#### Use isometric dot paper to sketch each prism.

1. triangular prism 2 units high, with two sides of the base that are 5 units long and 4 units long

#### SOLUTION:

Mark the corner of the solid. Draw 2 units down, 4 units to the left, and 5 units to the right. Then draw a triangle for the top of the solid.

Draw segments 2 units down from each vertex for the vertical edges. Connect the appropriate vertices using a dashed line for the hidden edge.



#### ANSWER:



2. rectangular prism 2 units high, 3 units wide, and 5 units long

#### SOLUTION:

Mark the front corner of the solid. Draw 2 units down, 3 units to the left, and 5 units to the right. Draw 3 units left from this last point. Then draw a rectangle for the top of the solid.

Draw segments 2 units down from each vertex for the vertical edges. Connect the appropriate vertices using dashed lines for the hidden edges.





#### Use isometric dot paper and each orthographic drawing to sketch a solid.



#### SOLUTION:

top view: There are two rows and three columns. left view: The figure is 4 units high in the back and 3 units high in the front. front view: The dark segment indicates a different height. right view: The figure is 4 units high in the back and 3 units high in the front.

Connect the dots on the isometric dot paper to represent the edges of the solid. Shade the tops of each column.







#### SOLUTION:

top view: There are three rows and three columns. The right-middle column is missing.

left view: The figure is 3 units high.

front view: The figure is 3 units wide.

right view: The dark segments indicate a change in depth, where the right-middle column is missing.

Connect the dots on the isometric dot paper to represent the edges of the solid. Shade the tops of each column.



ANSWER:



- 5. **FOOD** Describe how the cheese can be sliced so that the slices form each shape.
  - a. rectangle
  - **b.** triangle
  - c. trapezoid



#### SOLUTION:

**a.** The cheese slice is in the shape of a triangular prism. The front, right or left view of the cheese slice is a rectangle. So, to get a rectangular shape for the slice, one should cut it vertically.



**b.** The top view of the cheese slice is a triangle. So, to get a triangular shape for the slice, one should cut it horizontally.



**c.** The right or left view of the cheese slice is a rectangle. So, to get a trapezoidal shape for the slice, one should cut it at an angle.



ANSWER:a. slice verticallyb. slice horizontallyc. slice at an angle

Describe each cross section.



SOLUTION:



A vertical plane will cut the prism into two parts with a cross section of a rectangle.

#### ANSWER:

rectangle



A horizontal plane passing through the vertex will cut the cone into two parts with a cross section of a triangle.

ANSWER: triangle

#### Use isometric dot paper to sketch each prism.

8. cube 3 units on each edge

#### SOLUTION:

Mark the front corner of the solid. Draw 3 units down, 3 units to the left, and 3 units to the right. Then draw 3 units left from this last point. Draw a square for the top of the solid.

Draw segments 3 units down from each vertex for the vertical edges. Connect the appropriate vertices using dashed lines for the hidden edges.



٠

.



9. triangular prism 4 units high, with two sides of the base that are 1 unit long and 3 units long

#### SOLUTION:

Mark the corner of the solid. Draw 4 units down, 1 unit to the left, and 3 units to the right. Then draw a triangle for the top of the solid.

Draw segments 4 units down from each vertex for the vertical edges. Connect the appropriate vertices using a dashed line for the hidden edge.



#### ANSWER:



10. triangular prism 4 units high, with two sides of the base that are 2 units long and 6 units long

#### SOLUTION:

Mark the corner of the solid. Draw 4 units down, 6 units to the left, and 2 units to the right. Then draw a triangle for the top of the solid.

Draw segments 4 units down from each vertex for the vertical edges. Connect the appropriate vertices using a dashed line for the hidden edge.



#### ANSWER:

#### Sample answer:



CCSS TOOLS Use isometric dot paper and each orthographic drawing to sketch a solid.



