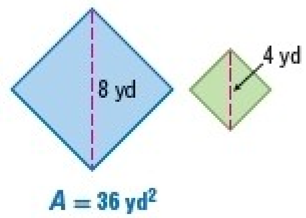
## 11-5 Areas of Similar Figures

**For each pair of similar figures, find the area of the green figure.**

1. 


*SOLUTION:*

The scale factor between the blue diamond and the green diamond is $\frac{8}{4} = \frac{2}{1}$, so the ratio of their areas is $\left(\frac{2}{1}\right)^2$.

$$\frac{\text{Area of the blue diamond}}{\text{Area of the green diamond}} = \left(\frac{2}{1}\right)^2$$

$$\frac{36}{\text{Area of the green diamond}} = 4$$

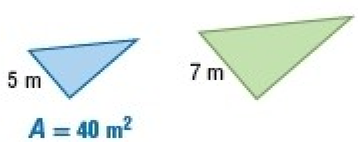
$$\text{Area of the green diamond} = \frac{36}{4}$$

$$\text{Area of the green diamond} = 9$$

The area of the green diamond is 9 $yd^2$.

*ANSWER:*

$9\ yd^2$

2. 


*SOLUTION:*

The scale factor between the blue triangle and the green triangle is $\frac{5}{7}$, so the ratio of their areas is $\left(\frac{5}{7}\right)^2$.

$$\frac{\text{Area of the blue triangle}}{\text{Area of the green triangle}} = \left(\frac{5}{7}\right)^2$$

$$\frac{40}{\text{Area of the green triangle}} = \frac{25}{49}$$

$$\text{Area of the green triangle} = \frac{49 \cdot 40}{25}$$

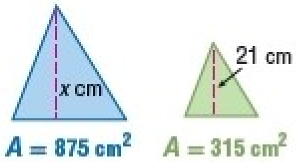
$$\text{Area of the green triangle} = 78.4$$

The area of the green triangle is 78.4 $m^2$.

*ANSWER:*

$78.4\ m^2$

**For each pair of similar figures, use the given areas to find the scale factor from the blue to the green figure. Then find *x*.**

3. 


*SOLUTION:*

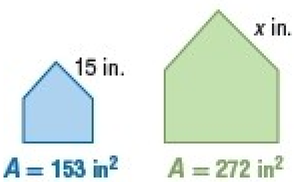
The scale factor between the blue triangle and the green triangle is $\frac{x}{21}$, so the ratio of their areas is $\left(\frac{x}{21}\right)^2$.

$$\frac{\text{Area of the blue triangle}}{\text{Area of the green triangle}} = \left(\frac{x}{21}\right)^2$$
$$\frac{875}{315} = \frac{x^2}{441}$$
$$315x^2 = 875 \cdot 441$$
$$x^2 = \frac{875 \cdot 441}{315}$$
$$x^2 = 1225$$
$$x = 35 \text{ cm}$$

The scale factor is $\frac{35}{21}$ or $\frac{5}{3}$.

*ANSWER:*

$\frac{5}{3}$; 35

4. 


*SOLUTION:*

The scale factor between the blue trapezoid and the green trapezoid is $\frac{15}{x}$, so the ratio of their areas is $\left(\frac{15}{x}\right)^2$.

$$\frac{\text{Area of the blue trapezoid}}{\text{Area of the green trapezoid}} = \left(\frac{15}{x}\right)^2$$
$$\frac{153}{272} = \frac{225}{x^2}$$
$$153x^2 = 225 \cdot 272$$
$$x^2 = \frac{225 \cdot 272}{153}$$
$$x^2 = 400$$
$$x = 20 \text{ in}$$

The scale factor is $\frac{15}{20}$ or $\frac{3}{4}$.

*ANSWER:*

$\frac{3}{4}$; 20

## 11-5 Areas of Similar Figures

5. **MEMORIES** Zola has a picture frame that holds all of her school pictures. Each small opening is similar to the large opening in the center. If the center opening has an area of 33 square inches, what is the area of each small opening?


*SOLUTION:*

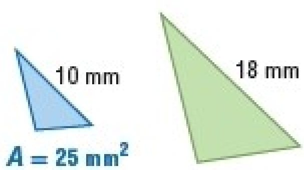
The scale factor between the center opening and the small opening is $\frac{3}{1.2}$, so the ratio of their areas is $\left(\frac{3}{1.2}\right)^2$.

$$\frac{\text{Area of the large opening}}{\text{Area of each small opening}} = \left(\frac{3}{1.2}\right)^2$$
$$\frac{33}{\text{Area of each small opening}} = \frac{9}{1.44}$$
$$\text{Area of each small opening} = 5.28 \text{ in}^2$$

*ANSWER:*

$5.28 \text{ in}^2$

**For each pair of similar figures, find the area of the green figure.**

6. 


*SOLUTION:*

The scale factor between the blue triangle and the green triangle is $\frac{10}{18}$ or $\frac{5}{9}$, so the ratio of their areas is $\left(\frac{5}{9}\right)^2$.

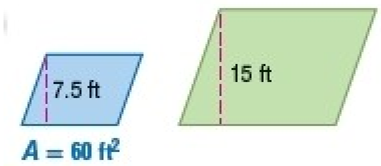
$$\frac{\text{Area of the blue triangle}}{\text{Area of the green triangle}} = \left(\frac{5}{9}\right)^2$$
$$\frac{25}{\text{Area of the green triangle}} = \frac{25}{81}$$
$$\text{Area of the green triangle} = \frac{25 \times 81}{25}$$
$$\text{Area of the green triangle} = 81$$

The area of the green triangle is 81 $\text{mm}^2$.

*ANSWER:*

$81 \text{ mm}^2$

## 11-5 Areas of Similar Figures

7. 


*SOLUTION:*

The scale factor between the blue parallelogram and the green parallelogram is $\frac{7.5}{15}$ or $\frac{1}{2}$, so the ratio of their areas is $\left(\frac{1}{2}\right)^2$.

$$\frac{\text{Area of the blue parallelogram}}{\text{Area of the green parallelogram}} = \left(\frac{1}{2}\right)^2$$

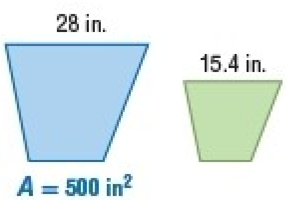
$$\frac{60}{\text{Area of the green parallelogram}} = \frac{1}{4}$$

$$\text{Area of the green parallelogram} = 240$$

The area of the green parallelogram is 240 $\text{ft}^2$.

*ANSWER:*

240 $\text{ft}^2$

8. 


*SOLUTION:*

The scale factor between the blue trapezoid and the green trapezoid is $\frac{28}{15.4}$, so the ratio of their areas is $\left(\frac{28}{15.4}\right)^2$.

$$\frac{\text{Area of the blue trapezoid}}{\text{Area of the green trapezoid}} = \left(\frac{28}{15.4}\right)^2$$

$$\frac{500}{\text{Area of the green trapezoid}} = \frac{784}{237.16}$$

$$\text{Area of the green trapezoid} = 151.25$$

The area of green triangle is 151.25 $\text{in}^2$.

*ANSWER:*

151.25 $\text{in}^2$

9. 


*SOLUTION:*

The scale factor between the blue pentagon and the green pentagon is $\frac{35}{28}$ or $\frac{5}{4}$, so the ratio of their areas is $\left(\frac{5}{4}\right)^2$.

$$\frac{\text{Area of the blue pentagon}}{\text{Area of the green pentagon}} = \left(\frac{5}{4}\right)^2$$

$$\frac{1050}{\text{Area of the green pentagon}} = \frac{25}{16}$$

$$\text{Area of the green pentagon} = 672$$

The area of green pentagon is 672 $cm^2$.

