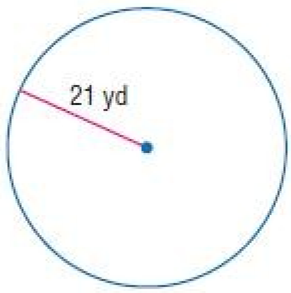
11-3 Areas of Circles and Sectors

CONSTRUCTION Find the area of each circle. Round to the nearest tenth.

1. Refer to the figure on page 800.



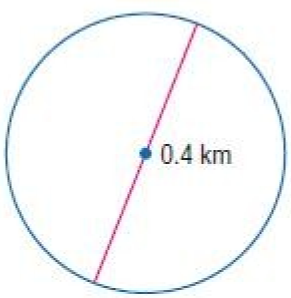
SOLUTION:

$$A = \pi r^2$$
$$= \pi(21)^2$$
$$\approx 1385.4$$

ANSWER:

1385.4 yd^2

2. Refer to the figure on page 800.



SOLUTION:

$$A = \pi r^2$$
$$= \pi(0.2)^2$$
$$\approx 0.1$$

ANSWER:

0.1 km^2

Find the indicated measure. Round to the nearest tenth.

3. Find the diameter of a circle with an area of 74 square millimeters.

SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ 74 &= \pi r^2 \\ \frac{74}{\pi} &= r^2 \\ \sqrt{\frac{74}{\pi}} &= r \\ 2\sqrt{\frac{74}{\pi}} &= d \\ 9.7 &\approx d \end{aligned}$$

ANSWER:

9.7 mm

4. The area of circle is 88 square inches. Find the radius.

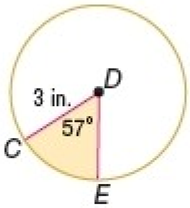
SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ 88 &= \pi r^2 \\ \frac{88}{\pi} &= r^2 \\ \sqrt{\frac{88}{\pi}} &= r \\ 5.3 &\approx r \end{aligned}$$

ANSWER:

5.3 in.

Find the area of each shaded sector. Round to the nearest tenth.

5. 

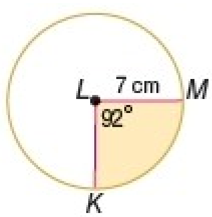
SOLUTION:

The ratio of the area A of a sector to the area of the whole circle, πr^2, is equal to the ratio of the degree measure of the intercepted arc x to 360.

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{57}{360}\pi(3)^2 \\ &= \frac{57}{360}\pi(9) \\ &\approx 4.5 \end{aligned}$$

ANSWER:

4.5 in^2

6. 

SOLUTION:

The ratio of the area A of a sector to the area of the whole circle, πr^2, is equal to the ratio of the degree measure of the intercepted arc x to 360.

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{92}{360}\pi(7)^2 \\ &= \frac{92}{360}\pi(49) \\ &\approx 39.3 \end{aligned}$$

ANSWER:

39.3 cm^2

7. **BAKING** Chelsea is baking pies for a fundraiser at her school. She divides each 9-inch pie into 6 equal slices.
a. What is the area, in square inches, for each slice of pie?
b. If each slice costs \$0.25 to make and she sells 8 pies at \$1.25 for each slice, how much money will she raise?

SOLUTION:

a. Since the pie is equally divided into 6 slices, each slice will have an arc measure of 360 ÷ 6 or 60.

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{60}{360}\pi(4.5)^2 \\ &= \frac{1}{6}\pi(20.25) \\ &\approx 10.6 \end{aligned}$$

b. The manufacturing cost for each slice is \$0.25 and she sells it for \$1.25. So, she makes a profit of \$1 from each slice of 8 pies. There are 6 slices in each pie. So, the total profit is 8(6)(1) = 48. Therefore, she will raise an amount of \$48.

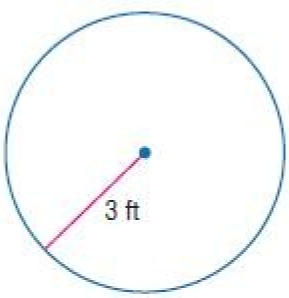
ANSWER:

a. 10.6 in^2

b. \$48

CCSS MODELING Find the area of each circle. Round to the nearest tenth.

8. Refer to the figure on page 801.



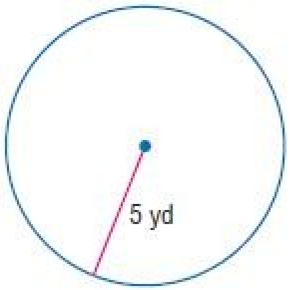
SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(3)^2 \\ &\approx 28.3 \end{aligned}$$

ANSWER:

28.3 ft^2

9. Refer to the figure on page 801.



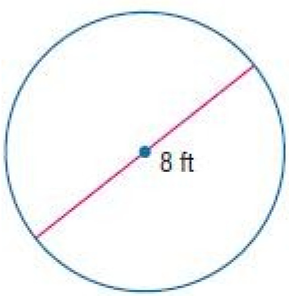
SOLUTION:

$$A = \pi r^2$$
$$= \pi(5)^2$$
$$\approx 78.5$$

ANSWER:

$78.5\ yd^2$

10. Refer to the figure on page 801.



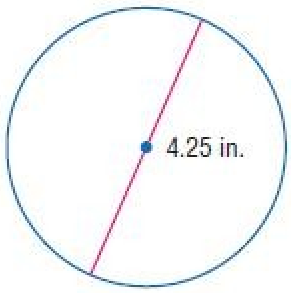
SOLUTION:

$$A = \pi r^2$$
$$= \pi(4)^2$$
$$\approx 50.3$$

ANSWER:

$50.3\ ft^2$

11. Refer to the figure on page 801.



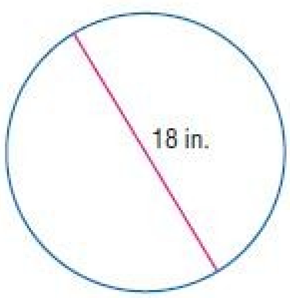
SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi\left(\frac{4.25}{2}\right)^2 \\ &\approx 14.2 \end{aligned}$$

ANSWER:

14.2 in^2

12. Refer to the figure on page 801.



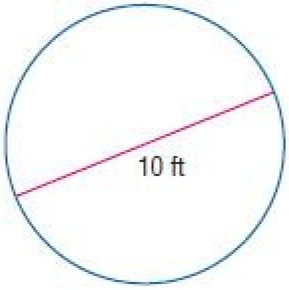
SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(9)^2 \\ &\approx 254.5 \end{aligned}$$

ANSWER:

254.5 in^2

13. Refer to the figure on page 801.



SOLUTION:

$$\begin{aligned} \text{area} &= \pi r^2 \\ &= \pi(5)^2 \\ &= 25\pi \\ &\approx 78.5 \end{aligned}$$

ANSWER:

78.5 ft^2

Find the indicated measure. Round to the nearest tenth, if necessary.

