10-8 Equations of Circle

Write the equation of each circle.

1. center at (9, 0), radius 5

SOLUTION:

$$(x-h)^2+(y-k)^2 = r^2 \quad \text{Equation of a circle}$$
$$(x-9)^2+(y-0)^2 = 5^2 \quad (h, k) = (9, 0), r = 5$$
$$(x-9)^2+y^2 = 25 \quad \text{Simplify.}$$

ANSWER:

$(x-9)^2 + y^2 = 25$

2. center at (3, 1), diameter 14

SOLUTION:

Since the radius is half the diameter, $r = \frac{1}{2}(14)$ or 7.

$$(x-h)^2+(y-k)^2 = r^2 \quad \text{Equation of a circle}$$
$$(x-3)^2+(y-1)^2 = 7^2 \quad (h, k) = (3, 1), r = 7$$
$$(x-3)^2+(y-1)^2 = 49 \quad \text{Simplify.}$$

ANSWER:

$(x-3)^2 + (y-1)^2 = 49$

3. center at origin, passes through (2, 2)

SOLUTION:

Find the distance between the points to determine the radius.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(2-0)^2 + (2-0)^2} \quad (x_1, y_1) = (2, 2) \text{ and } (x_2, y_2) = (0, 0)$$
$$= \sqrt{4+4} \quad \text{Simplify.}$$
$$= \sqrt{8} \quad \text{Simplify.}$$

Write the equation using h = 0, k = 0, and $r = \sqrt{8}$.

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Equation of a circle}$$
$$(x-0)^2 + (y-0)^2 = \left(\sqrt{8}\right)^2 \quad h = 0, k = 0, r = \sqrt{8}$$
$$x^2 + y^2 = 8 \quad \text{Simplify.}$$

ANSWER:

$x^2 + y^2 = 8$

4. center at (–5, 3), passes through (1, –4)

SOLUTION:

Find the distance between the points to determine the radius.

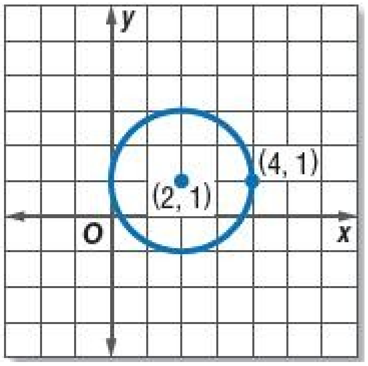
$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(1-(-5))^2 + (-4-3)^2} \quad (x_1, y_1) = (1, -4) \text{ and } (x_2, y_2) = (-5, 3)$$
$$= \sqrt{36+49} \quad \text{Simplify.}$$
$$= \sqrt{85} \quad \text{Simplify.}$$

Write the equation using h = -5, k = 3, and $r = \sqrt{85}$.

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Equation of a circle}$$
$$(x-(-5))^2 + (y-3)^2 = \left(\sqrt{85}\right)^2 \quad h = -5, k = 3, r = \sqrt{85}$$
$$(x+5)^2 + (y-3)^2 = 85 \quad \text{Simplify.}$$

ANSWER:

$(x+5)^2 + (y-3)^2 = 85$

5. 

SOLUTION:

Find the distance between the points to determine the radius.

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(4-2)^2 + (1-1)^2} && (x_1, y_1) = (4, 1) \text{ and } (x_2, y_2) = (2, 1) \\ &= \sqrt{4+0} \text{ or } 2 && \text{Simplify.} \end{aligned}$$

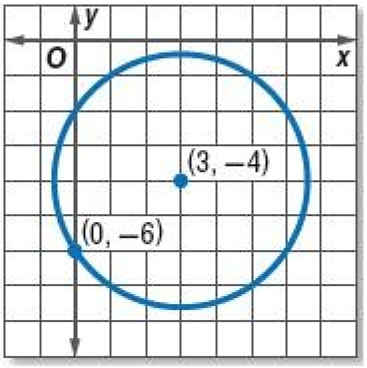
Write the equation using $h = 2$, $k = 1$, and $r = 2$.