*ANSWER:*

672 $cm^2$

**CCSS STRUCTURE For each pair of similar figures, use the given areas to find the scale factor of the blue to the green figure. Then find *x*.**

10. 


*SOLUTION:*

The scale factor between the blue figure and the green figure is $\frac{12}{x}$, so the ratio of their areas is $\left(\frac{12}{x}\right)^2$.

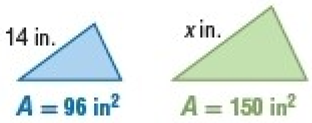
$$\frac{\text{Area (blue)}}{\text{Area (green)}} = \left(\frac{12}{x}\right)^2$$
$$\frac{72}{50} = \frac{144}{x^2}$$
$$72x^2 = 144 \cdot 50$$
$$x^2 = \frac{144 \cdot 50}{72}$$
$$x^2 = 100$$
$$x = 10 \text{ m}$$

The scale factor is $\frac{12}{10}$ or $\frac{6}{5}$.

*ANSWER:*

$\frac{6}{5}$;10

# 11-5 Areas of Similar Figures

11. 


*SOLUTION:*

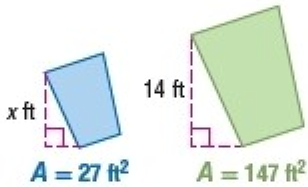
The scale factor between the blue triangle and the green triangle is $\frac{14}{x}$, so the ratio of their areas is $\left(\frac{14}{x}\right)^2$.

$$\begin{aligned}\frac{\text{Area (blue)}}{\text{Area (green)}} &= \left(\frac{14}{x}\right)^2\\ \frac{96}{150} &= \frac{196}{x^2}\\ 96x^2 &= 196 \cdot 150\\ x^2 &= \frac{196 \cdot 150}{96}\\ x^2 &= 306.25\\ x &= 17.5 \text{ in}\end{aligned}$$

The scale factor is $\frac{14}{17.5}$ or $\frac{4}{5}$.

*ANSWER:*

$\frac{4}{5}$;17.5

12. 


*SOLUTION:*

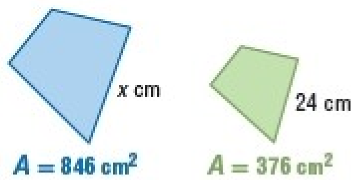
The scale factor between the blue figure and the green figure is $\frac{x}{14}$, so the ratio of their areas is $\left(\frac{x}{14}\right)^2$.

$$\begin{aligned}\frac{\text{Area (blue)}}{\text{Area (green)}} &= \left(\frac{x}{14}\right)^2 \\ \frac{27}{147} &= \frac{x^2}{196} \\ 147x^2 &= 196 \cdot 27 \\ x^2 &= \frac{196 \cdot 27}{147} \\ x^2 &= 36 \\ x &= 6 \text{ ft}\end{aligned}$$

The scale factor is $\frac{6}{14}$ or $\frac{3}{7}$.

*ANSWER:*

$\frac{3}{7}$;6

13. 


*SOLUTION:*

The scale factor between the blue figure and the green figure is $\frac{x}{24}$, so the ratio of their areas is $\left(\frac{x}{24}\right)^2$.

$$\frac{\text{Area (blue)}}{\text{Area (green)}} = \left(\frac{x}{24}\right)^2$$
$$\frac{846}{376} = \frac{x^2}{576}$$
$$376x^2 = 576 \cdot 846$$
$$x^2 = \frac{576 \cdot 846}{376}$$
$$x = 36\,\text{m}$$

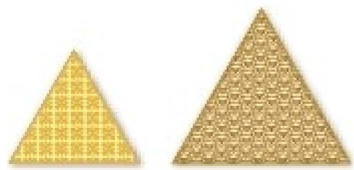
The scale factor is $\frac{36}{24}$ or $\frac{3}{2}$.

*ANSWER:*

$\frac{3}{2}$;36

## 11-5 Areas of Similar Figures

14. **CRAFTS** Marina crafts unique trivets and other kitchenware. Each trivet is an equilateral triangle. The perimeter of the small trivet is 9 inches, and the perimeter of the large trivet is 12 inches. If the area of the small trivet is about 3.9 square inches, what is the approximate area of the large trivet?



*SOLUTION:*

Since the given triangles are equilateral, the sides are congruent.
The perimeter of the small trivet is 9 inches, so the length of each side of the small trivet is 3 inches.
The perimeter of the large trivet is 12 inches, so the length of each side of the large trivet is 4 inches.

The scale factor between the small trivet and the large trivet is $\frac{3}{4}$, so the ratio of their areas is $\left(\frac{3}{4}\right)^2$.

$$\frac{\text{Area of small trivet}}{\text{Area of large trivet}} = \left(\frac{3}{4}\right)^2$$
$$\frac{3.9}{\text{Area of large trivet}} = \frac{9}{16}$$
$$\text{Area of large trivet} \approx 6.9$$

So, the area of large trivet is 6.9 $\text{in}^2$.

*ANSWER:*

6.9 $\text{in}^2$

15. **BAKING** Kaitlyn wants to use one of two regular hexagonal cake pans for a recipe she is making. The side length of the larger pan is 4.5 inches, and the area of the base of the smaller pan is 41.6 square inches.
**a.** What is the side length of the smaller pan?
**b.** The recipe that Kaitlyn is using calls for a circular cake pan with an 8-inch diameter. Which pan should she choose? Explain your reasoning.

*SOLUTION:*

**a.** Area of the smaller pan = 41.6

We know that the area of the regular hexagon is $A = \frac{1}{2}a(ns)$ where *s* is the length of the side, *A* is the area, and *n* is the number of sides.

The interior angle of a hexagon is 60° and the apothem bisects the side length creating a 30°-60°-90° triangle with a shorter leg that measures $\frac{s}{2}$. The length of the apothem is therefore $\frac{s\sqrt{3}}{2}$.

$$A = \frac{1}{2} \cdot \frac{s\sqrt{3}}{2}(6s)$$
$$41.6 = \frac{3\sqrt{3}}{2}s^2$$
$$s^2 = 16.01$$
$$s \approx 4$$

**b.** The area of the circular pie pan is

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(4)^2 \\ &= 16\pi \\ &\approx 50 \end{aligned}$$

The area of the smaller pan is given 41.6 in$^2$, and the area of the larger pan can be determined using ratios.

$$\frac{\text{Area of the larger pan}}{\text{Area of the smaller pan}} = \left(\frac{\text{larger side legnth}}{\text{smaller side length}}\right)^2$$
$$\frac{\text{Area of the larger pan}}{41.6} = \left(\frac{4.5}{4}\right)^2$$
$$\text{Area of the larger pan} = \left(\frac{4.5}{4}\right)^2 \cdot 41.6$$
$$\text{Area of the larger pan} = 52.6$$

The larger pan should be used since the larger pan is closer to the area of the circular pan.

*ANSWER:*

**a.** 4 in.

**b.** Larger; sample answer: The area of a circular pie pan with an 8 in. diameter is about $50 \text{ in}^2$. The area of the larger pan is $52.6 \text{ in}^2$, and the area of the smaller pan is $41.6 \text{ in}^2$. The area of the larger pan is closer to the area of the circle, so Kaitlyn should choose the larger pan to make the recipe.

16. **CHANGING DIMENSIONS** A polygon has an area of 144 square meters.
**a.** If the area is doubled, how does each side length change?
**b.** How does each side length change if the area is tripled?
**c.** What is the change in each side length if the area is increased by a factor of *x*?