14. The area of a circle is 68 square centimeters. Find the diameter.

SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ 68 &= \pi r^2 \\ \frac{68}{\pi} &= r^2 \\ \sqrt{\frac{68}{\pi}} &= r \\ 2\sqrt{\frac{68}{\pi}} &= d \\ 9.3 &\approx d \end{aligned}$$

ANSWER:

9.3 cm

15. Find the diameter of a circle with an area of 94 square millimeters.

SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ 94 &= \pi r^2 \\ \frac{94}{\pi} &= r^2 \\ \sqrt{\frac{94}{\pi}} &= r \\ 2\sqrt{\frac{94}{\pi}} &= d \\ 10.9 &\approx d \end{aligned}$$

ANSWER:

10.9 mm

16. The area of circle is 112 square inches. Find the radius.

SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ 112 &= \pi r^2 \\ \frac{112}{\pi} &= r^2 \\ \sqrt{\frac{112}{\pi}} &= r \\ 6.0 &\approx r \end{aligned}$$

ANSWER:

6 in.

17. Find the radius of a circle with an area of 206 square feet.

SOLUTION:

The area A of a circle is equal to π times the square of the radius r.

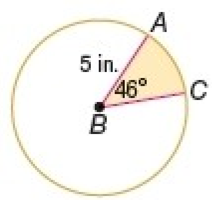
$$\begin{aligned} \text{area} &= \pi r^2 \\ 206 &= \pi r^2 \\ \frac{206}{\pi} &= r^2 \\ \sqrt{\frac{206}{\pi}} &= r \\ 8.1 &\approx r \end{aligned}$$

The radius of the circle is about 8.1 in.

ANSWER:

8.1 ft

Find the area of each shaded sector. Round to the nearest tenth, if necessary.

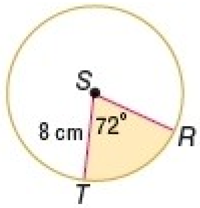
18. 

SOLUTION:

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{46}{360}\pi(5)^2 \\ &= \frac{46}{360}\pi(25) \\ &\approx 10.0 \end{aligned}$$

ANSWER:

10 in^2

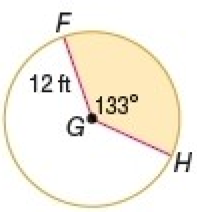
19. 

SOLUTION:

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{72}{360}\pi(8)^2 \\ &= \frac{1}{5}\pi(64) \\ &\approx 40.2 \end{aligned}$$

ANSWER:

40.2 cm^2

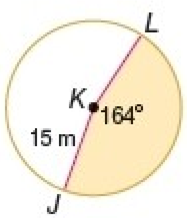
20. 

SOLUTION:

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{133}{360}\pi(12)^2 \\ &= \frac{133}{360}\pi(144) \\ &\approx 167.1 \end{aligned}$$

ANSWER:

167.1 ft^2

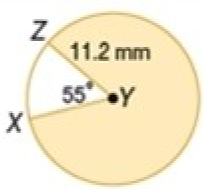
21. 

SOLUTION:

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{164}{360}\pi(15)^2 \\ &= \frac{164}{360}\pi(225) \\ &\approx 322.0 \end{aligned}$$

ANSWER:

322 m^2

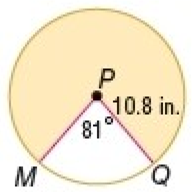
22. 

SOLUTION:

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{360-55}{360}\pi(11.2)^2 \\ &= \frac{305}{360}\pi(125.44) \\ &\approx 333.9 \end{aligned}$$

ANSWER:

333.9 mm^2

23. 

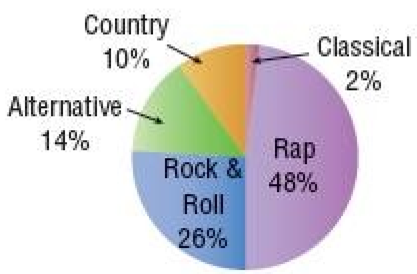
SOLUTION:

$$\begin{aligned} \text{sector area} &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{360-81}{360} \cdot \pi (10.8)^2 \\ &= \frac{279}{360} \cdot \pi \cdot 116.64 \\ &\approx 284.0 \end{aligned}$$

ANSWER:

284 in^2

24. **MUSIC** The music preferences of students at Thomas Jefferson High are shown in the circle graph. Find the area of each sector and the degree measure of each intercepted arc if the radius of the circle is 1 unit.



SOLUTION:

Multiply each percentage by 360° to find the measure of the corresponding arc.

The area of the circle is **π** units. Multiply each percentage by this to find the area of each corresponding sector.

rap:

$0.48 \cdot 360 = 172.8$

$0.48\pi \approx 1.51$

rock & roll:

$0.26 \cdot 360 = 93.6$

$0.26\pi \approx 0.82$

alternative:

$0.14 \cdot 360 = 50.4$

$0.14\pi \approx 0.44$

country:

$0.10 \cdot 360 = 36$

$0.10\pi \approx 0.31$

classical:

$0.02 \cdot 360 = 7.2$

$0.02\pi \approx 0.06$

ANSWER:

rap: 172.8°, 1.51 units^2;
rock & roll: 93.6°, 0.82 units^2;
alternative: 50.4°, 0.44 units^2;
country: 36°, 0.31 units^2;
classical: 7.2°, 0.06 units^2

25. **JEWELRY** A jeweler makes a pair of earrings by cutting two 50° sectors from a silver disk.
a. Find the area of each sector.
b. If the weight of the silver disk is 2.3 grams, how many milligrams does the silver wedge for each earring weigh?

SOLUTION:

a.

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{50}{360}\pi(2)^2 \\ &= \frac{5}{36}\pi(4) \\ &\approx 1.7 \end{aligned}$$

b. $\frac{50}{360}$ of the disc has been removed to make each earring. So, the weight of each earring is $\frac{50}{360} \cdot 2.3 \approx 0.3194$ gm or 319.4 mg.

ANSWER:

a. 1.7 cm^2
b. 319.4 mg

26. **PROM** The table shows the results of a survey of students to determine their preference for a prom theme.