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 && \text{Equation of a circle} \\ (x-2)^2 + (y-1)^2 &= 2^2 && h = 2, k = 1, r = 2 \\ (x-2)^2 + (y-1)^2 &= 4 && \text{Simplify.} \end{aligned}$$

ANSWER:

$(x-2)^2 + (y-1)^2 = 4$

10-8 Equations of Circle

6. 

SOLUTION:

Find the distance between the points to determine the radius.

$$
\begin{aligned}
r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(0-3)^2 + (-6-(-4))^2} && (x_1, y_1) = (0, -6) \text{ and } (x_2, y_2) = (3, -4) \\
&= \sqrt{9+4} \text{ or } \sqrt{13} && \text{Simplify.}
\end{aligned}
$$

Write the equation using $h = 3$, $k = –4$, and $r = \sqrt{13}$.

$$
\begin{aligned}
(x-h)^2 + (y-k)^2 &= r^2 && \text{Equation of a circle} \\
(x-3)^2 + (y-(-4))^2 &= \left(\sqrt{13}\right)^2 && h = 3, k = -4, r = \sqrt{13} \\
(x-3)^2 + (y+4)^2 &= 13 && \text{Simplify.}
\end{aligned}
$$

ANSWER:

$(x-3)^2 + (y+4)^2 = 13$

10-8 Equations of Circle

For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.

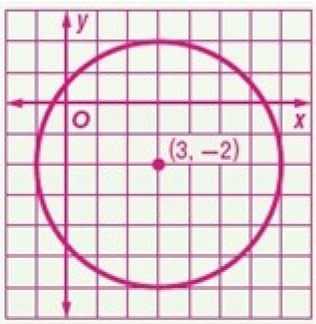
7. $x^2 - 6x + y^2 + 4y = 3$

SOLUTION:

The standard form of the equation of a circle with center at (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.
Use completing the square to rewrite the given equation in standard form.

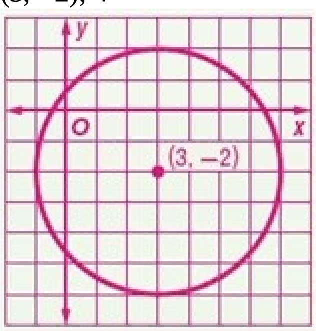
$$x^2 - 6x + y^2 + 4y = 3 \quad \text{Original equation}$$
$$x^2 - 6x + 9 + y^2 + 4y + 4 = 3 + 9 + 4 \quad \text{Add 9 and 4 to each side.}$$
$$(x-3)^2 + (y+2)^2 = 16 \quad \text{Factor and simplify.}$$
$$(x-3)^2 + (y+2)^2 = 4^2 \quad 16 = 4^2$$

So, $h = 3$, $k = -2$, and $r = 4$. The center is at (3, –2), and the radius is 4.



ANSWER:

(3, –2); 4



10-8 Equations of Circle

8. $x^2 + (y+1)^2 = 4$

SOLUTION:

The standard form of the equation of a circle with center at (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.

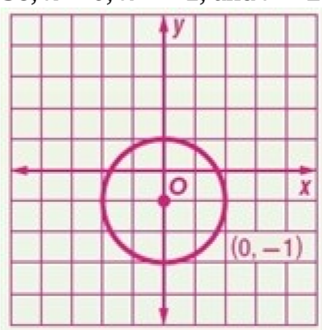
Rewrite $x^2 + (y+1)^2 = 4$ to find the center and the radius.

$$(x-0)^2 + (y-(-1))^2 = 2^2$$

$$\downarrow \qquad\qquad \downarrow \qquad \downarrow$$

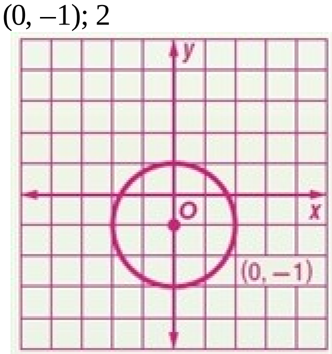
$$(x-h)^2 + (y- \quad k)^2 = r^2$$

So, $h = 0$, $k = -1$, and $r = 2$. The center is at (0, –1), and the radius is 2.