#### SOLUTION:



#### ANSWER:









- 15. ART A piece of clay in the shape of a rectangular prism is cut in half as shown at the right.a. Describe the shape of the cross section.
  - **b.** Describe how the clay could be cut to make the cross section a triangle.





a.



The front view of the prism is a rectangle, so when it is cut vertically, the cross section will be a rectangle.



Three edges meet at each vertex. So, if you cut off a corner of the clay, you get a triangular cross section.

#### ANSWER:

a. rectangleb. Cut off a corner of the clay.

#### Describe each cross section.



16.

#### SOLUTION:

A vertical plane will cut the prism into two parts with a cross section of a rectangle.



ANSWER: rectangle



#### SOLUTION:

The bases of the prism are hexagons. So, a plane parallel to the base will give a cross section of a hexagon.



# ANSWER: hexagon



#### 18.

#### SOLUTION:

The base of the cone is a circle. So, a plane parallel to the base will give a cross section of a circle.



ANSWER: circle



#### SOLUTION:

The two opposite sides of the cross section which comes at the adjacent sides of the prism will be parallel to each other. The other pair of opposite sides which cut the bases of the prism will be non-parallel. Therefore, the cross section will be a trapezoid.



ANSWER: trapezoid

#### 20. ARCHITECTURE Draw a top view, front view, and side view of the house.



#### SOLUTION:

While the depth is different, the top view is two rectangles adjacent to each other resembles a larger rectangle. The front view is a triangle (roof) on top of a rectangle. While the depth is different, the top view is a large rectangle above a smaller rectangle, resembling an even larger rectangle.



**COOKIES** Describe how to make a cut through a roll of cookie dough in the shape of a cylinder to make each shape.

21. circle

#### SOLUTION:

The vertical cross-section of a cylinder is a circle, so you would want to make a vertical cut in a cylinder to create a circle.





#### 22. longest rectangle

#### SOLUTION:

If we cut along the length of the cylinder the result will be a rectangle. The largest rectangle will come from cutting the cylinder along its length, through the center.



# ANSWER:

Make a horizontal cut through the center of the bases.

#### 23. oval

#### SOLUTION:

An oval is an elongated circle. Cut the cookie dough similar to how you would cut it to get a circle, but at an angle.



#### ANSWER: Make an angled cut

#### 24. shorter rectangle

#### SOLUTION:

To get a shorter rectangle, you want to make a cut along the length of the cylinder, but don't cut through the center.



ANSWER: Make a horizontal cut not through the center of the bases.

#### CCSS TOOLS Sketch the cross section from a vertical slice of each figure.



#### SOLUTION:

The cross section from a vertical slice will look just like the front view of the figure, which appears to be a rectangle and triangle. The rectangle represents the front view of the cylinder. The triangle represents the front view of the cone.



ANSWER: Sample answer:



26.

#### SOLUTION:

The cross section from a vertical slice will look just like the front view of the figure, which appears to be a square on top of a rectangle. The rectangle represents the front view of the prism. The square represents the front view of the cube.



ANSWER: Sample answer:





#### SOLUTION:

The cross section from a vertical slice will look just like the front view of the figure, which appears to be a rectangle with a small square cut out of every corner. The square holes represent the indentations in the figure.



## ANSWER: Sample answer:



EARTH SCIENCE Crystals are solids in which the atoms are arranged in regular geometrical patterns. Sketch a cross section from a horizontal slice of each crystal. Then describe the rotational symmetry about the vertical axis.
 a. tetragonal



**b.** hexagonal



c. monoclinic



#### SOLUTION:

#### a.

The cross section from a horizontal slice will look just like the top view of the figure, which appears to be a square.



Like all squares, a  $90^{\circ}$  rotation will produce an image identical to the preimage. The crystal appears the same for every  $90^{\circ}$  rotation about the axis.

**b.** The cross section from a horizontal slice will look just like the top view of the figure, which appears to be a regular hexagon.