*SOLUTION:*

**a.** Let $A$ be the original area. The the new area is $2A$. From theorem 11.1:

$$\frac{A_{\text{new}}}{A} = \left(\frac{s_{\text{new}}}{s}\right)^2$$
$$\frac{2A}{A} = \left(\frac{s_{\text{new}}}{s}\right)^2$$
$$\frac{s_{\text{new}}}{s} = \sqrt{2}$$

If the area is doubled, each side length will increase by a factor of $\sqrt{2}$.

**b.** Let $A$ be the original area. The the new area is $3A$. From theorem 11.1:

$$\frac{A_{\text{new}}}{A} = \left(\frac{s_{\text{new}}}{s}\right)^2$$
$$\frac{3A}{A} = \left(\frac{s_{\text{new}}}{s}\right)^2$$
$$\frac{s_{\text{new}}}{s} = \sqrt{3}$$

If the area is tripled, each side length will increase by a factor of $\sqrt{3}$.

**c.** Let $A$ be the original area. The the new area is $xA$. From theorem 11.1:

$$\frac{A_{\text{new}}}{A} = \left(\frac{s_{\text{new}}}{s}\right)^2$$
$$\frac{xA}{A} = \left(\frac{s_{\text{new}}}{s}\right)^2$$
$$\frac{s_{\text{new}}}{s} = \sqrt{x}$$

If the area changes by a factor of *x*, then each side length will change by a factor of $\sqrt{x}$.

*ANSWER:*

**a.** If the area is doubled, each side length will increase by a factor of $\sqrt{2}$.
**b.** If the area is tripled, each side length will increase by a factor of $\sqrt{3}$.
**c.** If the area changes by a factor of *x*, then each side length will change by a factor of $\sqrt{x}$.

17. **CHANGING DIMENSIONS** A circle has a radius of 24 inches.
**a.** If the area is doubled, how does the radius change?
**b.** How does the radius change if the area is tripled?
**c.** What is the change in the radius if the area is increased by a factor of *x*?

*SOLUTION:*

**a.** Let $A$ be the area. The new area is $2A$. From theorem 11.1:

$$\frac{A_{\text{new}}}{A} = \left(\frac{r_{\text{new}}}{r}\right)^2$$
$$\frac{2A}{A} = \left(\frac{r_{\text{new}}}{r}\right)^2$$
$$\frac{r_{\text{new}}}{r} = \sqrt{2}$$
$$r_{\text{new}} = \sqrt{2}r$$
$$r_{\text{new}} = \sqrt{2}(24)$$
$$r_{\text{new}} = 33.9$$

If the area is doubled, the radius changes from 24 in. to 33.9 in.

**b.** Let $A$ be the area. The new area is $3A$. From theorem 11.1:

$$\frac{A_{\text{new}}}{A} = \left(\frac{r_{\text{new}}}{r}\right)^2$$
$$\frac{3A}{A} = \left(\frac{r_{\text{new}}}{r}\right)^2$$
$$\frac{r_{\text{new}}}{r} = \sqrt{3}$$
$$r_{\text{new}} = \sqrt{3}r$$
$$r_{\text{new}} = \sqrt{3}(24)$$
$$r_{\text{new}} = 41.6$$

If the area is tripled, the radius changes from 24 in. to 41.6 in.

**c.** Let $A$ be the area. The new area is $xA$. From theorem 11.1:

$$\frac{A_{\text{new}}}{A} = \left(\frac{r_{\text{new}}}{r}\right)^2$$
$$\frac{xA}{A} = \left(\frac{r_{\text{new}}}{r}\right)^2$$
$$\frac{r_{\text{new}}}{r} = \sqrt{x}$$
$$r_{\text{new}} = \sqrt{x}r$$
$$r_{\text{new}} = 24\sqrt{x}$$

If the area changes by a factor of $x$, then the radius changes from 24 in. to $24\sqrt{x}$ in.

*ANSWER:*

**a.** If the area is doubled, the radius changes from 24 in. to 33.9 in.
**b.** If the area is tripled, the radius changes from 24 in. to 41.6 in.
**c.** If the area changes by a factor of $x$, then the radius changes from 24 in. to $24\sqrt{x}$ in.

18. **CCSS MODELING** Federico's family is putting hardwood floors in the two geometrically similar rooms shown. If the cost of flooring is constant and the flooring for the kitchen cost $2000, what will be the total flooring cost for the two rooms? Round to the nearest hundred dollars.


*SOLUTION:*

The scale factor between the living room and kitchen is $\frac{40}{24}$ or $\frac{5}{3}$, so the ratio of their areas is $\left(\frac{5}{3}\right)^2$.

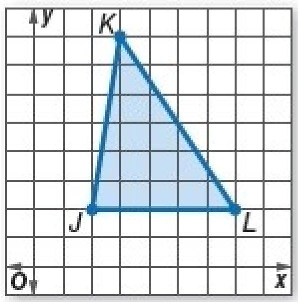
$$\begin{aligned}\frac{\text{Area (L)}}{\text{Area (K)}} &= \left(\frac{5}{3}\right)^2 \\ \frac{\text{Cost of flooring (L)}}{\text{Cost of flooring (K)}} &= \left(\frac{5}{3}\right)^2 \\ \frac{\text{Cost of flooring (L)}}{2000} &= \frac{25}{9} \\ 9[\text{Cost of flooring (L)}] &= 25(2000) \\ \text{Cost of flooring (L)} &= \frac{25(2000)}{9} \\ \text{Cost of flooring (L)} &\approx 5555.6\end{aligned}$$

$$\begin{aligned}\text{Totalcost} &\approx 5555.6 + 2000 \\ &= 7555.6 \\ &\approx \$7600\end{aligned}$$

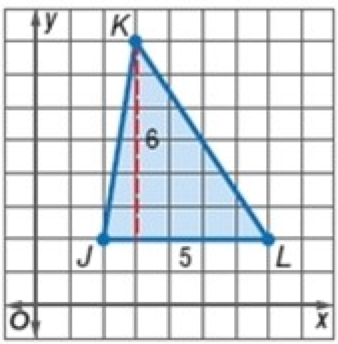
*ANSWER:*
$7600

**COORDINATE GEOMETRY Find the area of each figure. Use the segment length given to find the area of a similar polygon.**

19. $J'L' = 3$




*SOLUTION:*




Area of triangle *JKL* = 0.5(5)(6) or 15

The scale factor between $\Delta JKL$ and $\Delta J'K'L'$ is $\frac{JL}{J'L'}$ or $\frac{5}{3}$, so the ratio of their areas is $\left(\frac{5}{3}\right)^2$.

$$\frac{\text{Area of } \Delta JKL}{\text{Area of } \Delta J'K'L'} = \left(\frac{5}{3}\right)^2$$

$$\frac{15}{\text{Area of } \Delta J'K'L'} = \frac{25}{9}$$

$$25 \cdot \text{Area of } \Delta J'K'L' = 15 \cdot 9$$

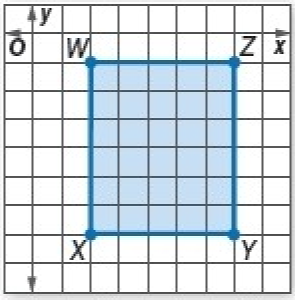
$$\text{Area of } \Delta J'K'L' = \frac{15 \cdot 9}{25}$$

$$\text{Area of } \Delta J'K'L' = 5.4$$

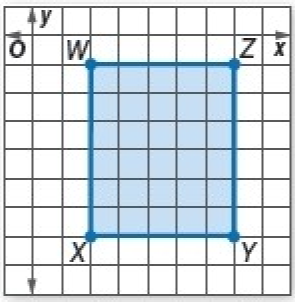
*ANSWER:*

area of $\Delta JKL = 15$; area of $\Delta J'K'L' \approx 5.4$

20. $WX' = 8$




*SOLUTION:*




Here, *XY* = *WZ* = 5 and *WX* = *YZ* = 6.
Area of the rectangle *WXYZ* = 5(6) or 30.
Given that $WX' = 8$.