ANSWER:

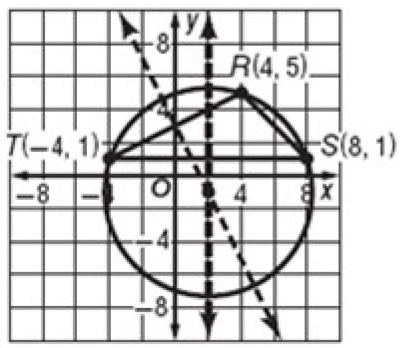
(0, –1); 2



9. **RADIOS** Three radio towers are modeled by the points *R*(4, 5), *S*(8, 1), and *T*(–4, 1). Determine the location of another tower equidistant from all three towers, and write an equation for the circle.

SOLUTION:

Step 1: You are given three points that lie on a circle. Graph triangle *RST* and construct the perpendicular bisectors of two sides to locate the center of the circle. Find the radius and then use the center and radius to write an equation. Construct the perpendicular bisectors of two sides. The center appears to be at (2, –1).



Step 2: Find the distance between the center and one of the points on the circle to determine the radius.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{(4-2)^2 + (5-(-1))^2} \quad (x_1, y_1) = (4, 5) \text{ and } (x_2, y_2) = (2, -1)$$

$$= \sqrt{4+36} \text{ or } \sqrt{40} \quad \text{Simplify.}$$

Step 3: Write the equation of the circle using $h = 2$, $k = –1$, and $r = \sqrt{40}$.

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Equation of a circle}$$

$$(x-2)^2 + (y-(-1))^2 = \left(\sqrt{40}\right)^2 \quad h = 2, k = -1, r = \sqrt{40}$$

$$(x-2)^2 + (y+1)^2 = 40 \quad \text{Simplify.}$$

The location of the tower equidistance from the other three is at (2, –1) and the equation for the circle is $(x-2)^2 + (y+1)^2 = 40$.

ANSWER:

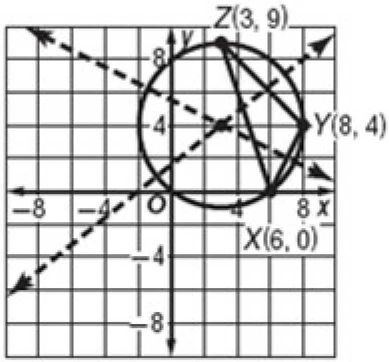
$(2, -1); (x-2)^2 + (y+1)^2 = 40$

10-8 Equations of Circle

10. **COMMUNICATION** Three cell phone towers can be modeled by the points $X(6, 0)$, $Y(8, 4)$, and $Z(3, 9)$. Determine the location of another cell phone tower equidistant from the other three, and write an equation for the circle.

SOLUTION:

Step 1: You are given three points that lie on a circle. Graph triangle XYZ and construct the perpendicular bisectors of two sides to locate the center of the circle. Find the radius and then use the center and radius to write an equation. Construct the perpendicular bisectors of two sides. The center appears to be at (3, 4).



Step 2: Find the distance between the center and one of the points on the circle to determine the radius.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{(6-3)^2 + (0-4)^2} \quad (x_1, y_1) = (6, 0) \text{ and } (x_2, y_2) = (3, 4)$$

$$= \sqrt{25} \text{ or } 5 \quad \text{Simplify.}$$

Step 3: Write the equation of the circle using $h = 3$, $k = 4$, and $r = 5$.

