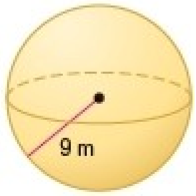
12-6 Surface Area and Volumes of Spheres

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

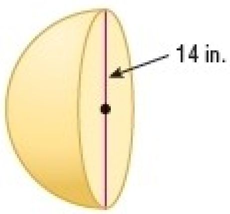
1. 

SOLUTION:

$$\begin{aligned} S &= 4\pi r^2 \\ &= 4\pi(9)^2 \\ &= 324\pi \\ &\approx 1017.9 \end{aligned}$$

ANSWER:

1017.9 m^2

2. 

SOLUTION:

$$\begin{aligned} S &= \frac{1}{2}\left(4\pi r^2\right) + \pi r^2 \\ &= \frac{1}{2}\left[4\pi(7)^2\right] + \pi(7)^2 \\ &= 98\pi + 49\pi \\ &= 147\pi \\ &\approx 461.8 \end{aligned}$$

ANSWER:

461.8 in^2

3. sphere: area of great circle = 36π yd^2

SOLUTION:

We know that the area of a great circle is πr^2. Find *r*.

$$\begin{aligned}\pi r^2 &= 36\pi \\ r^2 &= 36 \\ r &= 6\end{aligned}$$

Now find the surface area.

$$\begin{aligned}S &= 4\pi r^2 \\ &= 4\pi(6)^2 \\ &= 144\pi \\ &\approx 452.4\end{aligned}$$

ANSWER:

452.4 yd^2

4. hemisphere: circumference of great circle ≈ 26 cm

SOLUTION:

We know that the circumference of a great circle is $2\pi r$.

$$\begin{aligned}2\pi r &= 26 \\ \pi r &= 13 \\ r &= \frac{13}{\pi}\end{aligned}$$

The area of a hemisphere is one-half the area of the sphere plus the area of the great circle.

$$\begin{aligned}A &= \frac{1}{2}\left(4\pi r^2\right) + \pi r^2 \\ &= \frac{1}{2}\left(4\pi\left(\frac{13}{\pi}\right)^2\right) + \pi\left(\frac{13}{\pi}\right)^2 \\ &= 2\left(\frac{169}{\pi}\right) + \left(\frac{169}{\pi}\right) \\ &= 3\left(\frac{169}{\pi}\right) \\ &\approx 161.4 \text{ cm}^2\end{aligned}$$

ANSWER:

161.4 cm^2

Find the volume of each sphere or hemisphere. Round to the nearest tenth.

5. sphere: radius = 10 ft

SOLUTION:

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(10)^3 \\ &= \frac{4000\pi}{3} \\ &\approx 4188.8 \end{aligned}$$

ANSWER:

4188.8 ft^3

6. hemisphere: diameter = 16 cm

SOLUTION:

$$\begin{aligned} V &= \frac{2}{3}\pi r^3 \\ &= \frac{2}{3}\pi(8)^3 \\ &= \frac{1024\pi}{3} \\ &\approx 1072.3 \end{aligned}$$

ANSWER:

1072.3 cm^3

7. hemisphere: circumference of great circle = 24π m

SOLUTION:

We know that the circumference of a great circle is $2\pi r$. Find *r*.

$$\begin{aligned} 2\pi r &= 24\pi \\ r &= 12 \end{aligned}$$

Now find the volume.

$$\begin{aligned} V &= \frac{2}{3}\pi r^3 \\ &= \frac{2}{3}\pi(12)^3 \\ &= \frac{3456\pi}{3} \\ &\approx 3619.1 \end{aligned}$$

ANSWER:

3619.1 m^3

8. sphere: area of great circle = 55π in^2

SOLUTION:

We know that the area of a great circle is πr^2. Find *r*.

$$\begin{aligned} \pi r^2 &= 55\pi \\ r^2 &= 55 \\ r &= \sqrt{55} \end{aligned}$$

Now find the volume.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi\left(\sqrt{55}\right)^3 \\ &\approx 1708.6 \end{aligned}$$

ANSWER:

1708.6 in^3

9. **BASKETBALL** Basketballs used in professional games must have a circumference of $29\frac{1}{2}$ inches. What is the surface area of a basketball used in a professional game?

SOLUTION:

We know that the circumference of a great circle is $2\pi r$. Find r.

$$2\pi r = 29\frac{1}{2}$$
$$2\pi r = \frac{59}{2}$$
$$r = \frac{59}{4\pi}$$

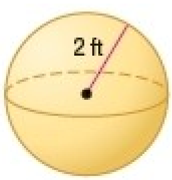
Find the surface area.

$$\begin{aligned} S &= 4\pi r^2 \\ &= 4\pi\left(\frac{59}{4\pi}\right)^2 \\ &= \frac{59^2}{4\pi} \\ &\approx 277.0 \end{aligned}$$

ANSWER:

277.0 in^2

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

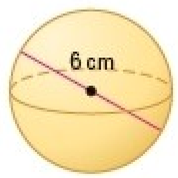
10. 

SOLUTION:

$$\begin{aligned} S &= 4\pi r^2 \\ &= 4\pi(2)^2 \\ &= 16\pi \\ &\approx 50.3 \end{aligned}$$

ANSWER:

50.3 ft^2

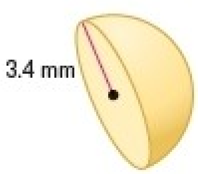
11. 

SOLUTION:

$$\begin{aligned} S &= 4\pi r^2 \\ &= 4\pi(3)^2 \\ &= 36\pi \\ &\approx 113.1 \end{aligned}$$

ANSWER:

113.1 cm^2

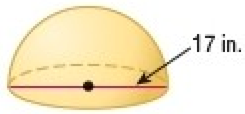
12. 

SOLUTION:

$$\begin{aligned} S &= \frac{1}{2}\left(4\pi r^2\right) + \pi r^2 \\ &= \frac{1}{2}\left[4\pi(3.4)^2\right] + \pi(3.4)^2 \\ &= 23.12\pi + 11.56\pi \\ &= 34.68\pi \\ &\approx 109.0 \end{aligned}$$

ANSWER:

109.0 mm^2



13.

SOLUTION:

$$\begin{aligned} S &= \frac{1}{2}\left(4\pi r^2\right) + \pi r^2 \\ &= \frac{1}{2}\left[4\pi(8.5)^2\right] + \pi(8.5)^2 \\ &= 144.5\pi + 72.25\pi \\ &= 216.75\pi \\ &\approx 680.9 \end{aligned}$$

ANSWER:

680.9 in^2

14. sphere: circumference of great circle = 2π cm

SOLUTION:

We know that the circumference of a great circle is $2\pi r$. Find r.

$$\begin{aligned} 2\pi r &= 2\pi \\ r &= 1 \end{aligned}$$

$$\begin{aligned} S &= 4\pi r^2 \\ &= 4\pi(1)^2 \\ &= 4\pi \\ &\approx 12.6 \end{aligned}$$

ANSWER:

12.6 cm^2

15. sphere: area of great circle ≈ 32 ft^2

SOLUTION:

We know that the area of a great circle is πr^2. Find r.

$$\pi r^2 = 32$$
$$r^2 = \frac{32}{\pi}$$
$$r = \sqrt{\frac{32}{\pi}}$$

$$S = 4\pi r^2$$
$$= 4\pi\left(\sqrt{\frac{32}{\pi}}\right)^2$$
$$= 128$$

ANSWER:

128 ft^2

16. hemisphere: area of great circle ≈ 40 in^2

SOLUTION:

We know that the area of a great circle is πr^2. Find r.

$$\pi r^2 \approx 40$$
$$r^2 \approx \frac{40}{\pi}$$

Substitute for $r2$ in the surface area formula.

$$S = \frac{1}{2}\left(4\pi r^2\right) + \pi r^2$$
$$S = \frac{1}{2}\left(4\pi\left(\frac{40}{\pi}\right)\right) + \pi\left(\frac{40}{\pi}\right)$$
$$= 3\pi\left(\frac{40}{\pi}\right)$$
$$= 120 \text{ in}^2$$

ANSWER:

120 in^2

17. hemisphere: circumference of great circle = 15π mm

SOLUTION:

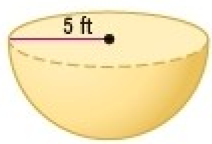
$$2\pi r = 15\pi$$
$$r = 7.5$$

$$\begin{aligned} S &= \tfrac{1}{2}\left(4\pi r^2\right) + \pi r^2 \\ &= \tfrac{1}{2}\left[4\pi(7.5)^2\right] + \pi(7.5)^2 \\ &= 112.5\pi + 56.25\pi \\ &= 168.75\pi \\ &\approx 530.1 \end{aligned}$$

ANSWER:

530.1 mm^2

CCSS PRECISION Find the volume of each sphere or hemisphere. Round to the nearest tenth.

