blue)}}{\text{Area(green)}} = \left(\frac{28}{19}\right)^2$$
$$\frac{700}{\text{Area(green)}} = \frac{784}{361}$$
$$\text{Area(green)} = \frac{700 \times 361}{784}$$
$$\text{Area(green)} \approx 322.3$$

The area of the green quadrilateral is about 322.3 m^2.

ANSWER:

322.3 m^2