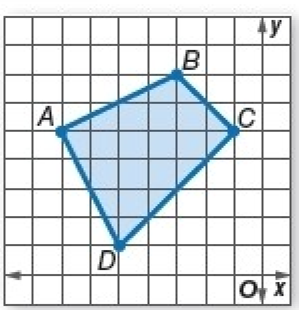
The scale factor between rectangle $WXYZ$ and quadrilateral $W'X'Y'Z'$ is $\frac{WX}{W'X'} = \frac{6}{8}$ or $\frac{3}{4}$, so the ratio of their areas is $\left(\frac{3}{4}\right)^2$.

$$\frac{\text{Area of } WXYZ}{\text{Area of } W'X'Y'Z'} = \left(\frac{3}{4}\right)^2$$
$$\frac{30}{\text{Area of } W'X'Y'Z'} = \frac{9}{16}$$
$$9 \cdot \text{Area of } W'X'Y'Z' = 30 \cdot 16$$
$$\text{Area of } W'X'Y'Z' = \frac{30 \cdot 16}{9}$$
$$\text{Area of } W'X'Y'Z' \approx 53.3$$

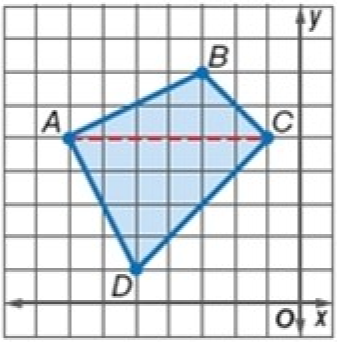
*ANSWER:*

area of *WXYZ* = 30; area of $W'X'Y'Z' \approx 53.3$

21. $B'C' = 5$



*SOLUTION:*




Area of a triangle = ½ $bh$
Substitute.

Area of triangle $ABC = \frac{1}{2}(6)(2)$ or 6.

Area of triangle $ADC = \frac{1}{2}(6)(4)$ or 12.

Area of quadrilateral $ABCD$ = Area of triangle $ABC$ + Area of triangle $ADC$
= 6 + 12
= 18

For the length of BC use the distance formula. $B = (–3, 7)$ and $C = (–1, 5)$.

$$\begin{aligned}\overline{BC} &= \sqrt{(-3-(-1))^2+(7-5)^2}\\ &= \sqrt{2^2+2^2}\\ &= \sqrt{4+4}\\ &= \sqrt{8}\end{aligned}$$

Here, $BC$ is $\sqrt{8}$.

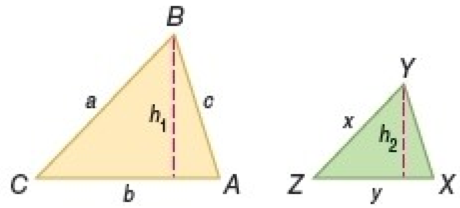
The scale factor between quadrilateral $ABCD$ and quadrilateral $A'B'C'D'$ is $\frac{BC}{B'C'}$ or $\frac{\sqrt{8}}{5}$, so the ratio of their areas is $\left(\frac{\sqrt{8}}{5}\right)^2$.

$$\begin{aligned}\frac{\text{Area of quadrilateral } ABCD}{\text{Area of quadrilateral } A'B'C'D'} &= \left(\frac{\sqrt{8}}{5}\right)^2\\ \frac{18}{\text{Area of } A'B'C'D'} &= \frac{8}{25}\\ 8\cdot\text{Area of } A'B'C'D' &= 18\cdot 25\\ \text{Area of } A'B'C'D' &= \frac{18\cdot 25}{8}\\ \text{Area of } A'B'C'D' &= 56.2\end{aligned}$$

*ANSWER:*

area of *ABCD* = 18; area of $A'B'C'D' \approx 56.2$

22. **PROOF** Write a paragraph proof.
**Given:** $\Delta ABC \sim \Delta XYZ$
**Prove:** $\dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta XYZ} = \dfrac{a^2}{x^2}$




*SOLUTION:*
We wish to prove that the ratio of the areas of two similar triangles is equal to the square of the ratios of corresponding side lengths. Begin by using the formula for the area of a triangle to compare the two areas in terms of their bases and heights. Then use the fact that the triangles are similar to make a substitution for the corresponding ratio.

The area of $\Delta ABC = \frac{1}{2} b \cdot h_1$ and the area of $\Delta XYZ = \frac{1}{2} y \cdot h_2$. The ratio of their areas is $\frac{\Delta ABC}{\Delta XYZ} = \frac{h_1 b}{h_2 y}$ or $\frac{h_1}{h_2} \cdot \frac{b}{y}$. Since the triangles are similar, we have the following equation for the ratio of corresponding side measures $\frac{a}{x} = \frac{b}{y} = \frac{h_1}{h_2}$. Therefore, by substitution $\frac{\Delta ABC}{\Delta XYZ} = \left(\frac{a}{x}\right)\left(\frac{a}{x}\right)$ or $\frac{a^2}{x^2}$.

*ANSWER:*
The area of $\Delta ABC = \frac{1}{2} b \cdot h_1$ and the area of $\Delta XYZ = \frac{1}{2} y \cdot h_2$. The ratio of the area of $\frac{\Delta ABC}{\Delta XYZ} = \frac{h_1 b}{h_2 y}$ or $\frac{h_1}{h_2} \cdot \frac{b}{y}$. The ratio of the side the corresponding measures are $\frac{a}{x} = \frac{b}{y} = \frac{h1}{h2}$. Therefore, by substitution $\frac{\Delta ABC}{\Delta XYZ} = \left(\frac{a}{x}\right)\left(\frac{a}{x}\right)$ or $\frac{a^2}{x^2}$.

## 11-5 Areas of Similar Figures

23. **STATISTICS** The graph shows the increase in high school tennis participation from 1995 to 2005.
**a.** Explain why the graph is misleading.
**b.** How could the graph be changed to more accurately represent the growth in high school tennis participation?


*SOLUTION:*

**a.** Sample answer: The graph is misleading because the tennis balls used to illustrate the number of participants are similar circles. When the diameter of the tennis ball increases, the area of the tennis ball also increases. The ratio of the areas of the similar circles is equal to the square of the ratio of their diameters. For instance, if the diameter of one circle is double another, then the ratio of their diameters is 2:1 and the ratio of their areas is 4:1.Thus, the increase in area is greater than the increase in the diameter. Since the area of the tennis ball increases at a greater rate than the diameter of the tennis ball, it looks like the number of participants in high school tennis is increasing more than it actually is.

**b.** Sample answer: If you use a figure with a constant width to represent the participation in each year and only change the height, the graph would not be misleading. For example, use stacks of tennis balls of the same size to represent the number of participants for each of the years or simply use bars of the same width. Be sure to keep the scale the same.

*ANSWER:*

**a.** Sample answer: The graph is misleading because the tennis balls used to illustrate the number of participants are similar circles. When the diameter of the tennis ball increases, the area of the tennis ball also increases. Since the area of the tennis ball increases at a greater rate than the diameter of the tennis ball, it looks like the number of participants in high school tennis is increasing more than it actually is.

**b.** Sample answer: If you use a figure with a constant width to represent the participation in each year and only change the height, the graph would not be misleading.

24. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate changing dimensions proportionally in three-dimensional figures.
**a. TABULAR** Copy and complete the table below for each scale factor of a rectangular prism that is 2 inches by 3 inches by 5 inches.

| Scale Factor | Length (in.) | Width (in.) | Height (in.) | Volume ($in^3$) | Ratio of Scaled Volume to Initial Volume |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 5 | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 10 | | | | | |

**b. VERBAL** Make a conjecture about the relationship between the scale factor and the ratio of the scaled volume to the initial volume.
**c. GRAPHICAL** Make a scatter plot of the scale factor and the ratio of the scaled volume to the initial volume using the STAT PLOT feature on your graphing calculator. Then use the STAT CALC feature to approximate the

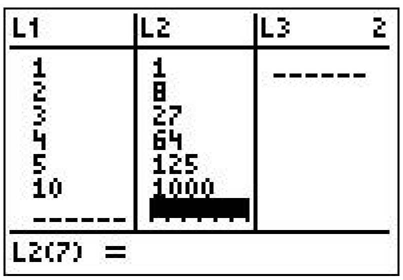
function represented by the graph.
**d. ALGEBRAIC** Write an algebraic expression for the ratio of the scaled volume to the initial volume in terms of scale factor $k$.

*SOLUTION:*
**a.** For the table, each dimension (length, width, or height) should be multiplied by the scale factor. The volume will be the product of all the dimensions.

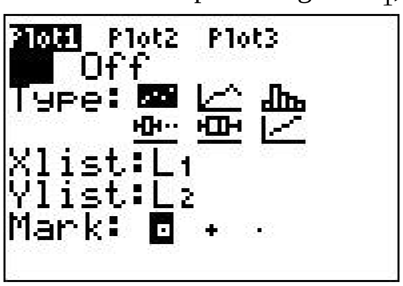
| Scale Factor | Length (in.) | Width (in.) | Height (in.) | Volume ($in^3$) | Ratio of Scaled Volume to Initial Volume |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 5 | 30 | 1 |
| 2 | 6 | 4 | 10 | 240 | 8 |
| 3 | 9 | 6 | 15 | 810 | 27 |
| 4 | 12 | 8 | 20 | 1920 | 64 |
| 5 | 15 | 10 | 25 | 3750 | 125 |
| 10 | 30 | 20 | 50 | 30,000 | 1000 |

**b.** Since each dimension is being multiplied by a scale factor, and then the volume is found by multiplying these terms together, we should suspect that the ratio of the scaled volume to the original volume will be much larger than a simple linear relationship.

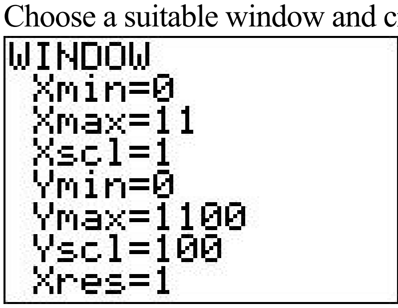
**c.** Enter the scale factors into $L_1$ of your calculator, and the ratio of scaled volume to the initial volume into $L_2$.

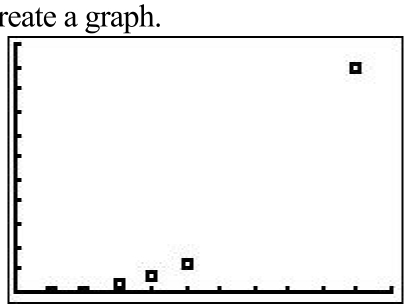



Create a scatter plot using the $L_1$, points as your $x$-values, and the $L_2$ values as your $y$-coordinates.

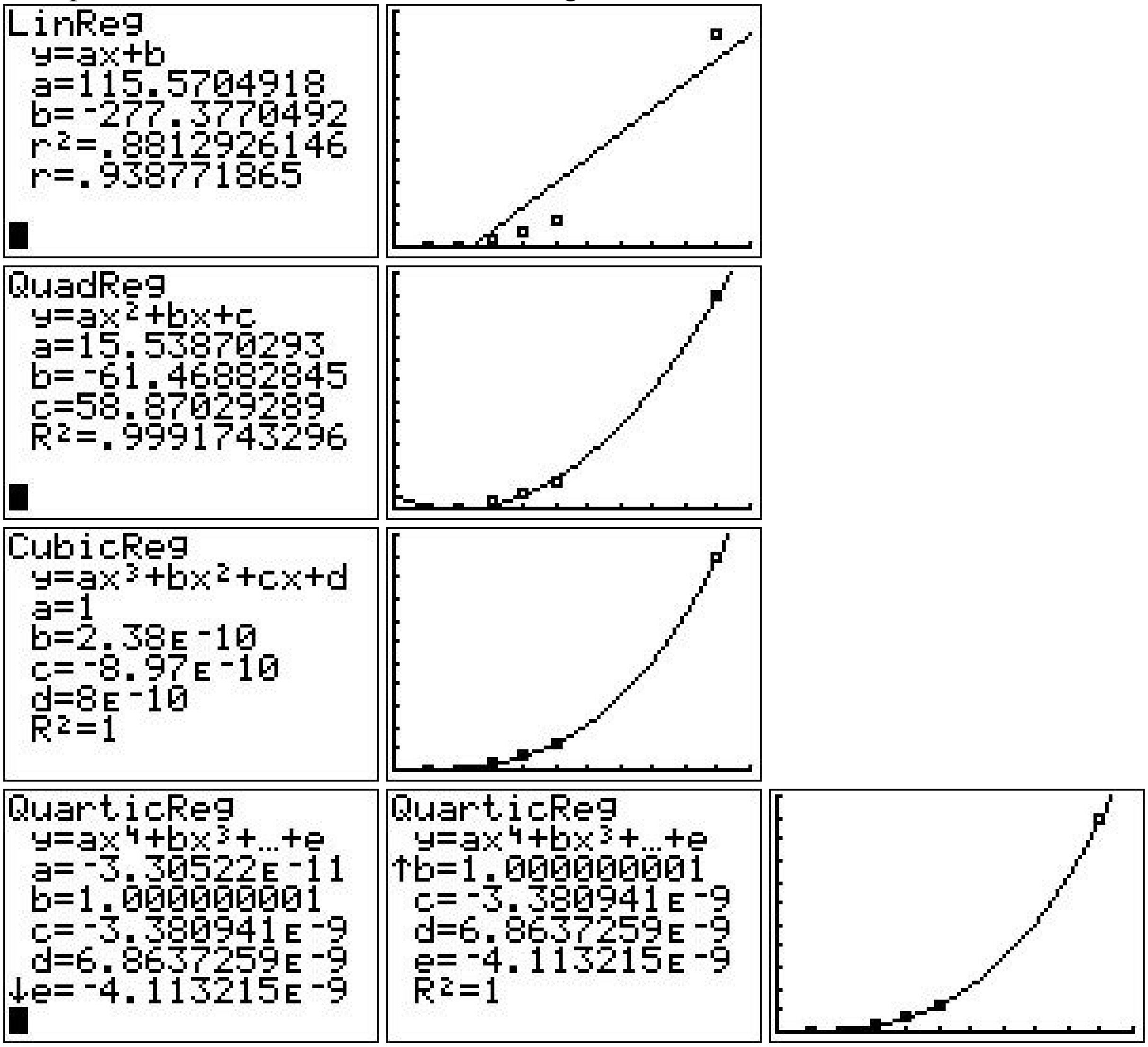



Choose a suitable window and create a graph.



Then press STAT, CALC, and check different regression curves to see which one fits best.