18. 

SOLUTION:

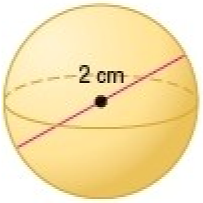
The volume *V* of a hemisphere is $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ or $V = \frac{2}{3}\pi r^3$, where *r* is the radius.

The radius is 5 cm.

$$\begin{aligned} V &= \frac{2}{3}\pi(5)^3 \\ &\approx 261.8 \text{ ft}^3 \end{aligned}$$

ANSWER:

261.8 ft^3

19. 

SOLUTION:

The volume V of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius.

The radius is 1 cm.

$$V = \frac{4}{3}\pi (1)^3$$
$$\approx 4.2 \text{ cm}^3$$

ANSWER:

4.2 cm^3

20. sphere: radius = 1.4 yd

SOLUTION:

The volume V of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius.

$$V = \frac{4}{3}\pi (1.4)^3$$
$$\approx 11.5 \text{ yd}^3$$

ANSWER:

11.5 yd^3

21. hemisphere: diameter = 21.8 cm

SOLUTION:

The radius is 10.9 cm. The volume V of a hemisphere is $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ or $V = \frac{2}{3}\pi r^3$, where r is the radius.

$$V = \frac{2}{3}\pi (10.9)^3$$
$$\approx 2712.3 \text{ cm}^3$$

ANSWER:

2712.3 cm^3

22. sphere: area of great circle = 49π m^2

SOLUTION:

The area of a great circle is πr^2.

$$\begin{aligned}\pi r^2 &= 49\pi \\ r^2 &= 49 \\ r &= 7\end{aligned}$$

The volume *V* of a sphere is $V = \frac{4}{3}\pi r^3$, where *r* is the radius.

$$\begin{aligned}V &= \frac{4}{3}\pi(7)^3 \\ &\approx 1436.8 \text{ m}^3\end{aligned}$$

ANSWER:

1436.8 m^3

23. sphere: circumference of great circle ≈ 22 in.

SOLUTION:

The circumference of a great circle is $2\pi r$.

$$\begin{aligned}2\pi r &= 22 \\ \pi r &= 11 \\ r &= \frac{11}{\pi}\end{aligned}$$

The volume *V* of a sphere is $V = \frac{4}{3}\pi r^3$, where *r* is the radius.

$$\begin{aligned}V &= \frac{4}{3}\pi\left(\frac{11}{\pi}\right)^3 \\ &\approx 179.8 \text{ in}^3\end{aligned}$$

ANSWER:

179.8 in^3

24. hemisphere: circumference of great circle ≈ 18 ft

SOLUTION:

The circumference of a great circle is $2\pi r$.

$$\begin{aligned} 2\pi r &= 18 \\ \pi r &= 9 \\ r &= \frac{9}{\pi} \end{aligned}$$

The volume V of a hemisphere is $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ or $V = \frac{2}{3}\pi r^3$, where r is the radius.

$$\begin{aligned} V &= \frac{2}{3}\pi\left(\frac{9}{\pi}\right)^3 \\ &\approx 49.2\ \text{ft}^3 \end{aligned}$$

ANSWER:

49.2 ft^3

25. hemisphere: area of great circle ≈ 35 m^2

SOLUTION:

The area of a great circle is πr^2.

$$\begin{aligned} \pi r^2 &= 35 \\ r^2 &= \frac{35}{\pi} \\ r &= \sqrt{\frac{35}{\pi}} \end{aligned}$$

The volume V of a hemisphere is $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ or $V = \frac{2}{3}\pi r^3$, where r is the radius.

$$\begin{aligned} V &= \frac{2}{3}\pi\left(\sqrt{\frac{35}{\pi}}\right)^3 \\ &\approx 77.9\ \text{m}^3 \end{aligned}$$

ANSWER:

77.9 m^3

26. **FISH** A puffer fish is able to "puff up" when threatened by gulping water and inflating its body. The puffer fish at the right is approximately a sphere with a diameter of 5 inches. Its surface area when inflated is about 1.5 times its normal surface area. What is the surface area of the fish when it is not puffed up?
Refer to the photo on Page 869.

SOLUTION:

$$\begin{aligned} S &= 4\pi r^2 \\ &= 4\pi(2.5)^2 \\ &= 25\pi \\ &\approx 78.5 \end{aligned}$$

Find the surface area of the non-puffed up fish, or when it is normal.

$$\begin{aligned} S_{\text{inflated}} &= 1.5S_{\text{normal}} \\ 25\pi &= 1.5S_{\text{normal}} \\ \frac{25\pi}{1.5} &= S_{\text{normal}} \\ 52.4 &\approx S_{\text{normal}} \end{aligned}$$

ANSWER:

about 52.4 in^2

27. **ARCHITECTURE** The Reunion Tower in Dallas, Texas, is topped by a spherical dome that has a surface area of approximately 13,924π square feet. What is the volume of the dome? Round to the nearest tenth.
Refer to the photo on Page 869.

SOLUTION:

Find *r*.

$$\begin{aligned} S(\text{dome}) &= 13,924\pi \\ 4\pi r^2 &= 13,924\pi \\ 4r^2 &= 13,924 \\ r^2 &= 3481 \\ r &= 59 \end{aligned}$$

Find the volume.

$$\begin{aligned} V &= \tfrac{4}{3}\pi(59)^3 \\ &\approx 860,289.5 \text{ ft}^3 \end{aligned}$$

ANSWER:

860,289.5 ft^3

28. **TREE HOUSE** The spherical tree house, or tree sphere, has a diameter of 10.5 feet. Its volume is 1.8 times the volume of the first tree sphere that was built. What was the diameter of the first tree sphere? Round to the nearest foot.

SOLUTION:

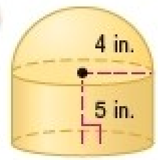
The volume of the spherical tree house is 1.8 times the volume of the first tree sphere.

$$\begin{aligned}
\text{V(new sphere)} &= 1.8 \times \text{V(first sphere)} \\
\frac{4}{3}\pi(5.25)^3 &= 1.8 \times \frac{4}{3}\pi(r)^3 \\
(5.25)^3 &= 1.8(r)^3 \\
\frac{5.25^3}{1.8} &= r^3 \\
\sqrt[3]{\frac{5.25^3}{1.8}} &= r \\
4.3 &\approx r
\end{aligned}$$

The diameter is 4.3(2) or about 9 ft.

ANSWER:
9 ft

CCSS SENSE-MAKING Find the surface area and the volume of each solid. Round to the nearest tenth.

29. 

