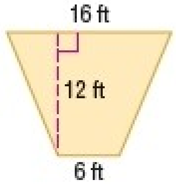
11-2 Areas of Trapezoids, Rhombi, and Kites

Find the area of each trapezoid, rhombus, or kite.

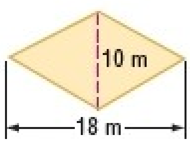
1. 

SOLUTION:

$$A = \frac{1}{2}(b_1 + b_2)h$$
$$= \frac{1}{2}(6 + 16)(12)$$
$$= 132$$

ANSWER:

132 ft^2

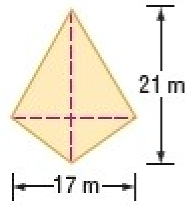
2. 

SOLUTION:

$$A = \frac{1}{2}d_1 d_2$$
$$= \frac{1}{2}(10)(18)$$
$$= 90$$

ANSWER:

90 m^2

3. 

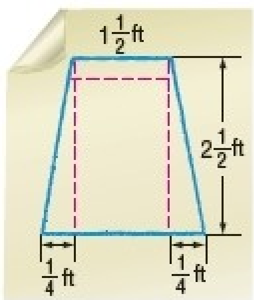
SOLUTION:

$$A = \frac{1}{2}d_1d_2$$
$$= \frac{1}{2}(17)(21)$$
$$= 178.5$$

ANSWER:

$178.5\ \text{m}^2$

4. **SHORT RESPONSE** Suki is doing fashion design at 4-H Club. Her first project is to make a simple A-line skirt. How much fabric will she need according to the design at the right?



SOLUTION:

The area A of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.

$h = 2\frac{1}{2}$ ft, $b_1 = 1\frac{1}{2}$ ft, and $b_2 = 1\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 2$ ft.

$$A = \frac{1}{2}h(b_1 + b_2)$$
$$= \frac{1}{2}(2.5)(1.5 + 2)$$
$$= 4.375 \text{ or } 4\frac{3}{8}$$

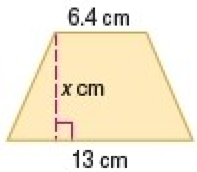
She will need $4\frac{3}{8} \cdot 2$ or $8\frac{3}{4}$ square feet of fabric.

ANSWER:

$8\frac{3}{4}\ \text{ft}^2$

ALGEBRA Find *x*.

5. $A = 78\text{ cm}^2$



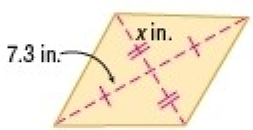
SOLUTION:

$$\begin{aligned} A &= \frac{1}{2}(b_1+b_2)h \\ 78 &= \frac{1}{2}(6.4+13)(x) \\ 78 &= 9.7x \\ \frac{78}{9.7} &= x \\ 8 &\approx x \end{aligned}$$

ANSWER:

8 cm

6. $A = 96\text{ in}^2$



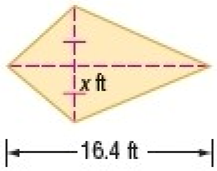
SOLUTION:

$$\begin{aligned} A &= \frac{1}{2}d_1 d_2 \\ 96 &= \frac{1}{2}(2x)(14.6) \\ \frac{96}{14.6} &= x \\ 6.6 &\approx x \end{aligned}$$

ANSWER:

6.6 in.

7. $A = 104 \text{ ft}^2$



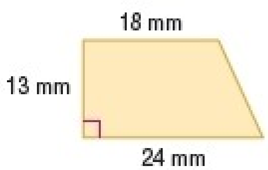
SOLUTION:

$$A = \frac{1}{2}d_1d_2$$
$$104 = \frac{1}{2}(2x)(16.4)$$
$$\frac{104}{16.4} = x$$
$$6.3 \approx x$$

ANSWER:

6.3 ft

CCSS STRUCTURE Find the area of each trapezoid, rhombus, or kite.

8. 

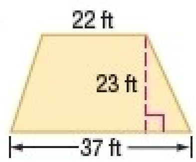
SOLUTION:

$h = 13$, $b_1 = 18$ and $b_2 = 24$

$$A = \frac{1}{2}(b_1 + b_2)h$$
$$= \frac{1}{2}(18 + 24)13$$
$$= \frac{13}{2}(42)$$
$$= 273$$

ANSWER:

273 mm^2

9. 

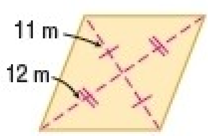
SOLUTION:

$h = 23$, $b_1 = 22$ and $b_2 = 37$

$$\begin{aligned} A &= \frac{1}{2}(b_1 + b_2)h \\ &= \frac{1}{2}(22 + 37)23 \\ &= \frac{23}{2}(59) \\ &= 678.5 \end{aligned}$$

ANSWER:

678.5 ft^2

10. 

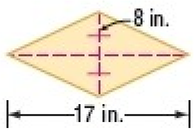
SOLUTION:

$d_1 = 22$ and $d_2 = 24$

$$\begin{aligned} A &= \frac{1}{2}(d_1 \times d_2) \\ &= \frac{1}{2}(22 \times 24) \\ &= \frac{1}{2}(528) \\ &= 264 \end{aligned}$$

ANSWER:

264 m^2

11. 

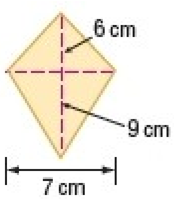
SOLUTION:

d_1 = 16 in. and d_2 = 17 in.

$$\begin{aligned} A &= \frac{1}{2}(d_1 \times d_2) \\ &= \frac{1}{2}(16 \times 17) \\ &= \frac{1}{2}(272) \\ &= 136 \end{aligned}$$

ANSWER:

136 in^2

12. 

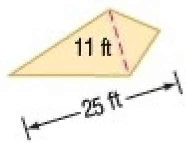
SOLUTION:

d_1 = 7 cm. and d_2 = 6 + 9 = 15 cm

$$\begin{aligned} A &= \frac{1}{2}(d_1 \times d_2) \\ &= \frac{1}{2}(7 \times 15) \\ &= \frac{1}{2}(105) \\ &= 52.5 \end{aligned}$$

ANSWER:

52.5 cm^2

13. 

SOLUTION:

d_1 = 11 ft. and d_2 = 25 ft

$$\begin{aligned} A &= \frac{1}{2}(d_1 \times d_2) \\ &= \frac{1}{2}(11 \times 25) \\ &= \frac{1}{2}(275) \\ &= 137.5 \end{aligned}$$

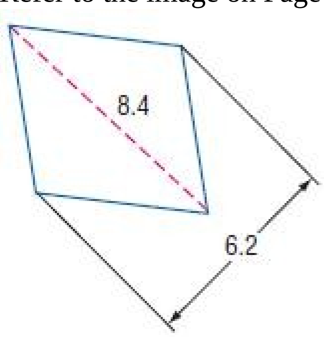
ANSWER:

137.5 ft^2

MICROSCOPES Find the area of the identified portion of each magnified image. Assume that the identified portion is either a trapezoid, rhombus, or kite. Measures are provided in microns.