As we can see the $R^2$ value for the cubic and quartic regression curves are equal to 1, which means they fit the data exactly. However the a, c, d, and e terms for the quartic regression are essentially 0 (the calculator reads $3.3 \times 10^{-11}$ as 0, which only arises from functional rounding errors) so the highest term is $x^3$. A cubic regression best fits the data.

$$\frac{\text{Scaled Volume}}{\text{Initial Volume}} = \frac{(k \cdot l)(k \cdot w)(k \cdot h)}{l \cdot w \cdot h}$$

$$= \frac{k^3 \cdot l \cdot w \cdot h}{l \cdot w \cdot h}$$

**d.** $= k^3$

*ANSWER:*

**a.**

| Scale Factor | Length (in.) | Width (in.) | Height (in.) | Volume (in³) | Ratio of Scaled Volume to Initial Volume |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 5 | 30 | 1 |
| 2 | 6 | 4 | 10 | 240 | 8 |
| 3 | 9 | 6 | 15 | 810 | 27 |
| 4 | 12 | 8 | 20 | 1920 | 64 |
| 5 | 15 | 10 | 25 | 3750 | 125 |
| 10 | 30 | 20 | 50 | 30,000 | 1000 |

**b.** Sample answer: The ratio increases at a greater rate than the scale factor, so the relationship is not linear.

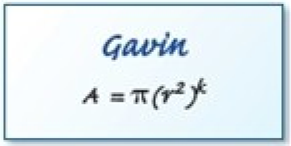
**c.**

[0, 11] scl: 1 by [0, 1100] scl: 100

$y \approx x^3$

**d.** $K^3$

25. **CCSS CRITIQUE** Violeta and Gavin are trying to come up with a formula that can be used to find the area of a circle with a radius $r$ after it has been enlarged by a scale factor $k$. Is either of them correct? Explain your reasoning.


*SOLUTION:*

Neither; sample answer: In order to find the area of the enlarged circle, you can multiply the radius by the scale factor and substitute it into the area formula, or you can multiply the area formula by the scale factor squared. The formula for the area of the enlargement is $A = \pi(kr)^2$ *or* $A = k^2\pi r^2$.

*ANSWER:*

Neither; sample answer: In order to find the area of the enlarged circle, you can multiply the radius by the scale factor and substitute it into the area formula, or you can multiply the area formula by the scale factor squared. The formula for the area of the enlargement is $A = \pi(kr)^2$ or $A = k^2\pi r^2$.

Violeta multiplied the entire area by the scale factor. This would have been correct if the circle was increased by the scale factor, and not the radius.

Gavin took the square of the radius to the $k$th power.

26. **CHALLENGE** If you want the area of a polygon to be $x$% of its original area, by what scale factor should you multiply each side length?

*SOLUTION:*

We want:

$$\frac{\text{Scaled area}}{\text{initial area}} = \frac{x}{100}$$

From theorem 11.1

$$\frac{\text{Scaled area}}{\text{initial area}} = \left(\frac{\text{Scaled side}}{\text{side}}\right)^2$$

Combining these two, we have

$$\left(\frac{\text{Scaled side}}{\text{side}}\right)^2 = \frac{x}{100}$$

$$\frac{\text{Scaled side}}{\text{side}} = \sqrt{\frac{x}{100}}$$

$$\frac{\text{Scaled side}}{\text{side}} = \frac{\sqrt{x}}{10}$$

*ANSWER:*

$\sqrt{\frac{x}{100}}$ or $\frac{1}{10}\sqrt{x}$

27. **REASONING** A regular $n$-gon is enlarged, and the ratio of the area of the enlarged figure to the area of the original figure is $R$. Write an equation relating the perimeter of the enlarged figure to the perimeter of the original figure $Q$.

*SOLUTION:*

Since we have a regular polygon with $n$ sides, and a perimeter of $Q$, the side length of the polygon is $\frac{Q}{n}$. We know from theorem 11.1 that if the scale factor for the areas is $R$, then the scale factor for corresponding sides is $\sqrt{R}$, so the side length of the scaled polygon is $\frac{Q}{n}\sqrt{R}$.

The enlarged polygon is still regular and has $n$ sides so the perimeter of the enalrged polygon is

$$\left(\frac{Q}{n}\sqrt{R}\right)n = Q\sqrt{R}$$

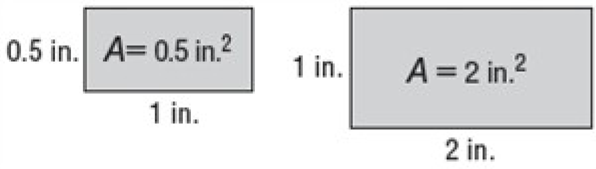
*ANSWER:*

$P_{\text{enlarged}} = Q\sqrt{R}$

28. **OPEN ENDED** Draw a pair of similar figures with areas that have a ratio of 4:1. Explain.

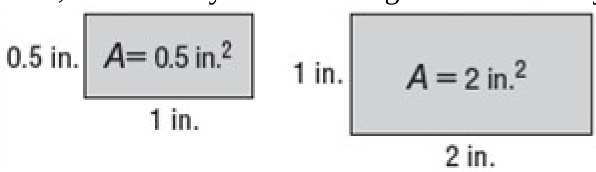
*SOLUTION:*

Sample answer: Draw two similar rectangles. Since the ratio of the areas should be 4:1, the ratio of the lengths of the corresponding sides will be $\sqrt{4}:\sqrt{1}$ or 2:1. Since $\frac{1}{0.5}=\frac{2}{1}$, draw a rectangle with a width of 0.5 inches and a length of 1 inch and a second rectangle with a width of 1 inch and a length of 2 inches. Thus, a 0.5-inch by 1-inch rectangle and a 1-inch by 2-inch rectangle are similar, and the ratio of their areas is 4:1.




*ANSWER:*

Sample answer: Since the ratio of the areas should be 4:1, the ratio of the lengths of the sides will be $\sqrt{4}:\sqrt{1}$ or 2:1. Thus, a 0.5-inch by 1-inch rectangle and a 1-inch by 2-inch rectangle are similar, and the ratio of their areas is 4:1.




29. **WRITING IN MATH** Explain how to find the area of an enlarged polygon if you know the area of the original polygon and the scale factor of the enlargement.

*SOLUTION:*

By theorem 11.1 the ratio of the areas of a scaled polygon and its original is equal to the square of the scale factor. If you know the area of the original polygon and the scale factor of the enlargement, you can find the area of the enlarged polygon by multiplying the original area by the scale factor squared.

For example, if a triangle has an area of 10 and a similar triangle is created by scaling each side of the original triangle by 2, then the area of the new triangle will be $4\times10=40$.

*ANSWER:*

Sample answer: If you know the area of the original polygon and the scale factor of the enlargement, you can find the area of the enlarged polygon by multiplying the original area by the scale factor squared.

30. $\Delta ABC \sim \Delta PRT$, $AC$ = 15 inches, $PT$ = 6 inches, and the area of $\Delta PRT$ is 24 square inches. Find the area of $\Delta ABC$.

**A** $9.6 \text{ in}^2$

**B** $60 \text{ in}^2$

**C** $66.7 \text{ in}^2$

**D** $150 \text{ in}^2$

*SOLUTION:*

The given triangles are similar, so the sides are proportional. $AC$ and $PT$ are corresponding sides, so the scale factor between $\Delta ABC$ and $\Delta PRT$ is $\frac{15}{6}$ or $\frac{5}{2}$.

The ratio of their areas is $\left(\frac{5}{2}\right)^2$.