SOLUTION:

To find the surface area of the figure, calculate the surface area of the cylinder (without the bases), hemisphere (without the base), and the base, and add them.

$$SA = \text{cylinder} + \text{hemisphere} + \text{base}$$

$$\begin{aligned} SA &= 2\pi rh + 2\pi r^2 + \pi r^2 \\ &= 2\pi r(h + r) + \pi r^2 \\ &= 2\pi(4)(9) + \pi(4)^2 \\ &= 72\pi + 16\pi \\ &= 88\pi \\ &\approx 276.5 \text{ in}^3 \end{aligned}$$

To find the volume of the figure, calculate the volume of the cylinder and the hemisphere and add them.

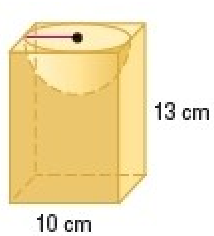
$$\begin{aligned} V(\text{cylinder}) &= \pi r^2 h \\ &= \pi(4)^2(5) \\ &= 80\pi \end{aligned}$$

$$\begin{aligned} V(\text{hemisphere}) &= \frac{2}{3}\pi r^3 \\ &= \frac{2}{3}\pi(4)^3 \\ &= \frac{128\pi}{3} \end{aligned}$$

$$\begin{aligned} V(\text{figure}) &= \text{cylinder} + \text{hemisphere} \\ &= 80\pi + \frac{128\pi}{3} \\ &\approx 385.4 \text{ in.}^3 \end{aligned}$$

ANSWER:

276.5 in^2; 385.4 in.^3

30. 

SOLUTION:

To find the surface area of the figure, we will need to calculate the surface area of the prism, the area of the top base, the area of the great circle, and one-half the surface area of the sphere.

First, find the surface area of the prism and subtract the area of the top base.

$$\begin{aligned} SA &= L + B \\ &= Ph + B \\ &= 40(13) + 10^2 \\ &= 520 + 100 \\ &= 620 \end{aligned}$$

This covers everything but the top of the figure. The corners of the top base can be found by subtracting the area of the great circle from the area of the base.

$$\begin{aligned} SA &= \text{Area(base)} - \text{Area(circle)} \\ &= 10^2 - \pi(5)^2 \\ &= 100 - 25\pi \end{aligned}$$

Next, find the surface area of the hemisphere.

$$\begin{aligned} SA &= 2\pi r^2 \\ &= 2\pi(5)^2 \\ &= 50\pi \end{aligned}$$

Now, find the total surface area.

$$\begin{aligned} SA &= \text{prism} + \text{corners} + \text{hemisphere} \\ &= 620 + (100 - 25\pi) + 50\pi \\ &= 720 + 25\pi \\ &\approx 798.5 \end{aligned}$$

The volume of the figure is the volume of the prism minus the volume of the hemisphere.

$$\begin{aligned} V(\text{cylinder}) &= lwh \\ &= 10 \cdot 10 \cdot 13 \\ &= 1300 \end{aligned}$$

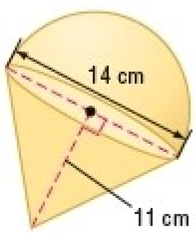
$$\begin{aligned} V(\text{hemisphere}) &= \tfrac{2}{3}\pi r^3 \\ &= \tfrac{2}{3}\pi(5)^3 \\ &= \tfrac{250\pi}{3} \end{aligned}$$

$$\begin{aligned} V(\text{figure}) &= \text{cylinder} - \text{hemisphere} \\ &= 1300 + \tfrac{250\pi}{3} \\ &\approx 1038.2 \end{aligned}$$

ANSWER:

798.5 cm^2; 1038.2 cm^3

31. **TOYS** The spinning top is a composite of a cone and a hemisphere.



a. Find the surface area and the volume of the top. Round to the nearest tenth.
b. If the manufacturer of the top makes another model with dimensions that are one-half of the dimensions of this top, what are its surface area and volume?

SOLUTION:

a. Use the Pythagorean Theorem to find the slant height.

$$\begin{aligned} l^2 &= 11^2 + 7^2 \\ l &= \sqrt{11^2 + 7^2} \\ &\approx 13.04 \end{aligned}$$

Lateral area of the cone:

$$\begin{aligned} L &= \pi r l \\ &= \pi(7)(13.04) \\ &\approx 286.76 \end{aligned}$$

Surface area of the hemisphere:

$$\begin{aligned} &= 2\pi(r)^2 \\ &= 2\pi(7)^2 \\ &\approx 307.876 \end{aligned}$$

$$\begin{aligned} SA(\text{figure}) &\approx 286.76 + 307.876 \\ &\approx 594.6 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} V(\text{figure}) &= \tfrac{1}{3}\pi r^2 h + \tfrac{2}{3}\pi r^3 \\ &= \tfrac{1}{3}\pi r^2 (h + 2r) \\ &= \tfrac{1}{3}\pi (7)^2 (25) \\ &\approx 1282.8 \text{ cm}^3 \end{aligned}$$

b. Use the Pythagorean Theorem to find the slant height.

$$\begin{aligned} l^2 &= 5.5^2 + 3.5^2 \\ l &= \sqrt{5.5^2 + 3.5^2} \\ &\approx 6.5 \end{aligned}$$

Lateral area of the cone:

$$\begin{aligned} L &= \pi r l \\ &= \pi(3.5)(6.5) \\ &\approx 71.6823 \end{aligned}$$

Surface area of the hemisphere:

$$\begin{aligned} &= 2\pi r^2 \\ &= 2\pi(3.5)^2 \\ &\approx 76.969 \end{aligned}$$

$$\begin{aligned} SA(\text{figure}) &\approx 71.6823 + 76.969 \\ &\approx 148.7 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} V(\text{figure}) &= \tfrac{1}{3}\pi r^2 h + \tfrac{2}{3}\pi r^3 \\ &= \tfrac{1}{3}\pi r^2 (h + 2r) \\ &= \tfrac{1}{3}\pi (3.5)^2 (12.5) \\ &\approx 160.4 \text{ cm}^3 \end{aligned}$$

ANSWER:

a. 594.6 cm^2 ; 1282.8 cm^3
b. 148.7 cm^2 ; 160.4 cm^3

32. **BALLOONS** A spherical helium-filled balloon with a diameter of 30 centimeters can lift a 14-gram object. Find the size of a balloon that could lift a person who weighs 65 kilograms. Round to the nearest tenth.

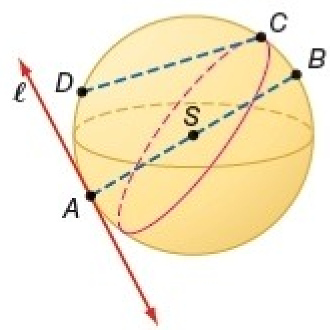
SOLUTION:
1 kg = 1000 g.
Form a proportion.
Let x be the unknown.

$$\frac{x}{65000} = \frac{30}{14}$$
$$x = \frac{30 \times 65000}{14}$$
$$x \approx 139,286 \text{ cm}$$

The balloon would have to have a diameter of approximately 139,286 cm.

ANSWER:
The balloon would have to have a diameter of approximately 139,286 cm.

Use sphere S to name each of the following.



33. a chord

SOLUTION:
A chord of a sphere is a segment that connects any two points on the sphere.

$\overline{DC}$

ANSWER:
$\overline{DC}$

34. a radius

SOLUTION:
A radius of a sphere is a segment from the center to a point on the sphere.

Sample answer: $\overline{SA}$

ANSWER:
Sample answer: $\overline{SA}$

35. a diameter

SOLUTION:
A diameter of a sphere is a chord that contains the center.

$\overline{AB}$

ANSWER:
$\overline{AB}$

36. a tangent

SOLUTION:
A tangent to a sphere is a line that intersects the sphere in exactly one point. Line *l* intersects the sphere at *A*.

ANSWER:
line l

37. a great circle

SOLUTION:
If the circle contains the center of the sphere, the intersection is called a great circle.

$\odot S$

ANSWER:
$\odot S$

38. **DIMENSIONAL ANALYSIS** Which has greater volume: a sphere with a radius of 2.3 yards or a cylinder with a radius of 4 feet and height of 8 feet?