14. human skin

Refer to the image on Page 777.



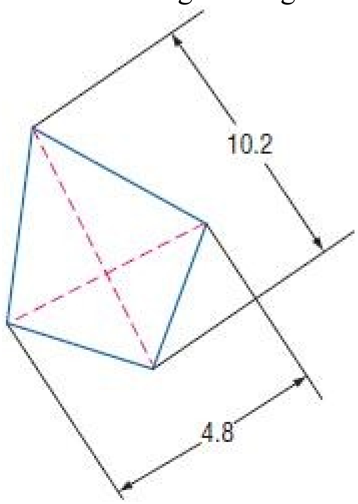
SOLUTION:

$$\begin{aligned} A &= \frac{1}{2}d_1 d_2 \\ &= \frac{1}{2}(6.2)(8.4) \\ &= 26.04 \end{aligned}$$

ANSWER:

26 square microns

15. heartleaf plant
    Refer to the image on Page 777.



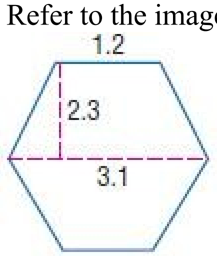
SOLUTION:

$$\begin{aligned} A &= \frac{1}{2}d_1d_2 \\ &= \frac{1}{2}(4.8)(10.2) \\ &= 24.48 \end{aligned}$$

ANSWER:

24.5 square microns

16. eye of a fly
    Refer to the image on Page 777.



SOLUTION:

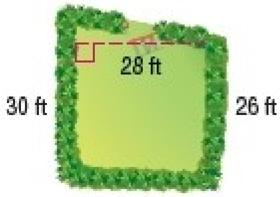
The figure consists of two trapezoids.

$$\begin{aligned} A &= \frac{1}{2}(b_1+b_2)h \\ &= 2\left[\frac{1}{2}(1.2+3.1)(2.3)\right] \\ &= (3.3)(2.3) \\ &= (3.3)(2.3) \end{aligned}$$

ANSWER:

9.9 square microns

17. **JOBS** Jimmy works on his neighbors' yards after school to earn extra money to buy a car. He is going to plant grass seed in Mr. Troyer's yard. What is the area of the yard?



SOLUTION:

$$\begin{aligned} A &= \frac{1}{2}(b_1+b_2)h \\ &= \frac{1}{2}(26+30)(28) \\ &= 784 \end{aligned}$$

ANSWER:

784 ft^2

ALGEBRA Find each missing length.

18. One diagonal of a kite is twice as long as the other diagonal. If the area of the kite is 240 square inches, what are the lengths of the diagonals?

SOLUTION:

The area A of a kite is one half the product of the lengths of its diagonals, d_1 and d_2.

$$\begin{aligned} A &= \frac{1}{2}d_1d_2 \\ 240 &= \frac{1}{2}(x)(2x) \\ 240 &= x^2 \\ \sqrt{240} &= x \\ 15.5 &\approx x \end{aligned}$$

Therefore, the diagonals are of length 15.5 in. and 31.0 in.

ANSWER:

15.5 in., 31.0 in.

19. The area of a rhombus is 168 square centimeters. If one diagonal is three times as long as the other, what are the lengths of the diagonals?

SOLUTION:

The area A of a rhombus is one half the product of the lengths of its diagonals, d_1 and d_2.

$$A = \frac{1}{2}d_1d_2$$
$$168 = \frac{1}{2}(x)(3x)$$
$$168 = \frac{3}{2}x^2$$
$$112 = x^2$$
$$\sqrt{112} = x$$
$$10.6 \approx x$$
$$31.7 \approx 3x$$

Therefore, the diagonals are of length 10.6 cm. and 31.7 cm.

ANSWER:

10.6 cm, 31.7 cm

20. A trapezoid has base lengths of 12 and 14 feet with an area of 322 square feet. What is the height of the trapezoid?

SOLUTION:

The area A of a trapezoid is one half the product of the height h and the sum of the lengths of its bases, b_1 and b_2.

$$A = \frac{1}{2}(b_1 + b_2)h$$
$$322 = \frac{1}{2}(12 + 14)(x)$$
$$322 = 13x$$
$$\frac{322}{13} = x$$
$$24.8 \approx x$$

.

ANSWER:

24.8 ft

21. A trapezoid has a height of 8 meters, a base length of 12 meters, and an area of 64 square meters. What is the length of the other base?

SOLUTION:

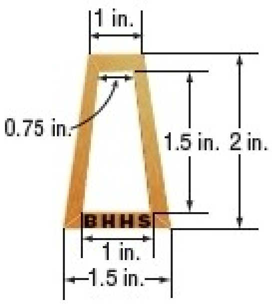
The area A of a trapezoid is one half the product of the height h and the sum of the lengths of its bases, b_1 and b_2.

$$\begin{aligned} A &= \frac{1}{2}(b_1+b_2)h \\ 64 &= \frac{1}{2}(12+x)(8) \\ 64 &= 48+4x \\ 16 &= 4x \\ 4 &= x \end{aligned}$$

ANSWER:

4 m

22. **HONORS** Estella has been asked to join an honor society at school. Before the first meeting, new members are asked to sand and stain the front side of a piece of wood in the shape of an isosceles trapezoid. What is the surface area that Allison will need to sand and stain?



SOLUTION:

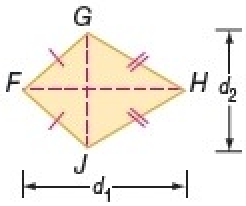
The required area is the difference between the larger trapezoid and the smaller trapezoid.
The area A of a trapezoid is one half the product of the height h and the sum of the lengths of its bases, b_1 and b_2.

$$\begin{aligned} A &= \frac{1}{2}(b_1+b_2)h \\ A &= \frac{1}{2}(1+1.5)(2) - \frac{1}{2}(1+0.75)(1.5) \\ A &= \frac{1}{2}(2.5)(2) - \frac{1}{2}(1.75)(1.5) \\ A &= 2.5-1.3125 \\ A &= 1.1875 \end{aligned}$$

ANSWER:

1.2 in^2

For each figure, provide a justification showing that $A = \frac{1}{2}d_1 d_2$.

23. 