Like all regular hexagons, a  $60^{\circ}$  rotation will produce an image identical to the preimage. The crystal appears the same for every  $60^{\circ}$  rotation about the axis.

**c.** The cross section from a horizontal slice will look just like the top view of the figure, which appears to be a rectangle with 2 triangular endpoints.

The crystal appears the same for every 180° rotation about the axis.

ANSWER:





The crystal appears the same for every 90° rotation about the axis.



The crystal appears the same for every 60° rotation about the axis.

c.

The crystal appears the same for every 180° rotation about the axis.

29. ART In a *perspective drawing*, a *vanishing point* is used to make the two-dimensional drawing appear three-dimensional. From one vanishing point, objects can be drawn from different points of view, as shown below.a. Draw a horizontal line and a vanishing point on the line. Draw a rectangle somewhere above the line and use the vanishing point to make a perspective drawing.

**b.** On the same drawing, draw a rectangle somewhere below the line and use the vanishing point to make a perspective drawing.

c. Describe the different views of the two drawings.



#### SOLUTION:

**a.** Draw the rectangle, horizontal line, and vanishing point. Connect the vertices of the rectangle to the vanishing point. The connection that will be hidden from view should be dashed. Draw a second rectangle closer to the vanishing point. Use the connections as the location of the vertices. This rectangle should be dashed.

Sample answer:



**b.** Draw the rectangle, horizontal line, and vanishing point. Connect the vertices of the rectangle to the vanishing point. The connection that will be hidden from view should be dashed. Draw a second rectangle closer to the vanishing point. Use the connections as the location of the vertices. This rectangle should be dashed.

Sample answer:



**c.** Sample answer: The first drawing shows a view of the object from the bottom. The second drawing shows a view of the object from the top.

#### ANSWER:



c. Sample answer: The first drawing shows a view of the object from the bottom. The second drawing shows a view of the object from the top.

#### Draw the top, left, front, and right view of each solid.



30.

SOLUTION:

Top view: One column of blocks is at a different height than the others, so a black bar needs to be shown between these columns to indicate the different height.

Left view: There is no change in depth with this view.

Right view: There is no change in depth with this view.

Front view: One column of blocks is at a different height than the others, so a black bar needs to be shown between these columns to indicate the different height.





SOLUTION:

Top view: Each column of blocks is at a different height than the others, so a black bar needs to be shown in between these columns to indicate the different height.

Left view: There is a different depth here, but it cannot be seen from this view, so 6 blocks are shown with no black bars.

Right view: Each row of blocks is at a different depth than the others, so a black bar needs to be shown in between these rows to indicate the different depth.

Front view: There is no change in depth with this view.





32.

#### SOLUTION:

Top view: The 2 blocks on the bottom left are lower than the others, so a black bar needs to be shown to indicate the different height.

Left view: The 2 blocks on the bottom right are at a different depth than the others, so a black bar needs to be shown to indicate the different depth.

Right view: The 2 blocks on the bottom left are at a different depth than the others, so a black bar needs to be shown to indicate the different depth.

Front view: The blocks on the bottom left are at a different depth than the others, so a black bar needs to be shown to indicate the different depth.



- 33. The top, front, and right views of a three-dimensional figure are shown.
  - **a.** Make a sketch of the solid.
  - **b.** Describe two different ways that a rectangular cross section can be made.
  - c. Make a connection between the front and right views of the solid and cross sections of the solid.



#### SOLUTION:

a.

top view: The dark segment indicates a different height. front view: The right side is higher than the left side. right view: The top piece, on the right side, has a curved top.

Connect the dots on the isometric dot paper to represent the edges of the solid. Shade the tops of each column. Sample answer:



**b.** Make a horizontal cut through the bottom part of the figure or make a vertical cut through the left side of the figure.

**c.** The front view of the solid is the cross section when a vertical cut is made lengthwise. The right view of the solid is the cross section when a vertical cut is made through the right side of the figure.