SOLUTION:

2.3 yards = 2.3(3) or 6.9 feet

$$\begin{aligned} \text{V(sphere)} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(6.9)^3 \\ &\approx 1376 \text{ ft}^3 \end{aligned}$$

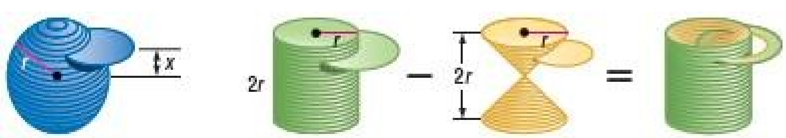
$$\begin{aligned} V(\text{cylinder}) &= \pi r^2 h \\ &= \pi(4)^2(8) \\ &\approx 402 \text{ ft}^3 \end{aligned}$$

Since 1376 ft^3 > 402 ft^3, the sphere has the greater volume.

ANSWER:

the sphere

39. **Informal Proof** A sphere with radius *r* can be thought of as being made up of a large number of discs or thing cylinders. Consider the disc shown that is *x* units above or below the center of the spheres. Also consider a cylinder with radius *r* and height 2*r* that is hollowed out by two cones of height and radius *r*.



a. Find the radius of the disc from the sphere in terms of its distance *x* above the sphere's center. (*Hint*: Use the Pythagorean Theorem.)

b. If the disc from the sphere has a thickness of *y* units, find its volume in terms of *x* and *y*.

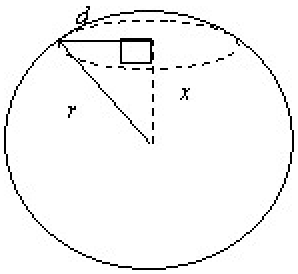
c. Show that this volume is the same as that of the hollowed-out disc with thickness of y units that is x units above the center of the cylinder and cone.

d. Since the expressions for the discs at the same height are the same, what guarantees that the hollowed-out cylinder and sphere have the same volume?

e. Use the formulas for the volumes of a cylinder and a cone to derive the formula for the volume of the hollowed-out cylinder and thus, the sphere.

SOLUTION:

a. Let *d* be the radius of the disc at a height *x*. From the diagram below, we can see how to use the Pythagorean Theorem to express *d* in terms of *x* and *r*.



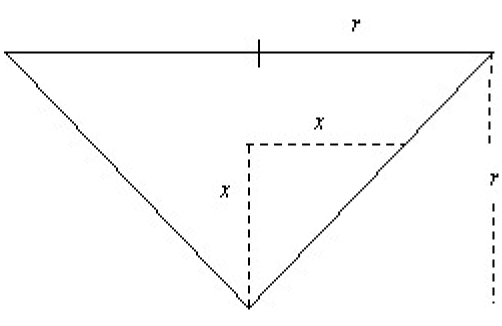
$$r^2 = x^2 + d^2$$
$$d^2 = r^2 - x^2$$
$$d = \sqrt{r^2 - x^2}$$

b. If each disc has a thickness of y, then the discs form tiny cylinders, and we can use the volume formula and the expression found in part a.

$$V = \pi d^2 y$$
$$V = \pi\left(\sqrt{r^2 - x^2}\right)^2 \cdot y$$
$$V = \pi\left(r^2 - x^2\right) \cdot y$$
$$V = \pi y r^2 - \pi y x^2$$

c. For a cylinder the radius of each disc is always r, at any height x. The volume of discs in the cylinder is thus:
$V = \pi r^2 y$.

For the cone, the height of the top half is r and the radius is r. By similar triangles, at any height x above the center, the radius of the disc at that height is also x.



The volume of a disc at height x is thus:
$$V = \pi x^2 y$$

Rearranging the terms and taking the difference we have:
$$V = \pi y r^2 - \pi y x^2$$

d. We have shown that each of the figures have the same height and that they have the same cross sectional area at each height, which implies, by Cavlieri's Principle that the two objects have the same volume.

e. The volume of the cylinder is:

$$V = \pi r^2(2r)$$
$$V = 2\pi r^3$$

The volume of the cones is:

$$V = 2 \cdot \tfrac{1}{3}\pi r^2(r)$$
$$V = \tfrac{2}{3}\pi r^3$$

From parts **c** and **d**, the volume of the sphere is just the difference of these two;

$$\begin{aligned} V &= 2\pi r^3 - \frac{2}{3}\pi r^3 \\ &= \frac{6}{3}\pi r^3 - \frac{2}{3}\pi r^3 \\ &= \frac{4}{3}\pi r^3 \end{aligned}$$

ANSWER:

a. $\sqrt{r^2 - x^2}$

b. $\pi\left(\sqrt{r^2 - x^2}\right)^2 \cdot y$ or $\pi y r^2 - \pi y x^2$

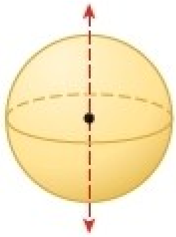
c. The volume of the disc from the cylinder is $\pi r^2 y$ or $\pi y r^2$. The volume of the disc from the two cones is $\pi x^2 y$ or $\pi y x^2$. Subtract the volumes of the discs from the cylinder and cone to get $\pi y r^2 - \pi y x^2$, which is the expression for the volume of the disc from the sphere at height x.

d. Cavalieri’s Principle

e. The volume of the cylinder is $\pi r^2(2r)$ or $2\pi r^3$. The volume of one cone is $\frac{1}{3}\pi r^2(r)$ or $\frac{1}{3}\pi r^3$, so the volume of the double napped cone is $2 \cdot \frac{1}{3}\pi r^3$ or $\frac{2}{3}\pi r^3$.

Therefore, the volume of the hollowed out cylinder, and thus the sphere, is $2\pi r^3 - \frac{2}{3}\pi r^3$ or $\frac{4}{3}\pi r^3$.

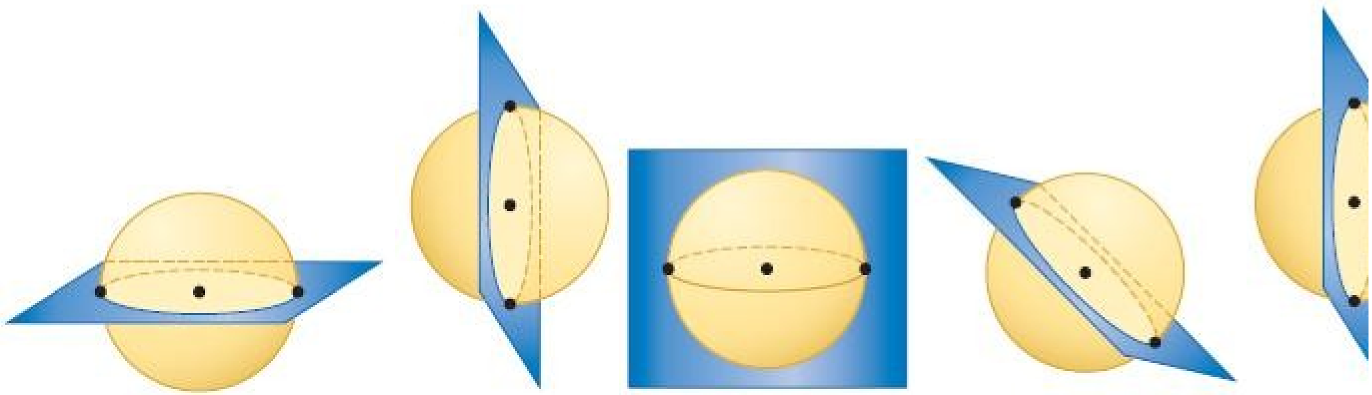
Describe the number and types of planes that produce reflection symmetry in each solid. Then describe the angles of rotation that produce rotation symmetry in each solid.

40. 