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PRT} = \left(\frac{5}{2}\right)^2$$
$$\frac{\text{Area of } \Delta ABC}{24} = \frac{25}{4}$$
$$\text{Area of } \Delta ABC = 150$$

The area of $\Delta ABC$ is 150 $\text{in}^2$. The correct choice is D.

*ANSWER:*

D

31. **ALGEBRA** Which of the following shows $2x^2 - 18xy - 72y^2$ factored completely?

**F** $(2x - 18y)(x + 4y)$

**G** $2(x - 9y)(x + 4y)$

**H** $(2x - 9y)(x + 4y)$

**J** $2(x - 12y)(x + 3y)$

*SOLUTION:*

$$\begin{aligned} 2x^2 - 18xy - 72y^2 &= 2(x^2 - 9xy - 36y^2) \\ &= 2(x^2 + 3xy - 12xy - 36y^2) \\ &= 2(x + 3y)(x - 12y) \end{aligned}$$

The correct choice is J.

*ANSWER:*

J

32. **EXTENDED RESPONSE** The measures of two complementary angles are represented by $2x + 1$ and $5x - 9$.
**a.** Write an equation that represents the relationship between the two angles.
**b.** Find the degree measure of each angle.

*SOLUTION:*

**a**. If two angles are complementary, then the sum of their angle measure is 90.
$(2x + 1) + (5x - 9) = 90$

**b**. If two angles are complementary, then the sum of their angle measure is 90.
$(2x + 1) + (5x - 9) = 90$

Solve for $x$.

$$\begin{aligned}(2x+1)+(5x-9)&=90\\7x-8&=90\\7x&=98\\x&=14\end{aligned}$$

So, the angle measures are 2(14) + 1 = 29° and 5(14) – 9 = 61°.

*ANSWER:*

**a.** $(2x + 1) + (5x - 9) = 90$
**b.** 61°; 29°

33. **SAT/ACT** Which of the following are the values of $x$ for which $(x + 5)(x - 4) = 10$?
**A** –5 and 4
**B** 5 and 6
**C** –4 and 5
**D** 6 and –5
**E** –6 and 5

*SOLUTION:*

$(x + 5)(x - 4) = 10$

Solve for $x$.

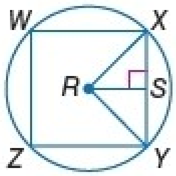
$$\begin{aligned}(x+5)(x-4)&=10\\x^2+x-20&=10\\x^2+x-30&=0\\(x+6)(x-5)&=0\\x&=-6 \text{ or } x=5\end{aligned}$$

So, the correct choice is E.

*ANSWER:*

E

34. In the figure, square *WXYZ* is inscribed in ⊙*R*. Identify the center, a radius, an apothem, and a central angle of the polygon. Then find the measure of a central angle.



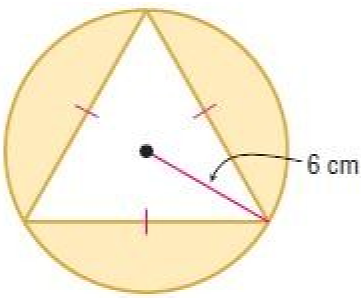

*SOLUTION:*

Center: point *R*, radius: $\overline{RX}$, apothem: $\overline{RS}$, central angle: $\overline{YRX}$, A square is a regular polygon with 4 sides. Thus, the measure of each central angle of square *WXYZ* is $\frac{360}{4}$ or 90.

*ANSWER:*

center: point *R*, radius: $\overline{RX}$, apothem: $\overline{RS}$, central angle: $\angle YRX$, 90°

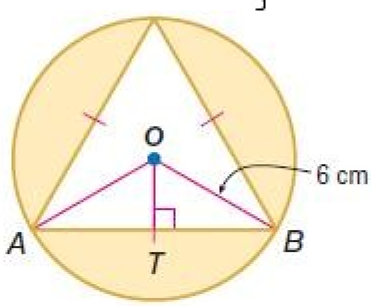
**Find the area of the shaded region. Round to the nearest tenth.**




35.

*SOLUTION:*

The inscribed equilateral triangle can be divided into three congruent isosceles triangles with each central angle having a measure of $\frac{360}{3}$ or 120 .




Apothem $\overline{OT}$ is the height of isosceles triangle *AOB*, so it bisects $\angle AOB$ and $\overline{AB}$. Thus, $m\angle BOT = \frac{120}{2}$ or 60 and *AT* = *BT*. Use trigonometric ratios to find the side length and apothem of the regular polygon.

$$\cos 60 = \frac{OT}{OB}$$

$$\cos 60 = \frac{OT}{6}$$

$$6\cos 60 = OT$$

$$\sin\ 60 = \frac{BT}{OB}$$
$$\sin\ 60 = \frac{BT}{6}$$
$$6\sin\ 60 = BT$$

$$\begin{aligned} AB &= 2(BT) \\ &= 2(6\sin\ 60) \\ &= 12\sin\ 60 \end{aligned}$$

Use the formula for finding the area of a regular polygon replacing *a* with *OT* and *P* with $3 \times AB$ to find the area of the triangle.

$$\begin{aligned} \text{Area of the triangle} &= \frac{1}{2}aP \\ &= \frac{1}{2}(OT)(3 \times AB) \\ &= \frac{1}{2}(6\cos 60)(3 \times 12\sin 60) \\ &\approx 46.77\text{ cm}^2 \end{aligned}$$

Use the formula for the area of a circle replacing *r* with *OB*.

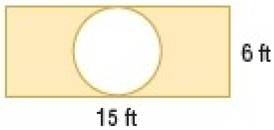
$$\begin{aligned} \text{Area of the circle} &= \pi r^2 \\ &= \pi(OB)^2 \\ &= \pi(6)^2 \\ &\approx 113.10\text{ cm}^2 \end{aligned}$$

Area of the shaded region = Area of the circle – Area of the given triangle
Substitute.

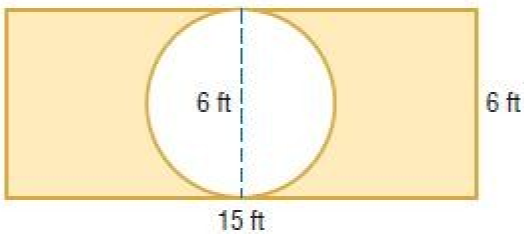
$$\begin{aligned} \text{Area of the shaded region} &= \text{Area of circle} - \text{Area of triangle} \\ &= 113.10 - 46.77 \\ &\approx 66.3\text{ cm}^2 \end{aligned}$$

*ANSWER:*

66.3 $\text{cm}^2$

36. 


*SOLUTION:*




The radius of the circle is half the diameter or 3 feet. The area of the shaded region is the difference of the area of the rectangle and the area of the circle.

$$\begin{aligned}\text{Area of the rectangle} &= \ell \times w\\ &= 15 \times 6\\ &= 90\end{aligned}$$

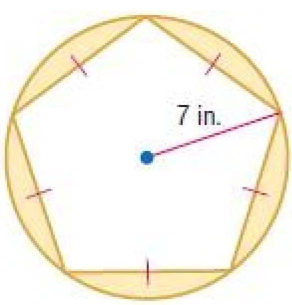
$$\begin{aligned}\text{Area of the circle} &= \pi r^2\\ &= \pi(3)^2\\ &= 9\pi\end{aligned}$$

Area of the shaded region = Area of the rectangle – Area of the circle
Substitute.

$$\begin{aligned}\text{Area of the shaded region} &= 90 - 9\pi\\ &\approx 61.7 \text{ ft}^2\end{aligned}$$

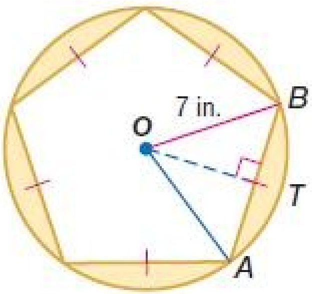
*ANSWER:*

61.7 $\text{ft}^2$

37. 