Theme	Percent
An Evening of Stars	11
Mardi Gras	32
Springtime in Paris	8
Night in Times Square	47
Undecided	2

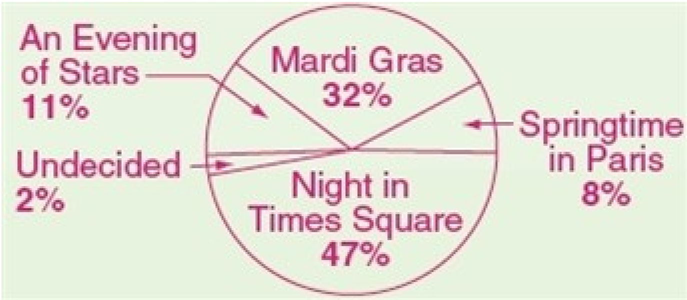
a. Create a circle graph with a diameter of 2 inches to represent these data.
b. Find the area of each theme's sector in your graph. Round to the nearest hundredth of an inch.

SOLUTION:

a. Multiply each % by 360 to find the degree measure of each sector.

$$\begin{aligned} &0.11 \cdot 360 = 39.6 \\ &0.02 \cdot 360 = 7.2 \\ &0.32 \cdot 360 = 115.2 \\ &0.08 \cdot 360 = 28.8 \\ &0.47 \cdot 360 = 169.2 \end{aligned}$$

Use these measures to create the sectors of the circle.



b. We are given the percentages, so multiply the area of the circle, π, by each percentage.

$0.11 \cdot \pi \approx 0.35$
$0.02 \cdot \pi \approx 0.06$
$0.32 \cdot \pi \approx 1.01$
$0.08 \cdot \pi \approx 0.25$
$0.47 \cdot \pi \approx 1.48$

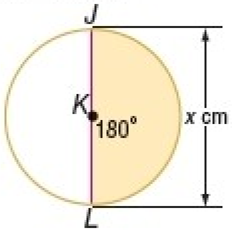
ANSWER:

a.

b. An Evening of Stars: 0.35 in^2; Mardi Gras: 1.01 in^2; Springtime in Paris: 0.25 in^2; Night in Times Square: 1.48 in^2; Undecided: 0.06 in^2

CCSS SENSE-MAKING The area A of each shaded region is given. Find x.

27. $A = 66\text{ cm}^2$



SOLUTION:

The ratio of the area A of a sector to the area of the whole circle, πr^2, is equal to the ratio of the degree measure of the intercepted arc x to 360. So, the area A of a sector is given by $A = \frac{x}{360} \cdot \pi r^2$.

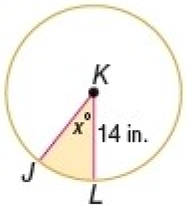
$$A = \frac{x}{360} \cdot \pi r^2$$
$$66 = \frac{180}{360} \cdot \pi r^2$$
$$66 = \frac{1}{2} \cdot \pi r^2$$
$$132 = \pi r^2$$
$$\frac{132}{\pi} = r^2$$
$$\sqrt{\frac{132}{\pi}} = r$$
$$6.5 \approx r$$

The value of x, which is the diameter of the circle, is about 13 cm.

ANSWER:

13

28. $A = 94\text{ in}^2$



SOLUTION:

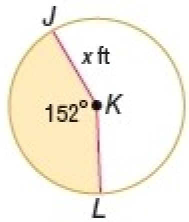
The ratio of the area A of a sector to the area of the whole circle, πr^2, is equal to the ratio of the degree measure of the intercepted arc x to 360. So, the area A of a sector is given by $A = \frac{x}{360} \cdot \pi r^2$.

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ 94 &= \frac{x}{360} \cdot \pi (14)^2 \\ 94 &= \frac{x}{360} \cdot \pi (196) \\ \frac{94}{196\pi} &= \frac{x}{360} \\ \frac{94 \cdot 360}{196\pi} &= x \\ 55 &\approx x \end{aligned}$$

ANSWER:

55

29. $A = 128 \text{ ft}^2$



SOLUTION:

The ratio of the area A of a sector to the area of the whole circle, πr^2, is equal to the ratio of the degree measure of the intercepted arc x to 360. So, the area A of a sector is given by $A = \frac{x}{360} \cdot \pi r^2$.

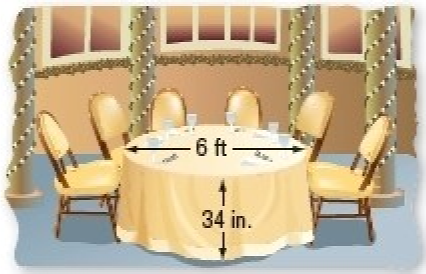
x in the diagram is the radius, r.

$$A = \frac{x}{360} \cdot \pi r^2$$
$$128 = \frac{152}{360} \cdot \pi r^2$$
$$\frac{128 \cdot 360}{152} = \pi r^2$$
$$\frac{128 \cdot 360}{152\pi} = r^2$$
$$\sqrt{\frac{128 \cdot 360}{152\pi}} = r$$
$$9.8 \approx r$$

ANSWER:

9.8

30. **CRAFTS** Luna is making tablecloths with the dimensions shown for a club banquet. Find the area of each tablecloth in square feet if each one is to just reach the floor.



SOLUTION:
34 in. is equivalent to about 2.83 ft.

The radius of the table cloth is about 3 + 2.83 = 5.83 ft.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(5.83)^2 \\ &\approx 107 \end{aligned}$$

ANSWER:
about 107 ft^2

31. **TREES** The age of a living tree can be determined by multiplying the diameter of the tree by its growth factor, or rate of growth.
a. What is the diameter of a tree with a circumference of 2.5 feet?
b. If the growth factor of the tree is 4.5, what is the age of the tree?

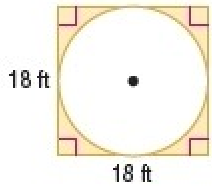
SOLUTION:
a.

$$\begin{aligned} C &= 2\pi r \\ 2.5 &= 2\pi r \\ \frac{2.5}{2\pi} &= r \\ 0.4 &\approx r \\ 0.8 &\approx d \end{aligned}$$

b. Multiply the growth factor by the diameter to find the age.
Age = (0.8)(4.5) = 3.6 years.

ANSWER:
a. 0.8 ft
b. 3.6 yr

Find the area of the shaded region. Round to the nearest tenth.

32. 

SOLUTION:

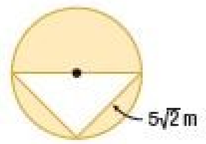
The area of the shaded region is the difference between the area of the square and that of the circle.