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Equation of a circle}$$

$$(x-3)^2 + (y-4)^2 = 5^2 \quad h = 3, k = 4, r = 5$$

$$(x-3)^2 + (y-4)^2 = 25 \quad \text{Simplify.}$$

The location of the cell phone tower equidistance from the other three is at (3, 4) and the equation for the circle is $(x - 3)^2 + (y - 4)^2 = 25$.

ANSWER:

$(3,4);(x-3)^2+(y-4)^2=25$

10-8 Equations of Circle

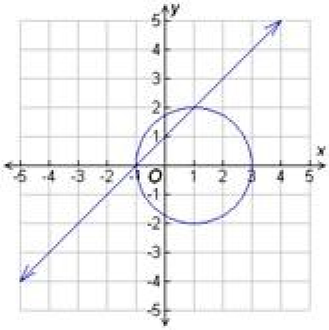
Find the point(s) of intersection, if any, between each circle and line with the equations given.

11. $(x-1)^2+y^2=4$

$y=x+1$

SOLUTION:

Graph these equations on the same coordinate plane. $(x - 1)^2 + y^2 = 4$ is a circle with center (1, 0) and a radius of 2. Draw a line through (0, 1) with a slope of 1 for $y = x + 1$.



The points of intersection are solutions of both equations. You can estimate these points on the graph to be at about (–1, 0) and (1, 2). Use substitution to find the coordinates of these points algebraically.

$$\begin{aligned}
(x-1)^2+y^2 &= 4 && \text{First equation} \\
(x-1)^2+(x+1)^2 &= 4 && \text{Since } y=x+1\text{, substitute } x+1 \text{ for } y. \\
x^2-2x+1+x^2+2x+1 &= 4 && \text{Multiply.} \\
2x^2+2 &= 4 && \text{Simplify.} \\
2x^2 &= 2 && \text{Subtract 2 from each side.} \\
x^2 &= 1 && \text{Divide each side by 2.} \\
x &= \pm 1 && \text{Take the square root of each side.}
\end{aligned}$$

So, $x = 1$ or -1. Use the equation $y = x + 1$ to find the corresponding y-values.
When $x = 1$, $y = 1 + 1$ or 2.
When $x = -1$, $y = -1 + 1$ or 0.
Therefore, the points of intersection are (1, 2) and (−1, 0).

ANSWER:

(1, 2), (–1, 0)

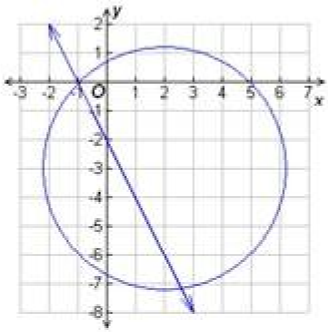
10-8 Equations of Circle

12. $(x-2)^2+(y+3)^2=18$

$y=-2x-2$

SOLUTION:

Graph these equations on the same coordinate plane. $(x-2)^2+(y+3)^2=18$ is a circle with center (2, –3) and a radius of $\sqrt{18}$. Draw a line through (0, 2) with a slope of –2 for $y=-2x-2$.



The points of intersection are solutions of both equations. You can estimate these points on the graph to be at about (–1, 0) and (2.6, –7.2). Use substitution to find the coordinates of these points algebraically.

$$
\begin{aligned}
(x-2)^2+(y+3)^2 &= 18 && \text{First equation}\\
(x-2)^2+(-2x-2+3)^2 &= 18 && \text{Since } y=-2x-2, \text{ substitute } -2x-2 \text{ for } y.\\
x^2-4x+4+4x^2-4x+1 &= 18 && \text{Multiply.}\\
5x^2-8x+5 &= 18 && \text{Simplify.}\\
5x^2-8x-13 &= 0 && \text{Subtract 18 from each side.}
\end{aligned}
$$

Use the quadratic formula to find the solutions for x.