*SOLUTION:*

The regular pentagon can be divided into 5 congruent isosceles triangles with each central angle having a measure of $\frac{360}{5}$ or 72.




Apothem $\overline{OT}$ is the height of the isosceles triangle $AOB$, so it bisects $\angle AOB$ and $\overline{AB}$. Thus, $m\angle BOT = \frac{72}{2}$ or 36 and $AT = BT$. Use trigonometric ratios to find the side length and apothem of the polygon.

$$\cos 36 = \frac{OT}{OB}$$
$$\cos 36 = \frac{OT}{7}$$
$$7\cos 36 = OT$$

$$\sin 36 = \frac{BT}{OB}$$
$$\sin 36 = \frac{BT}{7}$$
$$7\sin 36 = BT$$

$$\begin{aligned} AB &= 2(AD) \\ &= 2(7\sin 36) \\ &= 14\sin 36 \end{aligned}$$

Use the formula for finding the area of a regular polygon replacing $a$ with $OT$ and $P$ with $5 \times AB$.

$$\begin{aligned} \text{Area of the pentagon} &= \frac{1}{2}aP \\ &= \frac{1}{2}(OT)(5 \times AB) \\ &= \frac{1}{2}(7\cos 36)(5 \times 14\sin 36) \\ &\approx 116.5 \text{ in}^2 \end{aligned}$$

Use the formula for the area of a circle replacing $r$ with $OB$.

$$\begin{aligned} \text{Area of the circle} &= \pi r^2 \\ &= \pi(OB)^2 \\ &= \pi(7)^2 \text{ or } 49\pi \text{ in}^2 \end{aligned}$$

Area of the shaded region = Area of the circle – Area of the pentagon
Substitute.

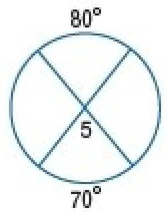
$$\text{Area of the shaded region} = 49\pi - 116.5$$
$$\approx 37.4 \text{ in.}^2$$

*ANSWER:*

37.4 in$^2$

**Find each measure.**

38. $m\angle 5$




*SOLUTION:*

By theorem 10.12, if two secants or chords intersect in the interior of a circle, then the measure of an angle formed is one half the sum of the measure of the arcs intercepted by the angle and its vertical angle.

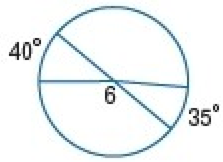
This means that

$$\begin{aligned} m\angle 5 &= \frac{1}{2}(80 + 70) \\ &= \frac{1}{2}(150) \\ &= 75 \end{aligned}$$

*ANSWER:*

75°

39. $m\angle 6$




*SOLUTION:*

By theorem 10.12, if two secants or chords intersect in the interior of a circle, then the measure of an angle formed is one half the sum of the measure of the arcs intercepted by the angle and its vertical angle.

We know that the sum of the arcs intercepted by $\angle 6$ and its vertical angle will have a measure of 360° – (40° + 35°) = 285°.

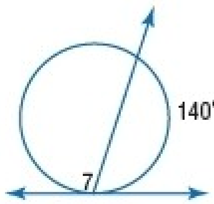
Theorem 10.12 tells us that $m\angle 6$ is half this:

$$\begin{aligned} m\angle 6 &= \frac{1}{2}(285) \\ &= 142.5 \end{aligned}$$

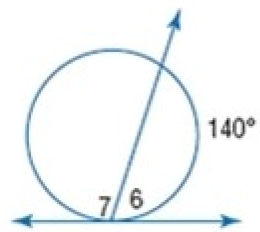
*ANSWER:*

142.5°

40. $m\angle 7$




*SOLUTION:*




Label the angle that forms a linear pair with $\angle 7$ as $\angle 6$.

By theorem 10.13, if a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.

$$m\angle 6 = \frac{1}{2}(140) = 70.$$

Since $\angle 7$ and $\angle 6$ are supplementary angles

$$\begin{aligned} m\angle 7 &= 180 - m\angle 6 \\ &= 180 - 70 \\ &= 110 \end{aligned}$$

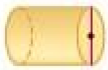
*ANSWER:*

110°

41. State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

*SOLUTION:*

Consider rotating the cylinder about its vertical axis. Any degree of rotation will result in a symmetrical shape. Rotating the cylinder about any other axis will not provide symmetry.



The cylinder has axis symmetric about its vertical axis.

Now consider a vertical plane that cuts through the cylinder along its axis. If the cylinder is reflected through this plane, it will be symmetrical. If a horizontal plane cuts through the center of the cylinder, then the reflection will again produce symmetry. Any other plane, such as a vertical plane cutting through the right side, will not produce symmetry.

The cylinder has plane symmetry about any vertical plane that cuts in line with the central axis, and also through the horizontal plane that cuts the cylinder in half.

The cylinder has both axis and plane symmetry.

*ANSWER:*
both

42. **YEARBOOKS** Tai resized a photograph that was 8 inches by 10 inches so that it would fit in a 4-inch by 4-inch area on a yearbook page.
**a.** Find the maximum dimensions of the reduced photograph.
**b.** What is the percent of reduction of the length?

*SOLUTION:*

**YEARBOOKS** Tai resized a photograph that was 8 inches by 10 inches so that it would fit in a 4-inch by 4-inch area on a yearbook page.
**a.** Find the maximum dimensions of the reduced photograph.
**b.** What is the percent of reduction of the length?

**a**. The photograph should be fit in to a 4-inch by 4-inch area so no side should exceed 4 in.

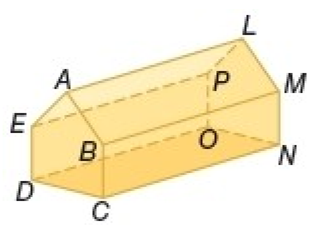
The longest side, 10 in, should be reduced to 4 in. Let *x* be the other dimension of the reduced photograph. The scale ratio of corresponding side lengths will be equal:

$$\frac{4}{10} = \frac{x}{8}$$
$$x = 3.2$$

The maximum dimensions of the reduced photograph is 3.2 in. by 4 in.

**b**. The side whose length is 10 inches should be reduced to 4, which is 40% of the original length. The original photo was reduced by
100% – 40% = 60%.

*ANSWER:*
a. 3.2 in. by 4 in.
b. 60%




**Refer to the figure to identify each of the following.**

43. Name all segments parallel to $\overline{AE}$ .

*SOLUTION:*
Two lines are parallel if they are coplanar and do not intersect. The only two planes that contain $AE$ are the pentagon $ABCDE$, and the rectangle $AELP$. If we imagine the lines extending infinitely in each plane, then the only line that does not intersect $AE$ is $LP$.

*ANSWER:*
$\overline{LP}$

44. Name all planes intersecting plane *BCN*.

*SOLUTION:*

The planes that intersect plane *BCN* are *ABM, OCN, ABC, LMN*.

*ANSWER:*

*ABM, OCN, ABC, LMN*

45. Name all segments skew to $\overline{DC}$.

*SOLUTION:*

Two lines are skew if they do not intersect and are not coplanar. The following segments meet this criteria:

$\overline{BM}, \overline{AL}, \overline{EP}, \overline{OP}, \overline{PL}, \overline{LM}, \overline{MN}$

*ANSWER:*

$\overline{BM}, \overline{AL}, \overline{EP}, \overline{OP}, \overline{PL}, \overline{LM}, \overline{MN}$