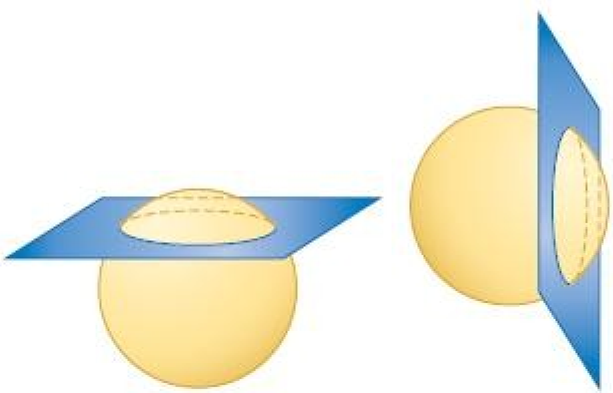
SOLUTION:

There is an infinite number of planes that produce reflection symmetry for spheres. Any plane that passes through th symmetry. Any plane that does not pass through the center will not produce symmetry.

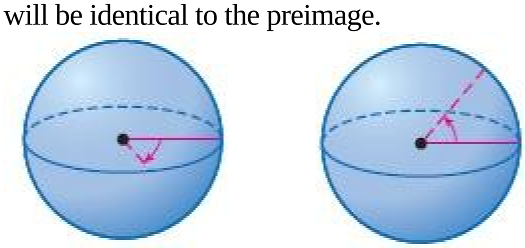
These are all symmetric



These are not:

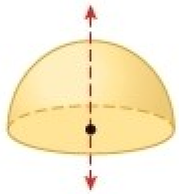


The angle of rotation is the angle through which a preimage is rotated to form the image. A sphere can be rotated at will be identical to the preimage.



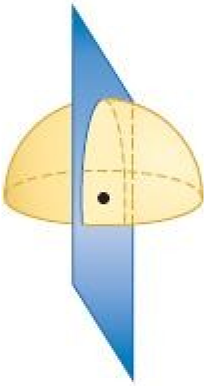
ANSWER:

There are infinitely many planes that produce reflection symmetry as long as they pass through the origin. Any angle symmetry.

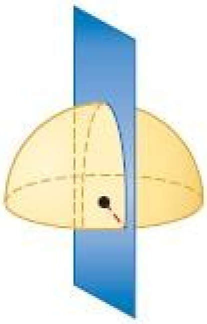
41. 

SOLUTION:

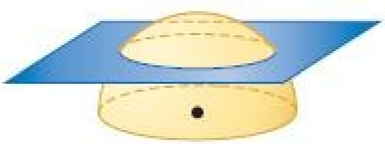
Any plane of symmetry must pass through the origin. Take a look at a few examples.



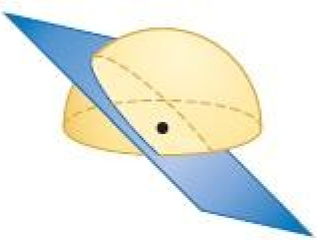
symmetric



Symmetric



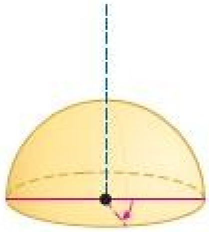
Not symmetric



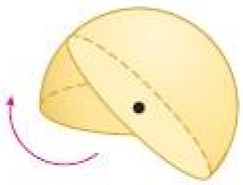
Not symmetric

We see that the only planes that produce reflective symmetry are vertical planes that pass through the origin.

The angle of rotation is the angle through which a preimage is rotated to form the image. A hemisphere can be rotated at any angle around the vertical axis, and the resulting image will be identical to the preimage. A hemisphere rotated about any other axis will not be symmetric.



Symmetric



Not Symmetric

ANSWER:

There are infinitely many planes that produce reflection symmetry as long as they are vertical planes. Only rotation about the vertical axis will produce rotation symmetry through infinitely many angles.

CHANGING DIMENSIONS A sphere has a radius of 12 centimeters. Describe how each change affects the surface area and the volume of the sphere.

42. The radius is multiplied by 4.

SOLUTION:

$$\begin{aligned} S &= 4\pi r^2 \\ &= 4\pi(12)^2 \\ &= 576\pi \end{aligned}$$

$$\begin{aligned} S &= 4\pi r^2 \\ &= 4\pi(48)^2 \\ &= 9216\pi \end{aligned}$$

$9216 \div 576 = 16$

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(12)^3 \\ &= 2304\pi \end{aligned}$$

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(48)^3 \\ &= 147,456\pi \end{aligned}$$

$$\frac{147,456}{2304} = 64$$

The surface area is multiplied by 4^2 or 16. The volume is multiplied by 4^3 or 64.

ANSWER:

The surface area is multiplied by 4^2 or 16. The volume is multiplied by 4^3 or 64.

43. The radius is divided by 3.

SOLUTION:

$$S = 4\pi r^2$$
$$= 4\pi(12)^2$$
$$= 576\pi$$
$$S = 4\pi r^2$$
$$= 4\pi(4)^2$$
$$= 64\pi$$

576 ÷ 64 = 9

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(12)^3$$
$$= 2304\pi$$
$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(4)^3$$
$$= \frac{256\pi}{3}$$
$$\frac{2304}{\frac{256}{3}} = 27$$

The surface area is divided by 3^2 or 9. The volume is divided by 3^3 or 27.

ANSWER:

The surface area is divided by 3^2 or 9. The volume is divided by 3^3 or 27.

44. **DESIGN** A standard juice box holds 8 fluid ounces.
a. Sketch designs for three different juice containers that will each hold 8 fluid ounces. Label dimensions in centimeters. At least one container should be cylindrical. (Hint: 1 fl oz ≈ 29.57353 cm^3)
b. For each container in part a, calculate the surface area to volume (cm^2 per fl oz) ratio. Use these ratios to decide which of your containers can be made for the lowest materials cost. What shape container would minimize this ratio, and would this container be the cheapest to produce? Explain your reasoning.

SOLUTION:

a. Sample answer: Choose two cylindrical containers one with height 9 cm and one with height 7 cm, and a third container that is a cube.
The radii of the cylindrical containers can be found using the formula for volume.

For 9 cm height:

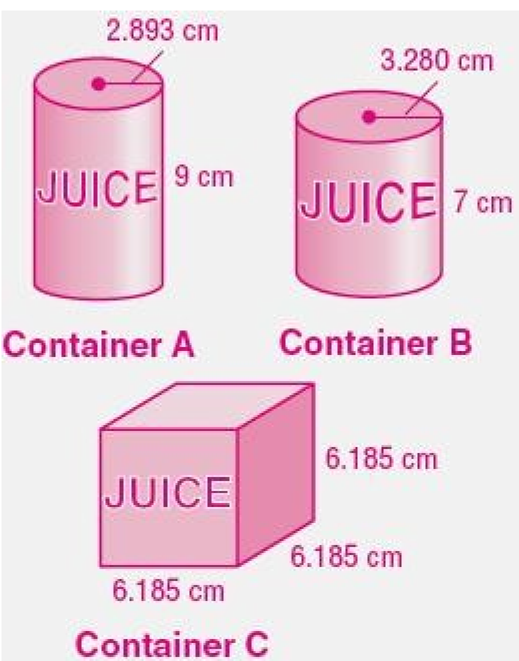
$$V = \pi r^2 h$$
$$r^2 = \frac{V}{\pi h}$$
$$r = \sqrt{\frac{V}{\pi h}}$$
$$r = \sqrt{\frac{8 \cdot 29.57353}{\pi \cdot 9}}$$
$$r = 2.893$$

For 7 cm height:

$$V = \pi r^2 h$$
$$r^2 = \frac{V}{\pi h}$$
$$r = \sqrt{\frac{V}{\pi h}}$$
$$r = \sqrt{\frac{8 \cdot 29.57353}{\pi \cdot 7}}$$
$$r = 3.280$$