The length of each side of the square is 18 ft and the radius of the circle is 9 ft.

$$\begin{aligned} A(shaded) &= A(square) - A(circle) \\ &= 18^2 - \pi(9)^2 \\ &= 324 - 81\pi \\ &\approx 69.5 \end{aligned}$$

ANSWER:

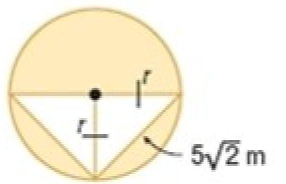
69.5 ft^2



33.

SOLUTION:

The area of the shaded region is the difference between the area of the circle and that of the triangle.



Draw a radius from to the bottom vertex of the triangle. This is an isosceles triangle where the legs are the radius. Using Pythagorean Theorem to find *r*.

$$\begin{aligned}(5\sqrt{2})^2 &= r^2 + r^2\\ 25 \cdot 2 &= 2r^2\\ 25 &= r^2\\ 5 &= r\end{aligned}$$

The height of the triangle is the radius of the circle: 5 cm.

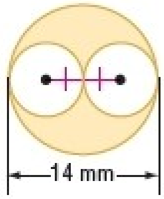
$$\begin{aligned}\text{area of triangle} &= \tfrac{1}{2}bh\\ &= \tfrac{1}{2}(10)(5)\\ &= 25\end{aligned}$$

$$\begin{aligned}\text{area of circle} &= \pi r^2\\ &= \pi(5)^2\\ &\approx 78.5\end{aligned}$$

The area of the shaded region is about 53.5 m^2.

ANSWER:

53.5 m^2

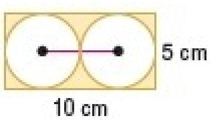
34. 

SOLUTION:
The area of the shaded region is the difference between the area of the larger circle and the sum of the areas of the smaller circles.

The diameter of the larger circle is 14 mm, so the radius is 7 mm. The two smaller circles are congruent to each other and the sum of their diameters is 14 mm. So, the radius of each of the congruent small circles is 3.5 mm.

$$\begin{aligned} A(shaded) &= A(\text{largecircle}) - A(\text{smallcircles}) \\ &= \pi(7)^2 - 2\pi(3.5)^2 \\ &= 49\pi - 24.5\pi \\ &= 24.5\pi \\ &\approx 77.0 \end{aligned}$$

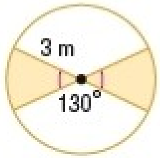
ANSWER:
77 mm^2

35. 

SOLUTION:
The two smaller circles are congruent to each other and the sum of their diameters is 10 cm, so the radius of each of the circles is 2.5 cm.

$$\begin{aligned} A(shaded) &= A(\text{rectangle}) - A(\text{smallcircles}) \\ &= 5(10) - 2\pi(2.5)^2 \\ &= 50 - 12.5\pi \\ &\approx 10.7 \end{aligned}$$

ANSWER:
10.7 cm^2

36. 

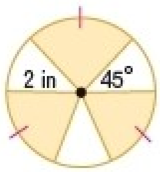
SOLUTION:

The angles of the sectors are each a linear pair with the 130° angle.

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{50}{360}\pi(3)^2 \\ &= \frac{5}{36}\pi(9) \\ &\approx 7.9 \end{aligned}$$

ANSWER:

7.9 m^2

37. 

SOLUTION:

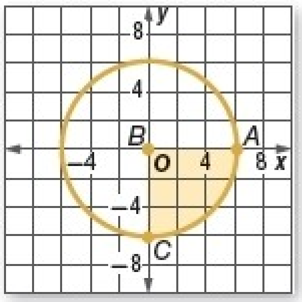
The sum of the central angles of the shaded sectors is 360 – 3(45) = 225.

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{225}{360}\pi(2)^2 \\ &= \frac{5}{8}\pi(4) \\ &\approx 7.9 \end{aligned}$$

ANSWER:

7.9 in^2

38. **COORDINATE GEOMETRY** What is the area of sector *ABC* shown on the graph?



SOLUTION:

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{90}{360}\pi(6)^2 \\ &= \frac{1}{4}\pi(36) \\ &\approx 28.3 \end{aligned}$$

ANSWER:

28.3 square units

39. **ALGEBRA** The figure shown below is a sector of a circle. If the perimeter of the figure is 22 millimeters, find its area in square millimeters.

SOLUTION:

The length of the arc is 22 – (6 + 6) = 10.

$$\begin{aligned} arclength &= \frac{x}{360} \cdot 2\pi r \\ 10 &= \frac{x}{360} \cdot 2\pi(6) \\ \frac{10}{12\pi} &= \frac{x}{360} \\ \frac{3600}{12\pi} &= x \\ 95.5 &\approx x \end{aligned}$$

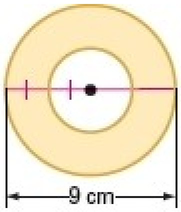
Use the measure of the central angle to find the area of the sector.

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{95.5}{360}\pi(6)^2 \\ &= \frac{95.5}{360}\pi(36) \\ &\approx 30.0 \end{aligned}$$

ANSWER:

30 mm^2

Find the area of each shaded region.

40. 

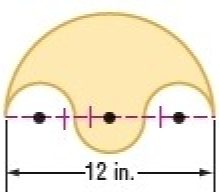
SOLUTION:

The radius of the larger circle is 4.5 cm and the radius of the small circle is 4.5 ÷ 2 = 2.25 cm.

$$\begin{aligned} A(shaded) &= A(large) - A(small) \\ &= \pi(4.5)^2 - \pi(2.25)^2 \\ &= 20.25\pi - 5.0625\pi \\ &\approx 47.7 \end{aligned}$$

ANSWER:

47.7 cm^2

41. 

SOLUTION:

The larger circle has a radius of 6 in.

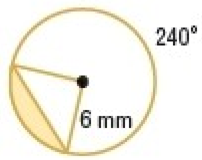
The three smaller circles are congruent and the sum of their diameters is 12 in. So, each has a radius of 2 in.

The area of the shaded region is half of the large circle minus half of one of the small circles. Note that the shaded half circle offsets one of the unshaded half circles.

$$\begin{aligned} A(shaded) &= \tfrac{1}{2}A(lg.circle) - \tfrac{1}{2}A(sm.circle) \\ &= \tfrac{1}{2}\pi(6)^2 - \tfrac{1}{2}\pi(2)^2 \\ &= 18\pi - 2\pi \\ &\approx 50.3 \end{aligned}$$

ANSWER:

50.3 in^2

42. 

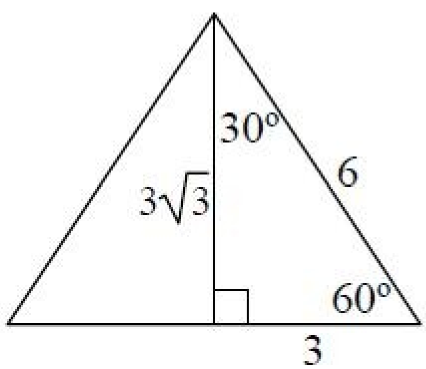
SOLUTION:

The area of the shaded region is the difference between the area covered by the minor arc and the area of the triangle.