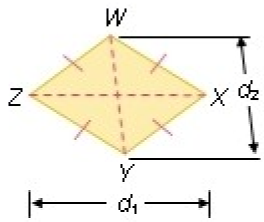
SOLUTION:

Set the area of the kite equal to the sum of the areas of the two triangles with bases d_1 and d_2.

$$\begin{aligned} Area(kite) &= Area(HJF) + Area(HGF) \\ &= \tfrac{1}{2}bh + \tfrac{1}{2}bh \\ &= \tfrac{1}{2}\left[d_1\left(\tfrac{1}{2}d_2\right)\right] + \tfrac{1}{2}\left[d_1\left(\tfrac{1}{2}d_2\right)\right] \\ &= \tfrac{1}{4}d_1 d_2 + \tfrac{1}{4}d_1 d_2 \\ &= \tfrac{1}{2}d_1 d_2 \end{aligned}$$

ANSWER:

The area of $\Delta HJF = \frac{1}{2}d_1\left(\frac{1}{2}d_2\right)$ and the area of $\Delta HGF = \frac{1}{2}d_1\left(\frac{1}{2}d_2\right)$. Therefore, the area of $\Delta HJF = \frac{1}{4}d_1 d_2$, and the area of $\Delta HGF = \frac{1}{4}d_1 d_2$. The area of kite *FGHJ* is equal to the area of $\Delta HJF +$ the area of ΔHGF or $\frac{1}{4}d_1 d_2 + \frac{1}{4}d_1 d_2$. After simplification, the area of kite *FGHJ* is equal to $\frac{1}{2}d_1 d_2$.



24.

SOLUTION:

Set the area of the kite equal to the sum of the areas of the two triangles with bases d_1 and d_2.

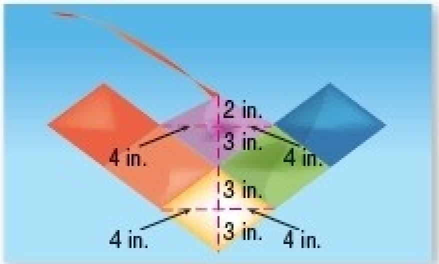
$ZWX = ZYX$ by SSS

$$\begin{aligned} Area(kite) &= Area(ZWX) + Area(ZYX) \\ &= \tfrac{1}{2}bh + \tfrac{1}{2}bh \\ &= \tfrac{1}{2}\left[d_1\left(\tfrac{1}{2}d_2\right)\right] + \tfrac{1}{2}\left[d_1\left(\tfrac{1}{2}d_2\right)\right] \\ &= \tfrac{1}{4}d_1d_2 + \tfrac{1}{4}d_1d_2 \\ &= \tfrac{1}{2}d_1d_2 \end{aligned}$$

ANSWER:

The area of $\Delta ZWX = \frac{1}{2}d_1\left(\frac{1}{2}d_2\right)$ and the area of $\Delta ZYX = \frac{1}{2}d_1\left(\frac{1}{2}d_2\right)$. Therefore, the area of $\Delta ZYX = \frac{1}{4}d_1d_2$, and the area of $\Delta ZWX = \frac{1}{4}d_1d_2$. The area of rhombus *WXYZ* is equal to the area of $\Delta ZWX +$ the area of ΔZYX or $\frac{1}{4}d_1d_2 + \frac{1}{4}d_1d_2$. After simplification, the area of rhombus *WXYZ* is equal to $\frac{1}{2}d_1d_2$.

25. **CRAFTS** Ashanti is in a kite competition. The yellow, red, orange, green, and blue pieces of her kite design shown are all congruent rhombi.
a. How much fabric of each color does she need to buy?
b. Competition rules require that the total area of each kite be no greater than 200 square inches. Does Ashanti's kite meet this requirement? Explain.



SOLUTION:

The area of the yellow rhombus is $\frac{1}{2}(6)(8)$ or 24 in^2. Since the yellow, red , orange, green, and blue pieces area all congruent rhombi, each have an area of 24 in^2.

The area of the purple kite-shaped piece is $\frac{1}{2}(5)(8)$ or 20 in^2.

The total area of the entire kite is 24(5) + 20 or 140 in^2, which is less than the maximum 200 in^2 allowed. Therefore, her kite meets this requirement.

ANSWER:

a. 24 in^2 each of yellow, red, orange, green, and blue; 20 in^2 of purple

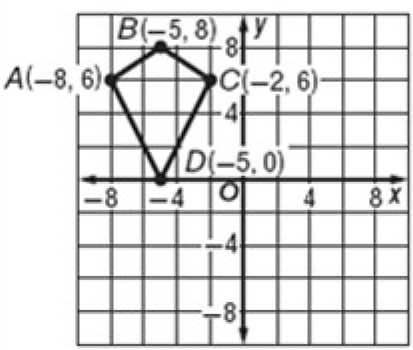
b. Yes; her kite has an area of 140 in^2, which is less than 200 in^2.

CCSS SENSE-MAKING Find the area of each quadrilateral with the given vertices.

26. A(–8, 6), B(–5, 8), C(–2, 6), and D(–5, 0)

SOLUTION:

Graph the quadrilateral.



The quadrilateral is a kite. The area A of a kite is one half the product of the lengths of its diagonals, d_1 and d_2. The lengths of the diagonals are 6 units and 8 units. Therefore, the area of the kite is $\frac{1}{2}(6)(8) = 24 \text{ units}^2$.

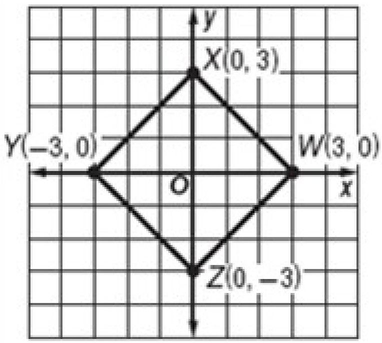
ANSWER:

24 sq. units

27. $W(3, 0)$, $X(0, 3)$, $Y(–3, 0)$, and $Z(0, –3)$

SOLUTION:

Graph the quadrilateral.



The quadrilateral is a rhombus. The area A of a rhombus is one half the product of the lengths of its diagonals, d_1 and d_2.

The lengths of the diagonals are 6 units each. Therefore, the area of the kite is $\frac{1}{2}(6)(6) = 18 \text{ units}^2$.

ANSWER:

18 sq. units

11-2 Areas of Trapezoids, Rhombi, and Kites

28. **METALS** When magnified in very powerful microscopes, some metals are composed of grains that have various polygonal shapes.

a. What is the area of figure 1 if the grain has a height of 4 microns and bases with lengths of 5 and 6 microns?
b. If figure 2 has perpendicular diagonal lengths of 3.8 microns and 4.9 microns, what is the area of the grain?