#### ANSWER:

**a.** Sample answer:



**b.** Make a horizontal cut through the bottom part of the figure or make a vertical cut through the left side of the figure.

**c.** The front view of the solid is the cross section when a vertical cut is made lengthwise. The right view of the solid is the cross section when a vertical cut is made through the right side of the figure.

#### 34. MULTIPLE REPRESENTATIONS In this problem, you will investigate isometric drawings.

a. GEOMETRIC Create isometric drawings of three different solids.

**b. TABULAR** Create a table that includes the number of cubes needed to construct the solid and the number of squares visible in the isometric drawing.

**c. VERBAL** Is there a correlation between the number of cubes needed to construct a solid and the number of squares visible in the isometric drawing? Explain.



SOLUTION:

a. Create three drawings that are different in size, shape, and the number of cubes used.

#### Sample answer:



b. Tally the number of cubes for each drawing. Then tally the number of squares that you can see.

Number of Cubes	Number of Squares
6	11
10	15
12	18

**c.** For the top drawing, we could move the cube in the very front and put it to the right of the cube 2 places behind it. We would still have 6 cubes, but we would then have 12 squares.

Sample answer: No; the number of squares in the isometric drawing will depend on the arrangement of the cubes that creates the figure.

#### ANSWER:

a. Sample answer:



-	•		
	,		
		3	Э

Number of Cubes	Number of Squares
6	11
10	15
12	18

**c.** Sample answer: No; the number of squares in the isometric drawing will depend on the arrangement of the cubes that creates the figure.

35. **CHALLENGE** The figure is a cross section of a geometric solid. Describe a solid and how the cross section was made.



SOLUTION:

Sample answer: The cross section could be created by slicing a cone at an angle through its lateral side and base as shown below.



#### ANSWER:

Sample answer: A cone is sliced at an angle through its lateral side and base.

#### 36. CCSS ARGUMENTS Determine whether the following statement is *true* or *false*. Explain your reasoning.

If the left, right, front, and back orthographic views of two objects are the same, then the objects are the same figure.

#### SOLUTION:

False; sample answer: Two solids could have the same left, right, back, and front views but be different solids. A solid made out of 3 layers of 9 blocks (3 x 3) and a similar solid missing either a stack of the two (or four) center blocks on one of the sides. All views are 3 x 3 squares. The top views would be different.

#### ANSWER:

False; sample answer: Two solids could have the same left, right, back, and front views but be different solids. A solid made out of 3 layers of 9 blocks  $(3 \times 3)$  and a similar solid missing either a stack of the two (or four) center blocks on one of the sides. All views are  $3 \times 3$  squares. The top views would be different.

37. **OPEN ENDED** Use isometric dot paper to draw a solid consisting of 12 cubic units. Then sketch the orthographic drawing for your solid.

#### SOLUTION:

Use 12 blocks. For the sample answer, we have two squares of 4 blocks each, sitting on top of each other. Then we have 2 sets of 2 blocks on top of the squares. Note that the front left view in the image represents the front view of the solid.

For the top view: There is a dark segment in the middle to represent the different heights.

For the left view: The 4-block-high column is in the back, so in the left view it will be on the left side with the 2-block column on the right.

For the front view: The dark segment in the middle represents the different depths.

For the right view: The 4-block-high column is in the back, so in the right view it will be on the right side with the 2-block column on the left.



38. CHALLENGE Draw the top view, front view, and left view of the solid figure at the right.



SOLUTION:

.

Top view: The 2 blocks in the middle left have holes, so a black bar needs to be shown to indicate the different depth.

Left view: The middle block and the top right block are both at different depth, so a black bar needs to be placed.

Front view: The 2 blocks in the middle top have more depth, so a black bar needs to be shown to indicate the different depth.