The central angle of the minor arc is 360 – 240 = 120.

$$\begin{aligned} A(sector) &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{120}{360}\pi(6)^2 \\ &= \frac{1}{3}\pi(36) \\ &= 12\pi \end{aligned}$$

The central angle is 60°, so the triangle is equilateral. Use 36-60-90 triangles to find the height.



Find the area of the triangle.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(6)\left(3\sqrt{3}\right) \\ &= 9\sqrt{3} \end{aligned}$$

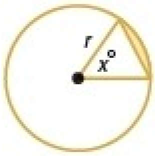
$$\begin{aligned} A(shaded) &= A(sector) - A(triangle) \\ &= 12\pi - 9\sqrt{3} \\ &\approx 22.1 \end{aligned}$$

ANSWER:

22.1 mm^2

43. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate segments of circles. A segment of a

circle is the region bounded by an arc and a chord.
a. ALGEBRAIC Write an equation for the area A of a segment of a circle with a radius r and a central angle of $x°$. (*Hint*: Use trigonometry to find the base and height of the triangle.)

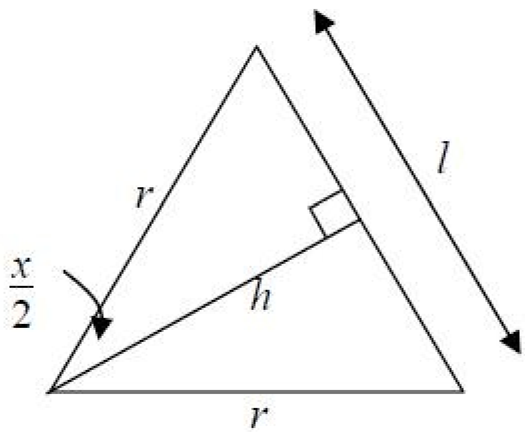


b. TABULAR Calculate and record in a table ten values of A for x-values ranging from 10 to 90 if r is 12 inches. Round to the nearest tenth.
c. GRAPHICAL Graph the data from your table with the x-values on the horizontal axis and the A-values on the vertical axis.
d. ANALYTICAL Use your graph to predict the value of A when x is 63. Then use the formula you generated in part **a** to calculate the value of A when x is 63. How do the values compare?

SOLUTION:

a. Draw a perpendicular from the center to the chord to get two congruent triangles whose hypotenuse is r units long.

Let the height of the triangle be h and the length of the chord, which is a base of the triangle be l.



Use trigonometry to find l and h in terms of r and x.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos\frac{x}{2} = \frac{h}{r}$$
$$r\cos\frac{x}{2} = h$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin\frac{x}{2} = \frac{\frac{l}{2}}{r}$$
$$r\sin\frac{x}{2} = \frac{l}{2}$$
$$2r\sin\frac{x}{2} = l$$

Now find the area of the triangle.

$$\begin{aligned} A &= \tfrac{1}{2}lh \\ &= \tfrac{1}{2}\left[2r\sin\left(\tfrac{x}{2}\right)\right]\left[r\cos\left(\tfrac{x}{2}\right)\right] \\ &= r^2\left[\sin\left(\tfrac{x}{2}\right)\cos\left(\tfrac{x}{2}\right)\right] \end{aligned}$$

Lastly, find the area of the segment.

$$\begin{aligned} A(\text{segment}) &= A(\text{sector}) - A(\text{triangle}) \\ &= \frac{x\pi r^2}{360} - r^2\left[\sin\left(\tfrac{x}{2}\right)\cos\left(\tfrac{x}{2}\right)\right] \end{aligned}$$

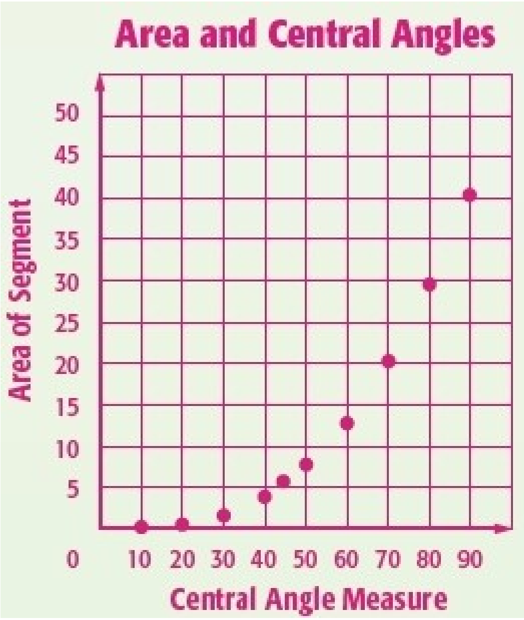
b. If $r = 12$, then the new formula is:

$$\begin{aligned} A(\text{segment}) &= A(\text{sector}) - A(\text{triangle}) \\ &= \frac{x\pi(12)^2}{360} - 12^2\left[\sin\left(\tfrac{x}{2}\right)\cos\left(\tfrac{x}{2}\right)\right] \\ &= \frac{144\pi x}{360}\cdot 144\pi - 144\left[\sin\left(\tfrac{x}{2}\right)\cos\left(\tfrac{x}{2}\right)\right] \\ &= \frac{2\pi x}{5} - 144\left[\sin\left(\tfrac{x}{2}\right)\cos\left(\tfrac{x}{2}\right)\right] \end{aligned}$$

Enter this formula into Y1 of your calculator. The select the table function and set the range for 10 to 90 by 10.

x	A
10	0.1
20	0.5
30	1.7
40	4.0
45	5.6
50	7.7
60	13.0
70	20.3
80	29.6
90	41.1

c. Plot the values. One other option would be to enter 10 through 90 by 10 in L1 and enter the formula for L2, replacing *x* with L1. Then, you can select STATPLOT L1, L2.



d. Sample answer: From the graph, it looks like the area would be about 15.5 when *x* is 63°.

$$\begin{aligned} A(\text{segment}) &= A(\text{sector}) - A(\text{triangle}) \\ &= \frac{2\pi x}{5} - 144\left[\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\right] \\ &= \frac{2\pi(63)}{5} - 144\left[\sin\left(\frac{63}{2}\right)\cos\left(\frac{63}{2}\right)\right] \\ &= 25.2\pi - 144\left[\sin(31.5)\cos(31.5)\right] \\ &\approx 15.02 \end{aligned}$$

Therefore, the area of the segment is about 15.0 when *x* is 63°.