SOLUTION:

a. Figure 1 is a trapezoid.

$$\begin{aligned}\text{Area of figure } 1 &= \frac{1}{2}(5+6)(4) \\ &= 22\end{aligned}$$

The area of figure 1 is 22 square microns.

b. Figure 2 is a rhombus.

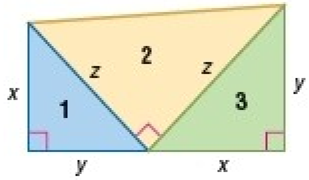
$$\begin{aligned}\text{Area of figure } 2 &= \frac{1}{2}(3.8)(4.9) \\ &= \frac{18.62}{2} \\ &\approx 9.1\end{aligned}$$

The area of figure 2 is about 9.3 square microns.

ANSWER:

a. 22 square microns
b. 9.3 square microns

29. **PROOF** The figure at the right is a trapezoid that consists of two congruent right triangles and an isosceles triangle. In 1876, James A. Garfield, the 20th president of the United States, discovered a proof of the Pythagorean Theorem using this diagram. Prove that $x^2 + y^2 = z^2$.



SOLUTION:

Following is an algebraic proof.

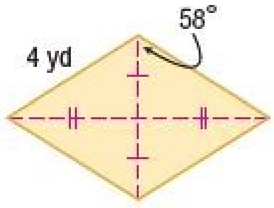
The trapezoid is on its side, so the bases are x and y and the height is $x + y$. Set the area of the trapezoid equal to the sum of the areas of the triangles and simplify.

$$\begin{aligned} Area(triangles) &= Area(trapezoid) \\ 1+2+3 &= \frac{1}{2}h(b_1+b_2) \\ \frac{1}{2}(xy)+\frac{1}{2}(z^2)+\frac{1}{2}(xy) &= \frac{1}{2}(x+y)(y+x) \\ \frac{1}{2}z^2+xy &= \frac{1}{2}x^2+xy+\frac{1}{2}y^2 \\ \frac{1}{2}z^2 &= \frac{1}{2}x^2+\frac{1}{2}y^2 \\ z^2 &= x^2+y^2 \end{aligned}$$

ANSWER:

The area of a trapezoid is $\frac{1}{2}h(b_1+b_2)$ So, $A=\frac{1}{2}(x+y)(x+y)$ or $\frac{1}{2}\left(x^2+2xy+y^2\right)$. The area of $\Delta 1=\frac{1}{2}(y)(x)$, $\Delta 2=\frac{1}{2}(z)(z)$, and $\Delta 3=\frac{1}{2}(x)(y)$. The area of $\Delta 1+\Delta 2+\Delta 3=\frac{1}{2}xy+\frac{1}{2}z^2+\frac{1}{2}xy$. Set the area of the trapezoid equal to the combined areas of the triangles to get $\frac{1}{2}\left(x^2+2xy+y^2\right)=\frac{1}{2}xy+\frac{1}{2}z^2+\frac{1}{2}xy$. Multiply by 2 on each side: $x^2+2xy+y^2=2xy+z^2$. When simplified, $x^2+y^2=z^2$.

DIMENSIONAL ANALYSIS Find the perimeter and area of each figure in feet. Round to the nearest tenth, if necessary.

30. 

SOLUTION:

Both diagonals are perpendicular bisectors, so the figure is a rhombus and all four triangles are congruent. All of the sides are 12 feet, so the perimeter is 48 feet.

Use trigonometry to find the lengths of the diagonals.

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 58 = \frac{x}{4}$$
$$4\sin 58 = x$$
$$8\sin 58 = 2x$$
$$8\sin 58 = d_1$$

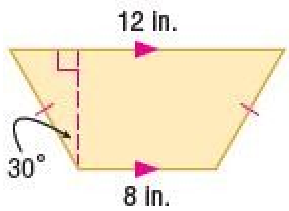
$$\cos y = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 58 = \frac{y}{4}$$
$$4\cos 58 = y$$
$$8\cos 58 = 2y$$
$$8\cos 58 = d_2$$

Now find the area.

$$A = \frac{1}{2}d_1 d_2$$
$$A = \frac{1}{2}(8\sin 58)(8\cos 58)$$
$$A = 14.4\text{yd}^2$$
$$A = 14.4 \times 9$$
$$A = 129.4\text{ft}^2$$

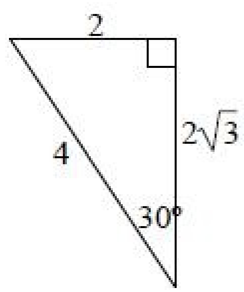
ANSWER:

48 ft; 129.4 ft^2

31. 

SOLUTION:

Use the 30-60-90 triangle to find the dimensions of the isosceles trapezoid. The base of the triangle is 0.5(12 – 8) = 2.



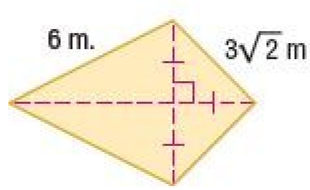
Don't forget to use dimensional analysis to convert the units to feet.

$$\begin{aligned} P &= 2(4) + 8 + 12 \\ &= 28\text{in} \\ &= \frac{28}{12}\text{ft} \\ &\approx 2.3\text{ft} \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}(b_1 + b_2)h \\ &= \frac{1}{2}(12 + 18)\left(2\sqrt{3}\right) \\ &= 30\sqrt{3} \\ &= \frac{30\sqrt{3}}{144}\text{in}^2 \\ &\approx 0.24\,\text{ft}^2 \end{aligned}$$

ANSWER:

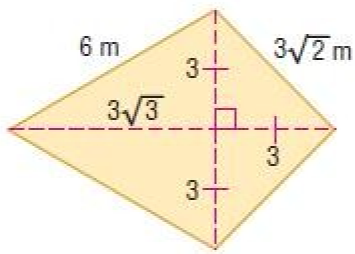
2.3 ft; 0.24 ft^2

32. 

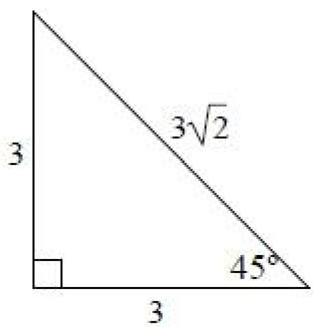
SOLUTION:

The figure is a kite because one of the diagonals is a perpendicular bisector. Find the perimeter. Convert the units to feet.

$$\begin{aligned} P &= 6+6+3\sqrt{2}+3\sqrt{2} \\ &= 12+6\sqrt{2}\,\mathrm{m} \\ &= \frac{12+6\sqrt{2}\mathrm{m}}{1} \cdot \frac{1.1\mathrm{yd}}{1\mathrm{m}} \cdot \frac{3\mathrm{ft}}{1\mathrm{yd}} \\ &\approx 67.6\mathrm{ft} \end{aligned}$$



Use the 45-45-90 triangle to find the lengths of the congruent parts of the diagonals.