We can similarly solve for the length of the cube:

$$V = l^3$$
$$l = \sqrt[3]{V}$$
$$l = \sqrt[3]{8 \cdot 29.57353}$$
$$l = 6.185$$



b. First calculate the surface area of each of the solids in part a.
For cylinders:

$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 \\ &= 2\pi\left(2.893\right)\left(9\right) + 2\pi(2.893)^2 \\ &= 216.2\text{cm}^2 \end{aligned}$$

$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 \\ &= 2\pi\left(3.280\right)\left(7\right) + 2\pi(3.280)^2 \\ &= 211.9\text{cm}^2 \end{aligned}$$

For the cube:

$$\begin{aligned} S &= 6l^2 \\ &= 6(6.185)^2 \\ &= 229.5\text{cm}^2 \end{aligned}$$

The ratios of surface area to volume are thus:
Container A

$$\begin{aligned} \frac{S}{V} &= \frac{216.2}{8} \\ &= 27.02\text{ cm}^2\text{ / fl oz} \end{aligned}$$

Container B

$$\begin{aligned} \frac{S}{V} &= \frac{211.5}{8} \\ &= 26.48\text{ cm}^2\text{ / fl oz} \end{aligned}$$

Container C

$$\begin{aligned} \frac{S}{V} &= \frac{229.5}{8} \\ &= 28.69\text{ cm}^2\text{ / fl oz} \end{aligned}$$

Container B has the lowest material cost. If we consider the ratio of surface area to volume for a sphere we get:

$$V = \frac{4}{3}\pi r^3$$

$$r^3 = \frac{3}{4\pi}V$$

$$r = \sqrt[3]{\frac{3}{4\pi}V}$$

$$r = \sqrt{\frac{3}{4\pi}\cdot 8\cdot 29.57353}$$

$$r = 3.837$$

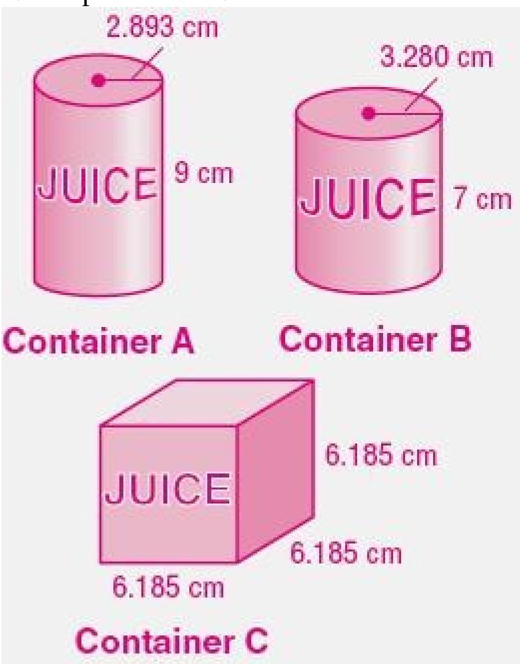
$$S = 4\pi r^2$$
$$= 4\pi(3.837)^2$$
$$= 185.0 \text{ cm}^2$$

$$\frac{S}{V} = \frac{185.0}{8}$$
$$= 23.13 \text{ cm}^2 / \text{fl oz}$$

A sphere has the lowest surface area to volume ratio of any three dimensional object, but the cost with trying to manufacture spherical containers would be greatly increased, since special machines would have to be used.

ANSWER:

a. Sample answers:

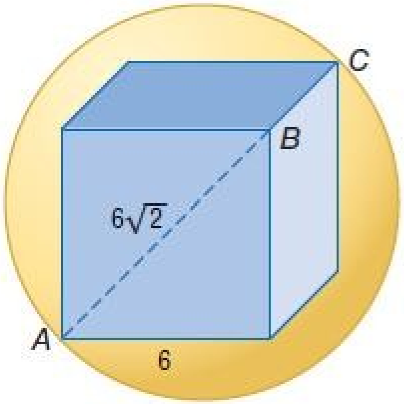


b. Sample answers: Container A, ≈ 27.02 cm^2 per fl oz; Container B, ≈26.48 cm^2 per fl oz; Container C, ≈ 28.69 cm^2 per fl oz; Of these three, Container B can be made for the lowest materials cost. The lower the surface area to volume ratio, the less packaging used for each fluid ounce of juice it holds. A spherical container with r = 3.837 cm would minimize this cost since it would have the least surface area to volume ratio of any shape, ≈ 23.13 cm per fl oz. However, a spherical container would likely be more costly to manufacture than a rectangular container since specially made machinery would be necessary.

45. **CHALLENGE** A cube has a volume of 216 cubic inches. Find the volume of a sphere that is circumscribed about the cube. Round to the nearest tenth.

SOLUTION:

Since the volume of the cube is 216 cubic inches, the length of each side is 6 inches and so, the length of a diagonal on a face is $6\sqrt{2}$ inches.



Use the length of the one side (*BC*) and this diagonal (*AB*) to find the length of the diagonal joining two opposite vertices of the cube (*AC*). The triangle that is formed by these sides is a right triangle. Apply the Pythagorean Theorem.

$$\begin{aligned} a^2+b^2 &= c^2 \\ 6^2+\left(6\sqrt{2}\right)^2 &= c^2 \\ 36+72 &= c^2 \\ \sqrt{108} &= c \\ 6\sqrt{3} &= c \end{aligned}$$

The sphere is circumscribed about the cube, so the vertices of the cube all lie on the sphere. Therefore, line *AC* is also the diameter of the sphere. The radius is $3\sqrt{3}$ inches. Find the volume.

$$\begin{aligned} \text{V (sphere)} &= \tfrac{4}{3}\pi r^3 \\ &= \tfrac{4}{3}\pi\left(3\sqrt{3}\right)^3 \\ &= \tfrac{4}{3}\pi(3)^3\left(\sqrt{3}\right)^3 \\ &= \tfrac{4}{3}\pi(27)(3\sqrt{3}) \\ &= 4\pi(27)\sqrt{3} \\ &\approx 587.7 \text{ in}^3 \end{aligned}$$

ANSWER:

587.7 in^3

46. **REASONING** Determine whether the following statement is true or false. If true, explain your reasoning. If false, provide a counterexample.

If a sphere has radius r, there exists a cone with radius r having the same volume.

SOLUTION:

Determine if there is a value of *h* for which the volume of the cone will equal the volume of the sphere.

The volume of a sphere with radius *r* is given by $V = \frac{4}{3}\pi r^3$. The volume of a cone with a radius of *r* and a height of *h* is given by $V = \frac{1}{3}\pi r^2 h$.

$$\begin{aligned} V_{\text{cone}} &= V_{\text{sphere}} \\ \frac{1}{3}\pi r^2 h &= \frac{4}{3}\pi r^3 \\ \pi r^2 h &= 4\pi r^3 \\ h &= \frac{4\pi r^3}{\pi r^2} \\ h &= 4r \end{aligned}$$

Thus, a cone and sphere with the same radius will have the same volume whenever the cone has a height equal to 4*r*.

Therefore, the statement is true.

ANSWER:

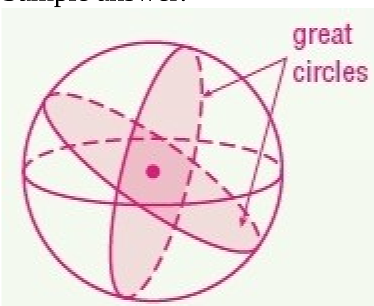
True; a cone of radius *r* and height 4*r* has the same volume, $\frac{4}{3}\pi r^3$, as a sphere with radius *r*.

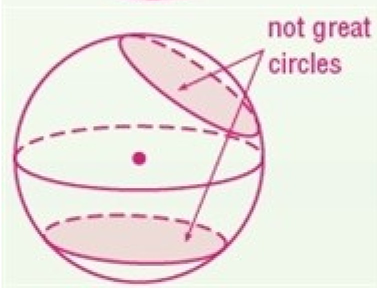
47. **OPEN ENDED** Sketch a sphere showing two examples of great circles. Sketch another sphere showing two examples of circles formed by planes intersecting the sphere that are not great circles.