ANSWER:

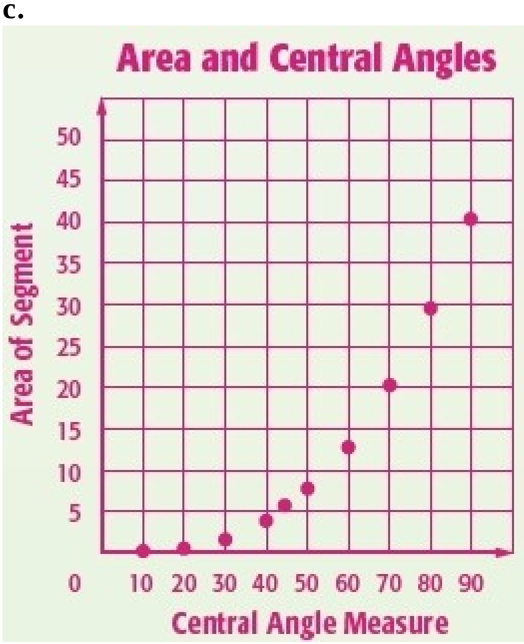
a.

$$A = \frac{x\pi r^2}{360} - r^2\left[\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\right]$$

b.

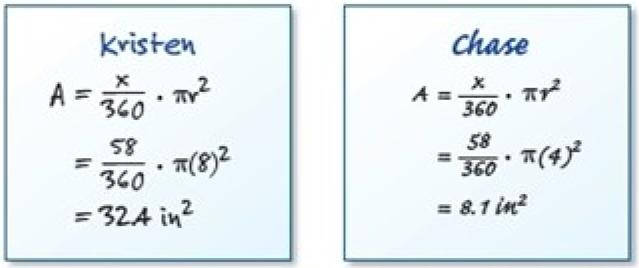
x	*A*
10	0.1
20	0.5
30	1.7
40	4.0
45	5.6
50	7.7
60	13.0
70	20.3
80	29.6
90	41.1

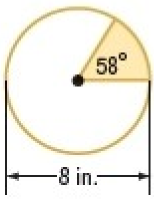
c.



d. Sample answer: From the graph, it looks like the area would be about 15.5 when *x* is 63°. Using the formula, the area is 15.0 when *x* is 63°. The values are very close because I used the formula to create the graph.

44. **ERROR ANALYSIS** Kristen and Chase want to find the area of the shaded region in the circle shown. Is either of them correct? Explain your reasoning.





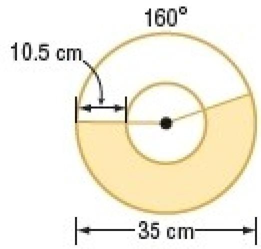
SOLUTION:

The diameter of the circle is given to be 8 in., so the radius is 4 in. Therefore, Chase is correct. Kristen used the diameter in the area formula instead of the radius.

ANSWER:

Chase; sample answer: Kristen used the diameter in the area formula instead of the radius.

45. **CHALLENGE** Find the area of the shaded region. Round to the nearest tenth.



SOLUTION:

The radius of the larger circle is 17.5 cm and that of the smaller circle is 7 cm.

The measure of the central angle of the shaded region is 360 – 160 = 200.

$$\begin{aligned} A(shaded) &= A(\text{lg } sector) - A(sm.sector) \\ &= \frac{200}{360}\pi(17.5)^2 - \frac{200}{360}\pi(7)^2 \\ &= \frac{5}{9}\pi(306.25) - \frac{5}{9}\pi(49) \\ &= \frac{1531.25-245}{9}\pi \\ &\approx 449.0 \end{aligned}$$

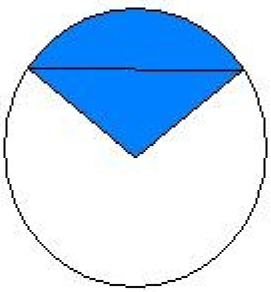
ANSWER:

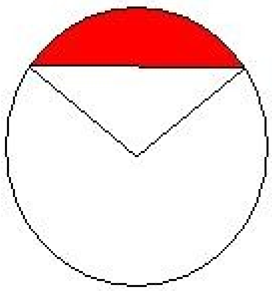
449.0 cm^2

46. **CCSS ARGUMENTS** Refer to Exercise 43. Is the area of a sector of a circle *sometimes*, *always*, or *never* greater than the area of its corresponding segment?

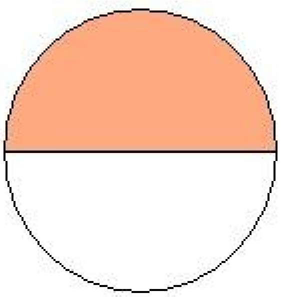
SOLUTION:

In most cases, the area of the sector (as designated by the blue region) is greater than the area of the segment (as designated by the red region) for the same central angle. The area of the segment is contained within the area of the sector. In fact, to calculate the area of the segment, you need to subtract the area of the triangle determined by the central angle and the chord from the area of the sector.





However, if the central angle and the chord both intercept a semicircle, the area of the sector and the area of the segment (as designated by the brown region) are equal.

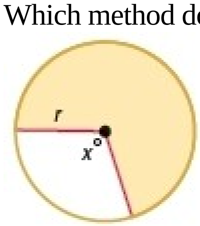


Therefore, the statement is *sometimes* true.

ANSWER:

Sometimes; when the arc is a semicircle, the areas are the same.

47. **WRITING IN MATH** Describe two methods you could use to find the area of the shaded region of the circle. Which method do you think is more efficient? Explain your reasoning.



SOLUTION:

Method 1:

You can find the shaded area of the circle by subtracting x from 360° and using the resulting measure in the formula for the area of a sector.

Let $x = 120$ and $r = 10$.

$360 - 120 = 240$

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{240}{360}\pi(10)^2 \\ &= \frac{2}{3}\pi(100) \\ &\approx 209.4 \end{aligned}$$

Method 2: You could find the shaded area by finding the area of the entire circle, finding the area of the un-shaded sector using the formula for the area of a sector, and subtracting the area of the un-shaded sector from the area of the entire circle.

Let x = 120 and r = 10.

$$\begin{aligned} A(circle) &= \pi r^2 \\ &= \pi(10)^2 \\ &= 100\pi \end{aligned}$$

$$\begin{aligned} A(sector) &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{120}{360}\pi(10)^2 \\ &= \frac{1}{3}\pi(100) \\ &= \frac{100}{3}\pi \end{aligned}$$

$$\begin{aligned} A(shaded) &= 100\pi - \frac{100}{3}\pi \\ &= \frac{300}{3}\pi - \frac{100}{3}\pi \\ &= \frac{200}{3}\pi \\ &= 209.4 \end{aligned}$$

The method in which you find the ratio of the area of a sector to the area of the whole circle is more efficient. It requires less steps, is faster, and there is a lower probability for error.