39. WRITING IN MATH A hexagonal pyramid is sliced through the vertex and the base so that the prism is separated into two congruent parts. Describe the cross section. Is there more than one way to separate the figure into two congruent parts? Will the shape of the cross section change? Explain.



If the regular hexagonal pyramid above is sliced in half through the vertex and through the blue dotted line connecting opposite vertices of the base, the cross section is a triangle as shown in the figure on the right. There are six different ways to slice the pyramid so that two equal parts are formed because the figure has six planes of symmetry. Each slice must go through the vertex of the pyramid and one of the six lines of symmetry for the hexagonal base as shown below. In each case, the cross section is an isosceles triangle. Only the side lengths of the triangles change. When the slice goes through a line of symmetry passing through two vertices of the hexagon, then two sides of the triangular cross section will have a length equal to the length of the sides of the hexagon, then two sides of the triangular cross section will have a length equal to the height of the triangular faces of the pyramid.



#### ANSWER:

The cross section is a triangle. There are six different ways to slice the pyramid so that two equal parts are formed because the figure has six planes of symmetry. In each case, the cross section is an isosceles triangle. Only the side lengths of the triangles change.

40. Which polyhedron is represented by the net shown below?



A cubeB octahedronC triangular prismD triangular pyramid

#### SOLUTION:

The net consists of 4 equilateral triangles. Joining the vertices of the larger triangle we will get a triangular pyramid Therefore, the correct choice is D.

ANSWER:

D

41. **EXTENDED RESPONSE** A homeowner wants to build a 3-foot-wide deck around his circular pool as shown below.



a. Find the outer perimeter of the deck to the nearest foot, if the circumference of the pool is about 81.64 feet.

**b.** What is the area of the top of the deck?

#### SOLUTION:

**a.** The length of each side of the inner side of the deck is equal to the diameter of the circle. Use the formula to find the diameter of the circle. circumference =  $\pi d$ 

$$81.64 = \pi d$$
$$\frac{81.64}{\pi} = d$$
$$26 \approx d$$

The width of the desk is 3 ft, so the length of each side of the outer side of the desk is 26 + 3 + 3 = 32 ft.

The perimeter of the outer side of the desk is 4(32) = 128 ft.

**b.** The area of the deck is the difference between the areas of the two squares.  $A = (32)^2 - (26)^2 = 1024 - 676 = 348$ 

#### ANSWER:

a. inner side of deck = circumference of pool =  $81.64 \div \pi \approx 26$  ft; outer side of deck = 26 + 3 + 3 = 32 ft; outer perimeter of deck =  $4 \times 32 = 128$  ft; b. area of deck =  $(2 \times 3 \times 32) + (2 \times 3 \times 26) = 348$  square feet

42. ALGEBRA Which inequality best describes the graph shown below?



#### SOLUTION:

The slope of the line is  $\frac{2}{3}$  and the *y*-intercept is -1. So, the equation of the line is  $y = \frac{2}{3}x - 1$ . The border is a dotted line, so the inequality symbol is < or >. The area below the line is shaded, so the inequality sign is <. Therefore, the inequality best describes the graph is  $y < \frac{2}{3}x - 1$ . The correct choice is F.

#### ANSWER:

F

43. SAT/ACT Expand  $(4\sqrt{5})^2$ . A 20 B  $8\sqrt{5}$ C  $16\sqrt{5}$ D 40 E 80 SOLUTION:  $(4\sqrt{5})^2 = 4\sqrt{5} \cdot 4\sqrt{5}$   $= 4 \cdot 4 \cdot \sqrt{5} \cdot \sqrt{5}$   $= 16 \cdot 5$  = 80The correct choice is E. ANSWER: For each pair of similar figures, use the given areas to find the scale factor from the blue to the green figure. Then find *x*.



SOLUTION:

The scale factor between the green trapezoid and the blue trapezoid is  $\frac{x}{12}$ , so the ratio of their areas is  $\left(\frac{x}{12}\right)^2$ .