SOLUTION:

The great circles need to contain the center of the sphere. The planes that are not great circles need to *not* contain the center of the sphere.

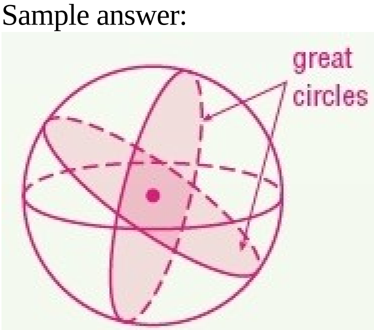
Sample answer:

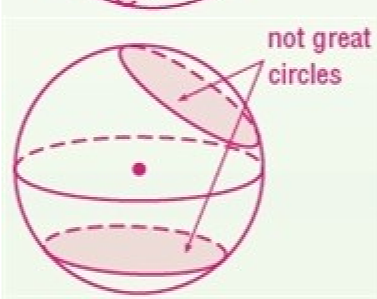




ANSWER:

Sample answer:





48. WRITING IN MATH Write a ratio comparing the volume of a sphere with radius r to the volume of a cylinder with radius r and height 2r. Then describe what the ratio means.

SOLUTION:

$$\begin{aligned}\frac{V(\text{sphere})}{V(\text{cylinder})} &= \frac{\left(\frac{4}{3}\pi r^3\right)}{\pi r^2(2r)} \\ &= \frac{\left(\frac{4}{3}\pi r^3\right)}{2\pi r^3} \\ &= \frac{4\pi r^3}{6\pi r^3} \\ &= \frac{2}{3}\end{aligned}$$

The ratio is 2:3.
The volume of the sphere is two thirds the volume of the cylinder.

ANSWER:

$\frac{2}{3}$; The volume of the sphere is two thirds the volume of the cylinder.

49. GRIDDED RESPONSE What is the volume of the hemisphere shown below in cubic meters?

SOLUTION:

The volume V of a hemisphere is $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ or $V = \frac{2}{3}\pi r^3$, where r is the radius.

Use the formula.

$$\begin{aligned}V &= \frac{2}{3}\pi(3.2)^3 \\ &\approx 68.6 \text{ m}^3\end{aligned}$$

ANSWER:

68.6

50. ALGEBRA What is the solution set of 3z + 4 < 6 + 7z?

A. $\{z | z > -0.5\}$

B. $\{z | z < -0.5\}$

C. $\{z | z > -2\}$

D. $\{z | z < -0.5\}$

SOLUTION:

$$\begin{aligned} 3z + 4 &< 6 + 7z \\ 4 &< 6 + 4z \\ -2 &< 4z \\ -0.5 &< z \end{aligned}$$

The correct choice is A.

ANSWER:

A

51. If the area of the great circle of a sphere is 33 ft^2, what is the surface area of the sphere?

F 42 ft^2

G 117 ft^2

H 132 ft^2

J 264 ft^2

SOLUTION:

We know that the area of a great circle is πr^2.

$$\begin{aligned} \pi r^2 &= 33 \\ r^2 &= \frac{33}{\pi} \end{aligned}$$

The surface area S of a sphere is $S = 4\pi r^2$, where r is the radius.

Use the formula.

$$\begin{aligned} S &= 4\pi\left(\frac{33}{\pi}\right) \\ &= 132 \text{ ft}^2 \end{aligned}$$

So, the correct choice is H.

ANSWER:

H

52. **SAT/ACT** If a line ℓ is perpendicular to a segment *AB* at *E*, how many points on line ℓ are the same distance from point *A* as from point *B*?
A none
B one
C two
D three
E all points

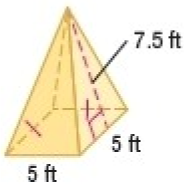
SOLUTION:

Since the segment *AB* is perpendicular to line ℓ, all points on ℓ are the same distance from point *A* as from *B*. The correct choice is E.

ANSWER:

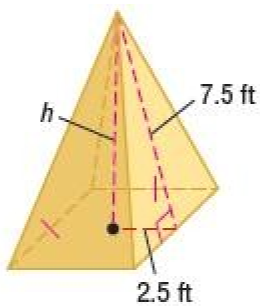
E

Find the volume of each pyramid. Round to the nearest tenth if necessary.

53. 

SOLUTION:

The base of the pyramid is a square with a side of 5 feet. The slant height of the pyramid is 7.5 feet. Use the Pythagorean Theorem to find the height *h*.



.

$$h^2 + 2.5^2 = 7.5^2$$
$$h^2 = 56.25 - 6.25$$
$$h = \sqrt{50} \text{ or } 5\sqrt{2}$$

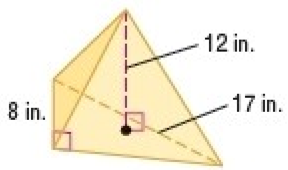
The volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the pyramid.

$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}(5 \times 5)\left(5\sqrt{2}\right)$$
$$\approx 58.9$$

Therefore, the volume of the pyramid is about 58.9 ft^3.

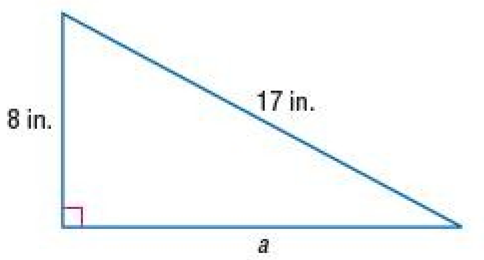
ANSWER:

58.9 ft^3

54. 

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the pyramid. The base of this pyramid is a right triangle with a leg of 8 inches and a hypotenuse of 17 inches. Use the Pythagorean Theorem to find the length of the other leg a.



$$a^2 + 8^2 = 17^2$$
$$a^2 = 289 - 64$$
$$a = \sqrt{225} \text{ or } 15$$

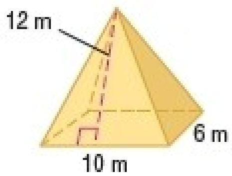
The height of the pyramid is 12 inches.

$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}\left[\frac{1}{2}(15)(8)\right](12)$$
$$= 240$$

Therefore, the volume of the pyramid is 240 in^3.

ANSWER:

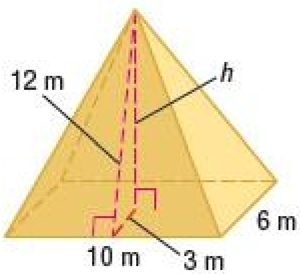
240 in^3



55.

SOLUTION:

The base of this pyramid is a rectangle of length 10 meters and width 6 meters. The slant height of the pyramid is 12 meters. Use the Pythagorean Theorem to find the height *h*.



$$3^2 + h^2 = 12^2$$
$$h^2 = 144 - 9$$
$$h = \sqrt{135} \text{ or } 3\sqrt{15}$$

The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid.

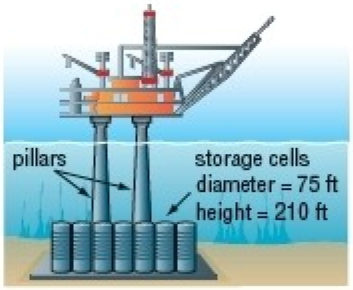
$$\begin{aligned} A &= \frac{1}{3}Bh \\ &= \frac{1}{3}(10 \times 6)(3\sqrt{15}) \\ &\approx 232.4 \end{aligned}$$

Therefore, the volume of the pyramid is about 232.4 m^3.

ANSWER:

232.4 m^3

56. **ENGINEERING** The base of an oil drilling platform is made up of 24 concrete cylindrical cells. Twenty of the cells are used for oil storage. The pillars that support the platform deck rest on the four other cells. Find the total volume of the storage cells.