ANSWER:

Sample answer: You can find the shaded area of the circle by subtracting x from 360° and using the resulting measure in the formula for the area of a sector. You could also find the shaded area by finding the area of the entire circle, finding the area of the un-shaded sector using the formula for the area of a sector, and subtracting the area of the un-shaded sector from the area of the entire circle. The method in which you find the ratio of the area of a sector to the area of the whole circle is more efficient. It requires less steps, is faster, and there is a lower probability for error.

48. **CHALLENGE** Derive the formula for the area of a sector of a circle using the formula for arc length.

SOLUTION:

Think of how the arc length and the area of a sector are related to the circle as a whole. For instance, half of a circle will have half of the arc length and half of the area of the whole circle. A quarter of a circle will have a quarter of the arc length and a quarter of the area. We can express each of these cases mathematically as follows:

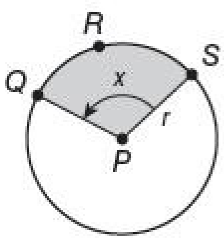
Half circle:

$$\frac{\text{area of sector}}{\text{Area of circle}} = \frac{1}{2} = \frac{\text{arc length}}{\text{Circumference of circle}}$$

Quarter circle:

$$\frac{\text{area of sector}}{\text{Area of circle}} = \frac{1}{4} = \frac{\text{arc length}}{\text{Circumference of circle}}$$

From this we should deduce that the ratio of the area of a sector to the area of the circle should be the same ratio as the arc length divided by the circumference. Next, we express this mathematically and using known formulas derive the area for a sector. Let A represent the area of the sector.



$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{arc length}}{\text{circumference}}$$

$$\frac{A}{\pi r^2} = \frac{\text{length of arc } QRS}{2\pi r}$$

$$\frac{A}{\pi r^2} = \frac{\frac{\pi r x}{180}}{2\pi r}$$

$$A = \frac{\frac{\pi r x}{180}}{2\pi r} \cdot \pi r^2$$

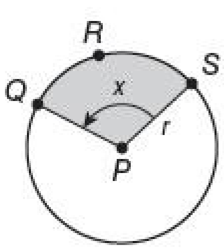
$$A = \frac{\frac{\pi r x}{180}}{2} \cdot r$$

$$A = \frac{\pi r x}{360} \cdot r$$

$$A = \frac{\pi r^2 x}{360}$$

ANSWER:

The ratio of the area of a sector to the area of a whole circle is equal to the ratio of the corresponding arc length to the circumference of the circle. Let A represent the area of the sector.



11-3 Areas of Circles and Sectors

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{arc length}}{\text{circumference}}$$

$$\frac{A}{\pi r^2} = \frac{\text{length of arc } QRS}{2\pi r}$$

$$\frac{A}{\pi r^2} = \frac{\frac{\pi r x}{180}}{2\pi r}$$

$$A = \frac{\frac{\pi r x}{180}}{2\pi r} \cdot \pi r^2$$

$$A = \frac{\frac{\pi r x}{180}}{2} \cdot r$$

$$A = \frac{\pi r x}{360} \cdot r$$

$$A = \frac{\pi r^2 x}{360}$$

49. **WRITING IN MATH** If the radius of a circle doubles, will the measure of a sector of that circle double? Will it double if the arc measure of that sector doubles?

SOLUTION:

If the radius of the circle doubles, the area will not double.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(3)^2 \\ &= 9\pi \end{aligned}$$

$$\begin{aligned} A &= \pi(2r)^2 \\ &= \pi(6)^2 \\ &= 36\pi \end{aligned}$$

If the radius of the circle doubles, the area will be four times as great. 36**π = 4(9π)**

If the arc length of a sector is doubled, the area of the sector is doubled.

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{60}{360}\pi(10)^2 \\ &= \frac{1}{6}\pi(100) \\ &= \frac{100}{6}\pi \end{aligned}$$

$$\begin{aligned} A &= \frac{2x}{360} \cdot \pi r^2 \\ &= \frac{120}{360}\pi(10)^2 \\ &= \frac{1}{3}\pi(100) \\ &= \frac{100}{3}\pi \end{aligned}$$

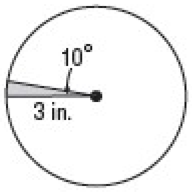
$$\frac{100}{3}\pi = 2\left[\frac{100}{6}\pi\right]$$

Since the arc length is not raised to a power, if the arc length is doubled, the area would also be twice as large.

ANSWER:

Sample answer: If the radius of the circle doubles, the area will not double. If the radius of the circle doubles, the area will be four times as great. Since the radius is squared, if you multiply the radius by 2, you multiply the area by 2^2, or 4. If the arc length of a sector is doubled, the area of the sector is doubled. Since the arc length is not raised to a power, if the arc length is doubled, the area would also be twice as large.

50. What is the area of the sector?



A $\frac{9\pi}{10}\text{in}^2$

B $\frac{3\pi}{5}\text{in}^2$

C $\frac{\pi}{4}\text{in}^2$

D $\frac{\pi}{6}\text{in}^2$

SOLUTION:

$$\begin{aligned} A &= \frac{x}{360} \cdot \pi r^2 \\ &= \frac{10}{360}\pi(3)^2 \\ &= \frac{1}{36}\pi(9) \\ &= \frac{\pi}{4} \end{aligned}$$

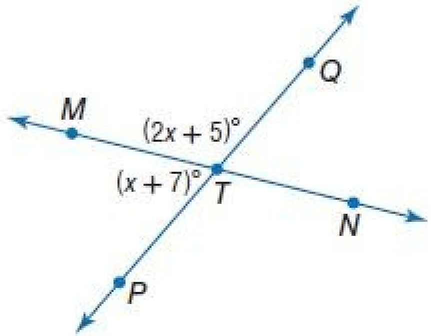
ANSWER:

C

51. **SHORT RESPONSE** $\overleftrightarrow{MN}$ and $\overleftrightarrow{PQ}$ intersect at T. Find the value of x for which $m\angle MTQ = 2x + 5$ and $m\angle PTM = x + 7$. What are the degree measures of $\angle MTQ$ and $\angle PTM$?

SOLUTION:

First draw a diagram.