Area of the green trapezoid  
Area of the blue trapezoid 
$$= \left(\frac{x}{12}\right)^2$$
  
 $\frac{25}{36} = \frac{x^2}{144}$   
 $144x^2 = 144 \cdot 25$   
 $x^2 = \frac{144 \cdot 25}{36}$   
 $x^2 = 100$   
 $x = 10 \text{ cm}$   
The scale factor is  $\frac{10}{12}$  or  $\frac{5}{6}$ .

ANSWER:

 $\frac{5}{6};10$ 



#### SOLUTION:

The scale factor between the green triangle and the blue triangle is  $\frac{x}{6}$ , so the ratio of their areas is  $\left(\frac{x}{6}\right)^2$ .

$$\frac{\text{Area of the green triangle}}{\text{Area of the blue triangle}} = \left(\frac{x}{6}\right)^2$$

$$\frac{108}{48} = \frac{x^2}{36}$$

$$48x^2 = 108 \cdot 36$$

$$x^2 = \frac{108 \cdot 36}{48}$$

$$x^2 = 81$$

$$x = 9 \text{ in}$$
The scale factor is  $\frac{9}{6}$  or  $\frac{3}{2}$ .

#### ANSWER:

 $\frac{3}{2};9$ 



SOLUTION:

The scale factor between the green parallelogram and the blue parallelogram is  $\frac{x}{22}$ , so the ratio of their areas is

 $\left(\frac{x}{22}\right)^{2}$ Area of the green parallelogram  $= \left(\frac{x}{22}\right)^{2}$   $\frac{98}{242} = \frac{x^{2}}{484}$   $242x^{2} = 484 \cdot 98$   $x^{2} = \frac{484 \cdot 98}{242}$   $x^{2} = 196$  x = 14 mThe scale factor is  $\frac{14}{22}$  or  $\frac{7}{11}$ .

ANSWER:

 $\frac{7}{11}$ ;14

47. **FURNITURE DESIGN** Jenna wants to cover the cushions of her papasan chair with new fabric. There are seven congruent circular cushions with a diameter of 12 inches around a center cushion with a diameter of 20 inches. Find the area of fabric in square yards that she will need to cover both sides of the cushions. Allow an extra 3 inches of fabric around each cushion.



#### SOLUTION:

To cover both sides of the smaller cushions with an extra 3 inches of fabric, Jenna will need to cut  $7 \times 2$  or 14 circular pieces of fabric with a radius of  $\frac{12}{2} + 3$  or 9 inches. To cover both sides of the center cushion, she will need to cut two circular pieces that have a radius of  $\frac{20}{2} + 3$  or 13 inches.

$$A_{\text{fabric}} = 14 \left[ \pi(9)^2 \right] + 2 \left[ \pi(13)^2 \right] \\ = 1134\pi + 338\pi \\ = 1472\pi$$

So, Jenna needs  $1472\pi in^2$  of fabric to cover the pillows.

Use dimensional analysis to convert the area to square yards.

$$\frac{1472\pi \text{ in}^2}{1} \cdot \frac{1\text{yd}}{36 \text{ in}} \cdot \frac{1\text{yd}}{36 \text{ in}} \approx 3.6 \text{yd}^2$$

Therefore, Jenna will need about  $3.6 \text{ yd}^2$  of fabric to cover both sides of all the cushions.

#### ANSWER:

about  $3.6 \text{ yd}^2$ 

Find the perimeter or circumference and area of each figure. Round to the nearest tenth.



ANSWER:

38 cm; 88 cm<sup>2</sup>



SOLUTION:  $C = 2\pi r$   $= 2\pi (4.6)$   $\approx 28.9$   $A = \pi r^{2}$   $= \pi (4.6)^{2}$   $\approx 66.5$ 

## ANSWER:

28.9 in.; 66.5 in<sup>2</sup>



36 m; 54 m<sup>2</sup>