$d_1 = 3 + 3 = 6$

Use the Pythagorean theorem to find the other piece of d_2.

$$\begin{aligned} a^2+b^2 &= c^2 \\ 3^2+b^2 &= 6^2 \\ b^2 &= 6^2-3^2 \\ b^2 &= 36-9 \\ b &= \sqrt{27} \\ b &= 3\sqrt{3} \end{aligned}$$

$$d_2 = 3+3\sqrt{3}$$

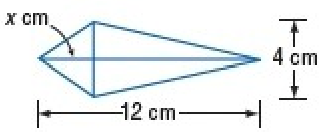
Now find the area of the kite.

$$A = \frac{1}{2}d_1 d_2$$
$$A = \frac{1}{2}(6)\left(3+3\sqrt{3}\right)$$
$$A = 9+9\sqrt{3}\text{m}^2$$
$$A = \frac{9+9\sqrt{3}\text{m}^2}{1}\cdot\frac{(1.1)^2\text{yd}^2}{1\text{m}^2}\cdot\frac{3^2\text{ft}^2}{1\text{yd}^2}$$
$$A \approx 267.8$$

ANSWER:

67.6 ft; 267.8 ft^2

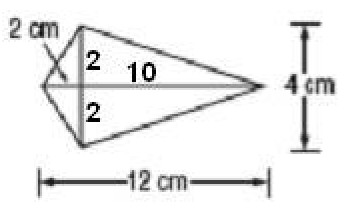
33. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate perimeters of kites.



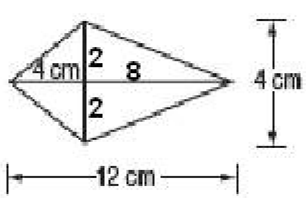
a. GEOMETRIC Draw a kite like the one shown if x = 2.
b. GEOMETRIC Repeat the process in part **a** for three x-values between 2 and 10 and for an x-value of 10.
c. TABULAR Measure and record in a table the perimeter of each kite, along with the x-value.
d. GRAPHICAL Graph the perimeter versus the x-value using the data from your table.
e. ANALYTICAL Make a conjecture about the value of x that will minimize the perimeter of the kite. What is the significance of this value?

SOLUTION:

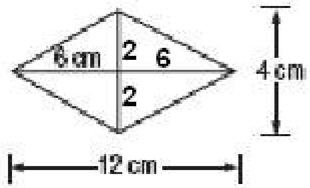
a. When x = 2, the long diagonal will be in pieces of length 2 and 10.



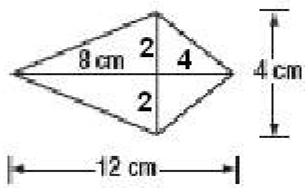
b. When x = 4, the long diagonal will be in pieces of length 4 and 8.



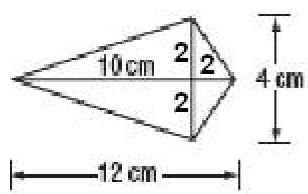
When x = 6, the long diagonal will be in pieces of length 6 and 6.



When x = 8, the long diagonal will be in pieces of length 8 and 4.



When x = 10, the long diagonal will be in pieces of length 10 and 2.



c. Use the Pythagorean Theorem to first find the lengths of the sides of the kite and then find its perimeter.

For x = 2: Two sides of the kite are the length of the hypotenuse of right triangle with legs of 2 cm each and the other two sides are the length of the hypotenuse of a right triangle with legs of 2 cm and 10 cm. Thus, the length of two legs = $\sqrt{2^2+2^2}$ or $\sqrt{8}$ and the length of the other two sides = $\sqrt{2^2+10^2}$ or $\sqrt{104}$. So, the perimeter of the kite is $P = 2\sqrt{8}+2\sqrt{104}$ or about 26.1 cm.

For x = 4: Two sides of the kite are the length of the hypotenuse of right triangle with legs of 2 cm and 4 cm and the other two sides are the length of the hypotenuse of a right triangle with legs of 2 cm and 8 cm. Thus, the length of two legs = $\sqrt{2^2+4^2}$ or $\sqrt{20}$ and the length of the other two sides = $\sqrt{2^2+8^2}$ or $\sqrt{68}$. So, the perimeter of the kite is $P = 2\sqrt{20}+2\sqrt{68}$ or about 25.4 cm.

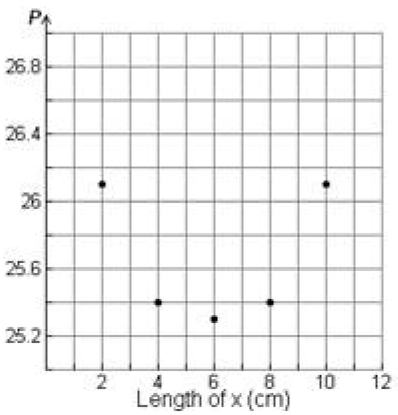
For x = 6: Two sides of the kite are the length of the hypotenuse of right triangle with legs of 2 cm and 6 cm and the other two sides are the length of the hypotenuse of a right triangle with legs of 2 cm and 6 cm. Thus, the length of two legs = $\sqrt{2^2+6^2}$ or $\sqrt{40}$ and the length of the other two sides = $\sqrt{2^2+6^2}$ or $\sqrt{40}$. So, the perimeter of the kite is $P = 2\sqrt{40}+2\sqrt{40}$ or about 25.3 cm.

For x = 8: Two sides of the kite are the length of the hypotenuse of right triangle with legs of 2 cm and 8 cm and the other two sides are the length of the hypotenuse of a right triangle with legs of 2 cm and 4 cm. Thus, the length of two legs = $\sqrt{2^2+8^2}$ or $\sqrt{68}$ and the length of the other two sides = $\sqrt{2^2+4^2}$ or $\sqrt{20}$. So, the perimeter of the kite is $P = 2\sqrt{68}+2\sqrt{20}$ or about 25.4 cm.

For x = 10: Two sides of the kite are the length of the hypotenuse of right triangle with legs of 2 cm and 10 cm and the other two sides are the length of the hypotenuse of a right triangle with legs of 2 cm each. Thus, the length of two legs = $\sqrt{2^2+10^2}$ or $\sqrt{104}$ and the length of the other two sides = $\sqrt{2^2+2^2}$ or $\sqrt{8}$. So, the perimeter of the kite is $P = 2\sqrt{104} + 2\sqrt{8}$ or about 26.1 cm.

x	*P*
2 cm	26.1
4 cm	25.4
6 cm	25.3
8 cm	25.4
10 cm	26.1

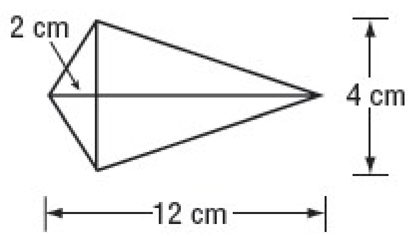
d.