SOLUTION:

The volume of a cylinder is $V = \pi r^2 h$, where *r* is the radius and *h* is the height of the cylinder.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \left(\frac{75}{2}\right)^2 210 \\ &= 295{,}312.5\pi \text{ ft}^3 \end{aligned}$$

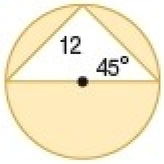
$$\begin{aligned} \text{total volume} &= 20(295{,}312.5\pi) \\ &\approx 18{,}555{,}031.6 \text{ ft}^3 \end{aligned}$$

The total volume of the storage cells is about 18,555,031.6 ft^3.

ANSWER:

18,555,031.6 ft^3

Find the area of each shaded region. Round to the nearest tenth.

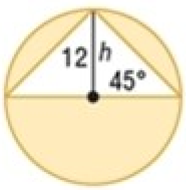
57. 

SOLUTION:

We are given the base of the triangle. Since the triangle is 45°-45°-90°, the height is also 12.

$$\begin{aligned}\text{Area(triangle)} &= \frac{1}{2}bh \\ &= \frac{1}{2}(12)(12) \\ &= 72\end{aligned}$$

The triangle is a 45°-45°-90° triangle, so the diameter of the circle is $12\sqrt{2}$and the radius *h* is $6\sqrt{2}$.

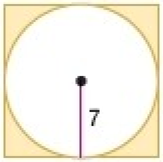


$$\begin{aligned}\text{Area(circle)} &= \pi\left(6\sqrt{2}\right)^2 \\ &= 72\pi\end{aligned}$$

$$\begin{aligned}\text{Area(shaded)} &= \text{Area(circle)} - \text{Area(triangle)} \\ &= 72\pi - 72 \\ &\approx 154.2 \text{ units}^2\end{aligned}$$

ANSWER:

154.2 units^2

58. 

SOLUTION:

The side length of the given square is 2(7) or 14 units.

$$\begin{aligned}\text{Area(square)} &= 14 \cdot 14\\ &= 196\end{aligned}$$

$$\begin{aligned}\text{Area(circle)} &= \pi(7)^2\\ &\approx 153.9\end{aligned}$$

$$\begin{aligned}\text{Area(shaded)} &= \text{Area(circle)} - \text{Area(square)}\\ &\approx 196 - 153.9\\ &\approx 42.1\text{units}^2\end{aligned}$$

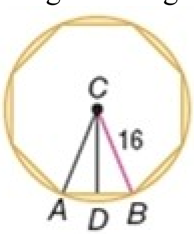
ANSWER:

42.1 units^2

59. 

SOLUTION:

A regular octagon has 8 congruent central angles, so the measure of each central angle is 360 ÷ 8 = 45.



Apothem $\overline{DC}$ is the height of the isosceles triangle ABC. Use the Trigonometric ratios to find the side length and apothem of the polygon.

$$\begin{aligned}\sin x &= \frac{\text{opposite}}{\text{hypotenuse}}\\ \sin 22.5 &= \frac{DB}{BC}\\ \sin 22.5 &= \frac{DB}{16}\\ 16\sin 22.5 &= DB\end{aligned}$$

$$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 22.5 = \frac{DC}{BC}$$
$$\cos 22.5 = \frac{DC}{16}$$
$$16\cos 22.5 = DC$$

$AB = 2(DB) = 32 \sin 22.5$

$$\begin{aligned}\text{Area(octagon)} &= \frac{1}{2}aP \\ &= \frac{1}{2}(CD)(8 \times AB) \\ &= \frac{1}{2}(16\cos 22.5)(8 \times 32\sin 22.5) \\ &\approx 724 \text{ units}^2\end{aligned}$$

$$\begin{aligned}\text{Area(circle)} &= \pi(16)^2 \\ &\approx 804.2\end{aligned}$$

$$\begin{aligned}\text{Area(shaded)} &= \text{Area(circle)} - \text{Area(octagon)} \\ &= 804.2 - 724 \\ &= 80.2 \text{ units}^2\end{aligned}$$

ANSWER:

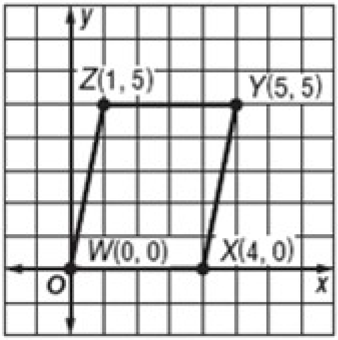
80.2 units^2

COORDINATE GEOMETRY Find the area of each figure.

60. ▱WXYZ with $W(0, 0)$, $X(4, 0)$, $Y(5, 5)$, and $Z(1, 5)$

SOLUTION:

Graph the diagram.



The length of the base of the parallelogram goes from (0, 0) to (0, 4), so it is 4 units.

The height of the parallelogram goes from (1, 0) to (1, 5), so it is 5 units.

The area of a parallelogram is the product of a base b and its corresponding height h. Therefore, the area is 20 units^2.

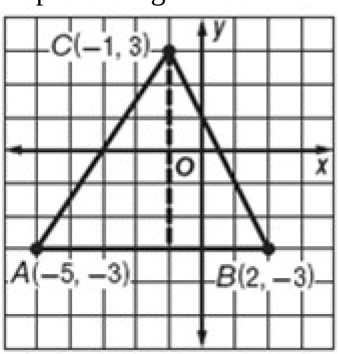
ANSWER:

20 units^2

61. △ABC with $A(2, –3)$, $B(–5, –3)$, and $C(–1, 3)$

SOLUTION:

Graph the diagram.



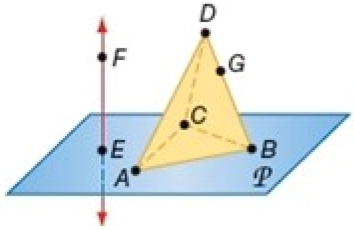
The length of the base of the triangle goes from (–5, –3) to (2, –3), so it is 7 units. The height of the triangle goes from (–1, –3) to (–1, 3), so it is 6 units.

Therefore, the area is 0.5(7)(6) = 21 units^2.

ANSWER:

21 units^2

Refer to the figure.



62. How many planes appear in this figure?

SOLUTION:

A plane is a flat surface made up of points that extends infinitely in all directions. There is exactly one plane through any three points not on the same line.

plane *P*
plane *ABD*
plane *ACD*
plane *ABC*

ANSWER:

4

63. Name three points that are collinear.

SOLUTION:

Collinear points are points that lie on the same line. Noncollinear points do not lie on the same line.

D, *B*, and *G* lie on line *DG*.

ANSWER:

D, B, and G

64. Are points *G*, *A*, *B*, and *E* coplanar? Explain.

SOLUTION:

Coplanar points are points that lie in the same plane. Noncoplanar points do not lie in the same plane.

Points *A*, *B*, and *E* lie in plane *P*, but point *G* does not lie in plane *P*. Thus, they are not coplanar. Points *A*, *G*, and *B* lie in plane *AGB*, but point *E* does not lie in plane *AGB*.

ANSWER:

Points *A*, *B*, and *E* lie in plane *P*, but point *G* does not lie in plane *P*. Thus, they are not coplanar. Points *A*, *G*, and *B* lie in a plane, but point *E* does not lie in plane *AGB*.

65. At what point do $\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ intersect?

SOLUTION:

$\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ do not intersect. $\overleftrightarrow{AB}$ lies in plane *P*, but only *E* lies in *P*. If *E* were on $\overleftrightarrow{AB}$, then they would intersect.

ANSWER:

$\overleftrightarrow{EF}$ and $\overleftrightarrow{AB}$ do not intersect. $\overleftrightarrow{AB}$ lies in plane *P*, but only *E* lies in *P*.