The two angles $\angle MTQ$ and $\angle PTM$ form a linear pair. So, the sum of their measures is 180.

$$2x + 5 + x + 7 = 180$$
$$3x = 168$$
$$x = 56$$
$$m\angle MTQ = 2(56) + 5 = 117$$
$$m\angle PTM = 56 + 7 = 63$$

ANSWER:

$x = 56$; $m\angle MTQ = 117$; $m\angle PTM = 63$

52. **ALGEBRA** Raphael bowled 4 games and had a mean score of 130. He then bowled two more games with scores of 180 and 230. What was his mean score for all 6 games?

F 90
G 155
H 180
J 185

SOLUTION:

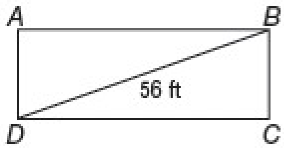
$$\frac{\text{Sum of score of first 4 games}}{4} = 130$$
$$\text{Sum of score of first 4 games} = 520$$
$$\text{Mean score of all 6 games} = \frac{520 + 180 + 230}{6} = \frac{930}{6} = 155$$

The correct choice is G.

ANSWER:

G

53. **SAT/ACT** The diagonals of rectangle *ABCD* each have a length of 56 feet. If $m\angle BAC$ = 42°, what is the length of $\overline{AB}$ to the nearest tenth of a foot?



A 80.5
B 75.4
C 56.3
D 50.4
E 41.6

SOLUTION:

ABCD is a rectangle, so $\angle ABD \cong \angle BAC = 42$.

Use the cosine ratio of an angle to find the length *AB*.

$$\cos x = \frac{adjacent}{hypotenuse}$$
$$\cos 42 = \frac{AB}{56}$$
$$56\cos 42 = AB$$
$$41.6 \approx AB$$

The correct choice is E.

ANSWER:

E

Find each missing length.

54. One diagonal of a kite is half as long as the other diagonal. If the area of the kite is 188 square inches, what are the lengths of the diagonals?

SOLUTION:

$$A = \frac{1}{2}d_1d_2$$
$$188 = \frac{1}{2}(x)\left(\frac{x}{2}\right)$$
$$188 = \frac{x^2}{4}$$
$$752 = x^2$$
$$27.4 \approx x$$

ANSWER:

27.4 in., 13.7 in.

55. The area of a rhombus is 175 square centimeters. If one diagonal is two times as long as the other, what are the lengths of the diagonals?

SOLUTION:

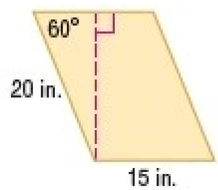
$$\begin{aligned} A &= \tfrac{1}{2}d_1d_2 \\ 175 &= \tfrac{1}{2}(x)(2x) \\ 175 &= x^2 \\ \sqrt{175} &= x \\ 13.2 &\approx x \end{aligned}$$

Therefore, the diagonals are of length 13.2 cm. and 26.4 cm.

ANSWER:

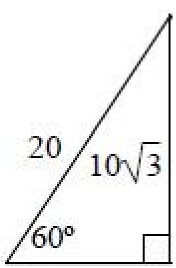
13.2 cm, 26.4 cm

Find the area of each parallelogram. Round to the nearest tenth if necessary.

56. 

SOLUTION:

Use the 30-60-90 triangle to find the height.



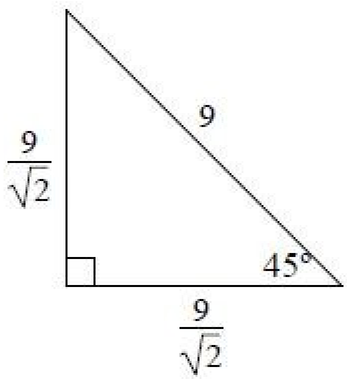
$$\begin{aligned} A &= bh \\ &= 15\left(10\sqrt{3}\right) \\ &\approx 259.8 \end{aligned}$$

ANSWER:

259.8 in^2

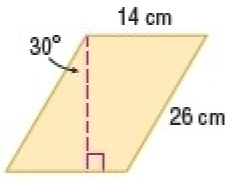
57. 

SOLUTION:
Use the 45-45-90 triangle to find the height.



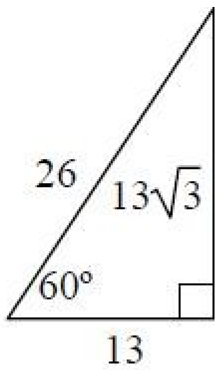
$$\begin{aligned} A &= bh \\ &= 28\left[\frac{9}{\sqrt{2}}\right] \\ &= 126\sqrt{2} \\ &= 126\sqrt{2} \end{aligned}$$

ANSWER:
178.2 ft^2

58. 

SOLUTION:

Use the 30-60-90 triangle to find the height.



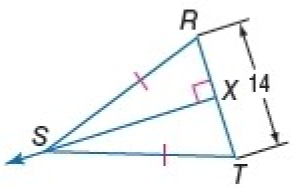
$$\begin{aligned} A &= bh \\ &= 14\left(13\sqrt{3}\right) \\ &\approx 315.2 \end{aligned}$$

ANSWER:

315.2 cm^2

Find each measure.

59. *XT*



SOLUTION:

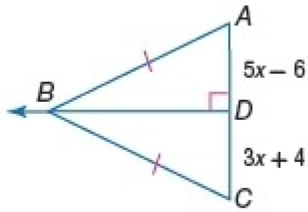
If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment. *SR* = *ST*, so point *S* is equidistant from *R* and *T*, and *SX* is the perpendicular bisector of *RT*.

Since *SX* is the perpendicular bisector of *RT*, *RX* = *XT* and *X* is the midpoint of *RT*. *RT* = 14, so *XT* = 7.

ANSWER:

7

60. AC



SOLUTION:

$AB = BC$ and $\overline{BD} \perp \overline{AC}$.

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment, so $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$. By the perpendicular bisector theorem, $AD = CD$.

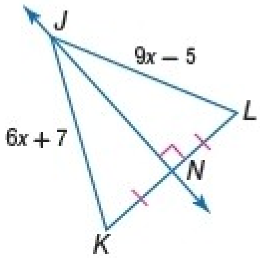
$$\begin{aligned} 5x - 6 &= 3x + 4 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

$$AC = 2(DC) = 2(3(5) + 4) = 38.$$

ANSWER:

38

61. *JK*



SOLUTION:

KN = *LN* and $\overline{JN} \perp \overline{KL}.$

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment, so $\overline{JN}$ is the perpendicular bisector of $\overline{KL}.$

By the perpendicular bisector theorem, *JK* = *JL*.

$$\begin{aligned} 9x - 5 &= 6x + 7 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

JK = 6(4) + 7 = 31

ANSWER:

31