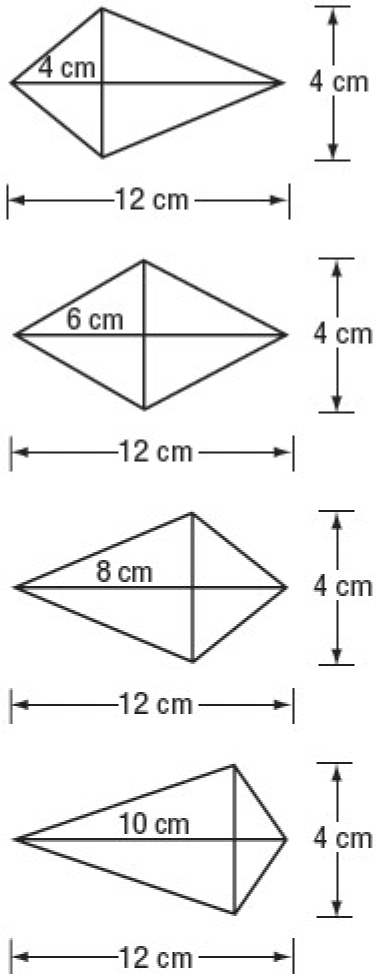
e. Sample answer: Based on the graph, the perimeter will be minimized when x = 6. This value is significant because when x = 6, the figure is a rhombus.

ANSWER:

a.



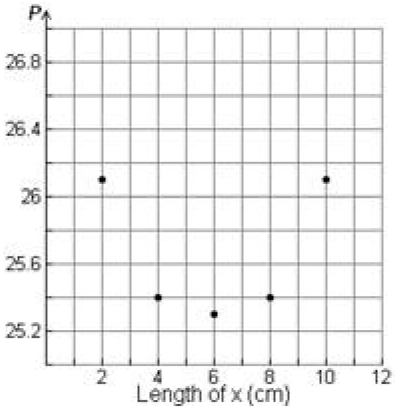
b.



c.

x	*P*
2 cm	26.1
4 cm	25.4
6 cm	25.3
8 cm	25.4
10 cm	26.1

d.

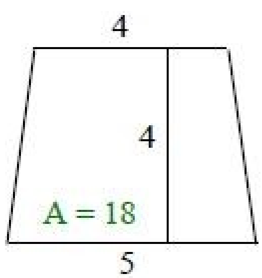


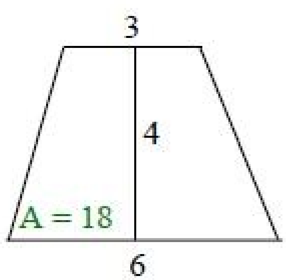
e. Sample answer: Based on the graph, the perimeter will be minimized when $x = 6$. This value is significant because when $x = 6$, the figure is a rhombus.

34. **CCSS CRITIQUE** Antonio and Madeline want to draw a trapezoid that has a height of 4 units and an area of 18 square units. Antonio says that only one trapezoid will meet the criteria. Madeline disagrees and thinks that she can draw several different trapezoids with a height of 4 units and an area of 18 square units. Is either of them correct? Explain your reasoning.

SOLUTION:

There is more than one trapezoid with a height of 4 units and an area of 18 square units. The sum of the bases of the trapezoid has to be 9, so one possibility is a trapezoid with bases of 4 and 5 units and a height of 4 units. Another is a trapezoid with bases of 3 and 6 units and a height of 4 units. Therefore, Madeline is correct.

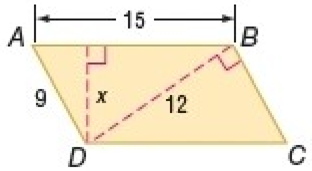




ANSWER:

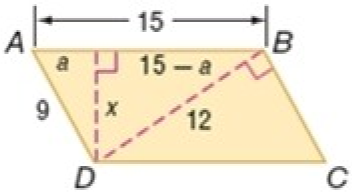
Madeline; sample answer: There is more than one trapezoid with a height of 4 units and an area of 18 square units. The sum of the bases of the trapezoid has to be 9, so one possibility is a trapezoid with bases of 4 and 5 units and a height of 4 units. Another is a trapezoid with bases of 3 and 6 units and a height of 4 units.

35. **CHALLENGE** Find x in parallelogram $ABCD$.



SOLUTION:

Let the two segments of the side $\overline{AB}$ be a and $15 - a$.



We have two variables, x and a. They are a part of two right triangles, so we can use the Pythagorean Theorem for each triangle and then combine the equations.

$$a^2 + x^2 = 9^2$$
$$x^2 = 81 - a^2$$

$$x^2 + (15 - a)^2 = 12^2$$
$$x^2 = 144 - (15 - a)^2$$

We now have x2 on the left side of each equation. Use substitution.

$$144 - (15 - a)^2 = 81 - a^2$$
$$144 - (225 - 30a + a^2) = 81 - a^2$$
$$144 - 225 + 30a - a^2 = 81 - a^2$$
$$-81 + 30a = 81$$
$$30a = 162$$
$$a = 5.4$$

Use the value of a to find the value of x.

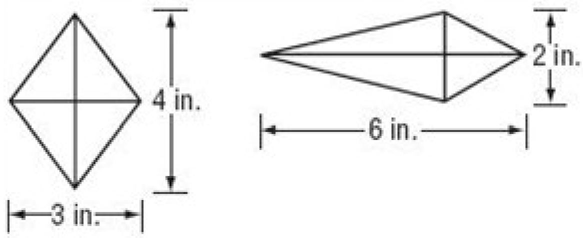
$$x^2 = 9^2 - 5.4^2$$
$$x = \sqrt{9^2 - 5.4^2}$$
$$= \sqrt{51.84}$$
$$= 7.2$$

ANSWER:

7.2

36. **OPEN ENDED** Draw a kite and a rhombus with an area of 6 square inches. Label and justify your drawings.

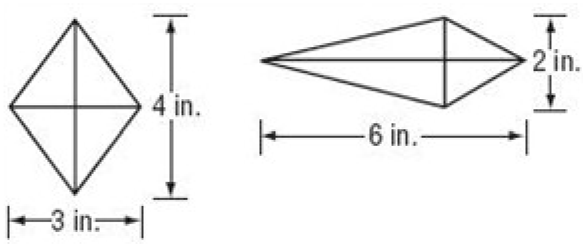
SOLUTION:



Sample answer: Since the area formula for both a rhombus and a kite is one half the product of the lengths of the two diagonals, if the area is 6 square inches, the product of the two diagonals must be 12. I used 3 and 4 inches for the diagonals of the rhombus and 2 and 6 inches for the diagonals of the kite.

Both diagonals must be perpendicular bisectors for the rhombus and one diagonal must be a perpendicular bisector for the kite.

ANSWER:



Sample answer: Since the area formula for both a rhombus and a kite is one half the product of the lengths of the two diagonals, if the area is 6 square inches, the product of the two diagonals must be 12. I used 3 and 4 inches for the diagonals of the rhombus and 2 and 6 inches for the diagonals of the kite.

37. **REASONING** If the areas of two rhombi are equal, are the perimeters *sometimes*, *always*, or *never* equal? Explain.

SOLUTION:

If the areas are equal, it means that the products of the diagonals are equal. The only time that the perimeters will be equal is when the diagonals are also equal, or when the two rhombi are congruent. Therefore, the statement is *sometimes* true.

For example,

When the diagonals are 6 and 8, the area is 24 and the perimeter is 26.

When the diagonals are 12 and 4, the area is 24 and the perimeter is 32.

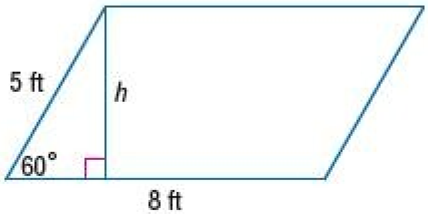
ANSWER:

Sometimes; sample answer: If the areas are equal, it means that the products of the diagonals are equal. The only time that the perimeters will be equal is when the diagonals are also equal, or when the two rhombi are congruent.

38. **WRITING IN MATH** How can you use trigonometry to find the area of a figure?

SOLUTION:

Trigonometry can be used with known angles and sides to find unknown sides. For example;



The height of the parallelogram can be found using the sine function:

$\sin(60°) = \frac{h}{5}$

$h = 5\sin\left(60°\right)$

And the area can be found as usual:

$A = bh$

$A = (8)(5\sin(60°))$

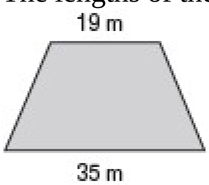
$A = 40\sin(60°)$

$A \approx 34.6\text{ft.}^2$

ANSWER:

Sample answer: You can use trigonometry and known angle and side measures to find unknown triangular measures that are required to calculate the area.

39. The lengths of the bases of an isosceles trapezoid are shown below.



If the perimeter is 74 meters, what is its area?

A 162 m^2

B 270 m^2

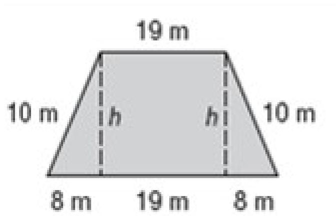
C 332.5 m^2

D 342.25 m^2

SOLUTION:

The lengths of the legs of an isosceles trapezoid are congruent. The perimeter is 74 and the sum of the lengths of the bases is 54 m, so the sides are each 10 m.

We can draw dashed lines for the height to produce two congruent triangles



The bases of the triangles are 8 because the middle figure is a rectangle and the entire base is 35.

Use the Pythagorean Theorem to find the height *h*.

$$\begin{aligned} 8^2 + h^2 &= 10^2 \\ h^2 &= 10^2 - 8^2 \\ h^2 &= 100 - 64 \\ h^2 &= 36 \\ h &= \sqrt{36} \\ h &= 6 \end{aligned}$$

Now find the area of the trapezoid.

$$\begin{aligned} A &= \frac{1}{2}(b_1 + b_2)h \\ &= \frac{1}{2}(19 + 35)(6) \\ &= 27(6) \\ &= 162 \end{aligned}$$

ANSWER:
A

40. **SHORT RESPONSE** One diagonal of a rhombus is three times as long as the other diagonal. If the area of the rhombus is 54 square millimeters, what are the lengths of the diagonals?

SOLUTION:

$$A = \frac{1}{2}d_1d_2$$
$$54 = \frac{1}{2}(x)(3x)$$
$$54 = \frac{3}{2}x^2$$
$$36 = x^2$$
$$6 = x$$

Therefore, the diagonals are of length 6 mm. and 18 mm.

ANSWER:

6 mm, 18 mm

41. **ALGEBRA** What is the effect on the graph of the equation $y = \frac{1}{2}x$ when the equation is changed to $y = -2x$?

F The graph is moved 1 unit down.
G The graph is moved 1 unit up.
H The graph is rotated 45° about the origin.
J The graph is rotated 90° about the origin.

SOLUTION:

The product of the slopes of the two lines is –1. So, the lines are perpendicular to each other.

When two lines are perpendicular, the first line is rotated 90° about the origin to get the second line.

Therefore, the correct choice is J.

ANSWER:

J

42. A regular hexagon is divided into 6 congruent triangles. If the perimeter of the hexagon is 48 centimeters, what is the height of each triangle?

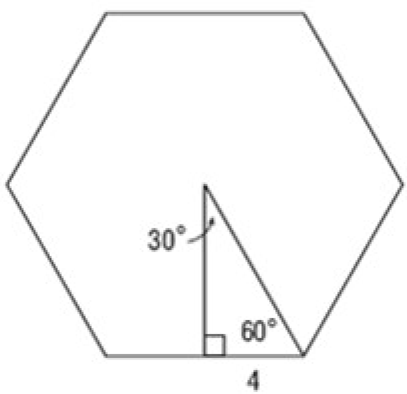
A 4 cm
B $4\sqrt{3}$cm
C $6\sqrt{3}$ cm
D 8 cm
E $8\sqrt{3}$cm

SOLUTION:

The perimeter of the hexagon is 48 cm, so the length of each side of the hexagon is 8 cm. The height of each triangle is equal to the apothem of the hexagon. Each interior angle of a regular hexagon measures $\frac{6-2}{6}\cdot 180 = 120.$ So, each diagonal will make an angle of 60° with the sides.

An apothem bisects the side of the polygon.

The triangle with the sides of one apothem, half the side of the polygon to which the apothem is drawn, and the line joining the center and the vertex at endpoints of the side form a $30° - 60° - 90°$ triangle.



Let *h* be the height of the triangle. Use the tangent ratio to find *h*.

$$\tan 60° = \frac{h}{4}$$
$$h = 4\tan 60°$$
$$h = 4\sqrt{3}$$

The height of each triangle is $4\sqrt{3}$ cm.

ANSWER:

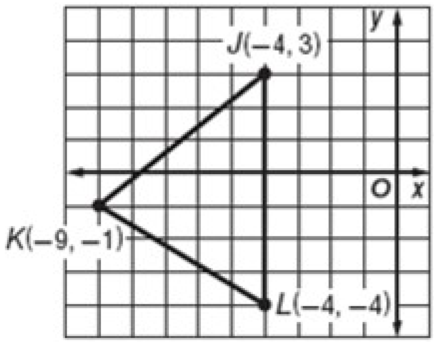
B

COORDINATE GEOMETRY Find the area of each figure.

43. $\triangle JKL$ with $J(–4, 3)$, $K(–9, –1)$, and $L(–4, –4)$

SOLUTION:

Plot the points and draw the triangle.



The length of base JL is 7 units. The corresponding height is from the line $x = –9$ to $x = –4$, so the height is 5 units.

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(7)(5) \\ &= 17.5 \end{aligned}$$

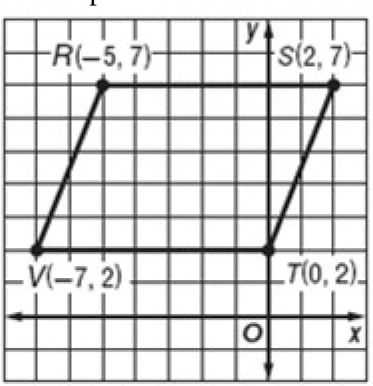
ANSWER:

17.5 units^2

44. ▱ $RSTV$ with $R(–5, 7)$, $S(2, 7)$, $T(0, 2)$, and $V(–7, 2)$

SOLUTION:

Plot the points.



The parallelogram has base length of 7 units.

The height is from $y = 2$ to $y = 7$, so the height is 5 units. The area of the parallelogram is 7(5) or 35 units^2.

ANSWER:

35 units^2

45. **WEATHER** Meteorologists track severe storms using Doppler radar. A polar grid is used to measure distances as the storms progress. If the center of the radar screen is the origin and each ring is 10 miles farther from the center, what is the equation of the fourth ring?
Refer to the image on Page 780.

SOLUTION:

The standard form of the equation of a circle with center at (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

$(h, k) = (0, 0)$

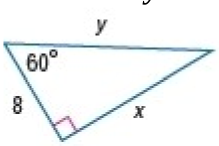
The fourth ring will be 40 miles away from the origin, so $r = 40$.

Therefore, the equation is $(x - 0)^2 + (y - 0)^2 = 40^2$ or $x^2 + y^2 = 1600$.

ANSWER:

$x^2 + y^2 = 1600$

Find *x* and *y*.

46. 

SOLUTION:

This is a 30°-60°-90° triangle.

Side opposite the 30° = a
Side opposite the 60° = $a\sqrt{3}$
Hypotenuse = $2a$

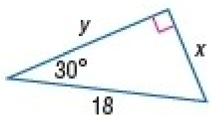
For this triangle:

Side opposite the 30° = 8
Side opposite the 60° = $8\sqrt{3}$
Hypotenuse = 16

Therefore, $x = 8\sqrt{3}$ and $y = 12$

ANSWER:

$x = 8\sqrt{3}; y = 16$

47. 

SOLUTION:
This is a 30°-60°-90° triangle.

Side opposite the 30° = *a*
Side opposite the 60° = $a\sqrt{3}$
Hypotenuse = 2*a*

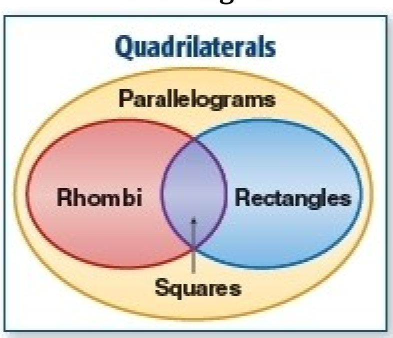
For this triangle:

Side opposite the 30° = 9
Side opposite the 60° = $9\sqrt{3}$
Hypotenuse = 18

Therefore, x = 9 and y = $9\sqrt{3}$

ANSWER:
$x = 9; y = 9\sqrt{3}$

Use the Venn diagram to determine whether each statement is *always, sometimes,* or *never* true.



48. A parallelogram is a square.

SOLUTION:
When a figure is a parallelogram, it can be anything inside the yellow sphere of the Venn diagram. The figure is a square only when it is inside the purple intersection in the middle of the diagram. Therefore, a figure that is a parallelogram is *sometimes* also a square.

ANSWER:
sometimes

49. A square is a rhombus.

SOLUTION:

When a figure is a square, it is inside the purple intersection in the middle of the diagram. This region is within the region of a rhombus, so when a figure is a square, it is *always* a rhombus as well.

ANSWER:

always

50. A rectangle is a parallelogram.

SOLUTION:

When a figure is a rectangle, it can be anything inside the blue sphere of the Venn diagram. This region is within the region of a parallelogram, so any figure that is a rectangle is *always* also a parallelogram.

ANSWER:

always

51. A rhombus is a rectangle but not a square.

SOLUTION:

When a figure is a rhombus, it can be anything inside the red sphere of the Venn diagram. The figure is a rectangle when it is inside the blue sphere. The intersection of these two spheres indicate when a figure is both a rhombus and a rectangle. However, everything within this region is also a square, so a figure can *never* be both a rhombus and a rectangle while not also being a square.

ANSWER:

never

52. A rhombus is a square.

SOLUTION:

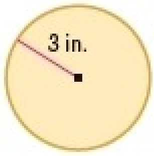
When a figure is a rhombus, it can be anything inside the red sphere of the Venn diagram. When a figure is within the purple region, it is a square. However, a figure within the red region but not within the purple region is a rhombus that is not a square.

Therefore, a figure can *sometimes* be both a rhombus and a square.

ANSWER:

sometimes

Find the circumference and area of each figure. Round to the nearest tenth.

53. 

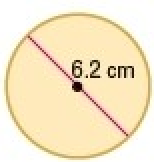
SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(3)^2 \\ &= 9\pi \\ &\approx 28.3 \end{aligned}$$

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(3) \\ &= 6\pi \\ &\approx 18.8 \end{aligned}$$

ANSWER:

18.8 in.;
28.3 in^2

54. 

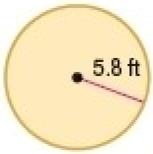
SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(3.1)^2 \\ &= 9.61\pi \\ &\approx 30.2 \end{aligned}$$

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(3.1) \\ &= 6.2\pi \\ &\approx 19.5 \end{aligned}$$

ANSWER:

19.5 cm;
30.2 cm^2

55. 

SOLUTION:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(5.8)^2 \\ &= 33.64\pi \\ &\approx 105.7 \end{aligned}$$

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(5.8) \\ &= 11.6\pi \\ &\approx 36.4 \end{aligned}$$

ANSWER:

36.4 ft;

105.7 ft^2