$$
\begin{aligned}
x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} && \text{Quadratic Formula}\\
x &= \frac{-(-8)\pm\sqrt{(-8)^2-4(5)(-13)}}{2(5)} && a=5, b=-8, c=-13\\
x &= \frac{-8\pm\sqrt{324}}{10} && \text{Simplify.}\\
x &= \frac{-8\pm18}{10} && \text{Use a calculator.}\\
x &= -1 \text{ or } 2\tfrac{3}{5} && \text{Simplify.}
\end{aligned}
$$

Use the equation $y=-2x-2$ to find the corresponding y-values.

When $x=-1$, $y=-2(-1)-2$ or 0.

When $x=2\frac{3}{5}$, $y=-2\left(2\frac{3}{5}\right)-2$ or $-7\frac{1}{5}$.

Therefore, the points of intersection are $(-1, 0)$ and $(2\frac{3}{5}, -7\frac{1}{5})$.

ANSWER:

$(-1, 0)$, $(2\frac{3}{5}, -7\frac{1}{5})$

CCSS STRUCTURE Write the equation of each circle.

13. center at origin, radius 4

SOLUTION:

$$
\begin{aligned}
(x-h)^2+(y-k)^2 &= r^2 && \text{Equation of a circle}\\
(x-0)^2+(y-0)^2 &= 4^2 && (h,k)=(0,0),\ r=4\\
x^2+y^2 &= 16 && \text{Simplify.}
\end{aligned}
$$

ANSWER:

$x^2+y^2=16$

14. center at (6, 1), radius 7

SOLUTION:

$$
\begin{aligned}
(x-h)^2+(y-k)^2 &= r^2 && \text{Equation of a circle}\\
(x-6)^2+(y-1)^2 &= 7^2 && (h,k)=(6,1),\ r=7\\
(x-6)^2+(y-1)^2 &= 49 && \text{Simplify.}
\end{aligned}
$$

ANSWER:

$(x-6)^2+(y-1)^2=49$

15. center at (–2, 0), diameter 16

SOLUTION:

Since the radius is half the diameter, $r = \frac{1}{2}(16)$ or 8.

$$
\begin{aligned}
(x-h)^2+(y-k)^2 &= r^2 && \text{Equation of a circle}\\
(x-(-2))^2+(y-0)^2 &= 8^2 && (h,k)=(-2,0),\ r=8\\
(x+2)^2+y^2 &= 64 && \text{Simplify.}
\end{aligned}
$$

ANSWER:

$(x+2)^2+y^2=64$

10-8 Equations of Circle

16. center at (8, –9), radius $\sqrt{11}$

SOLUTION:

$$(x-h)^2+(y-k)^2=r^2 \quad \text{Equation of a circle}$$
$$(x-8)^2+(y-(-9))^2=\left(\sqrt{11}\right)^2 \quad (h,k)=(8,-9), r=\sqrt{11}$$
$$(x-8)^2+(y+9)^2=11 \quad \text{Simplify.}$$

ANSWER:

$(x-8)^2+(y+9)^2=11$

17. center at (–3, 6), passes through (0, 6)

SOLUTION:

Find the distance between the points to determine the radius.

$$r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \quad \text{Distance Formula}$$
$$=\sqrt{(-3-0)^2+(6-6)^2} \quad (x_1,y_1)=(-3,6) \text{ and } (x_2,y_2)=(0,6)$$
$$=\sqrt{9} \text{ or } 3 \quad \text{Simplify.}$$

Write the equation of the circle using h = –3, k = 6, and r = 3.

$$(x-h)^2+(y-k)^2=r^2 \quad \text{Equation of a circle}$$
$$(x-(-3))^2+(y-6)^2=3^2 \quad h=-3, k=6, r=3$$
$$(x+3)^2+(y-6)^2=9 \quad \text{Simplify.}$$

ANSWER:

$(x+3)^2+(y-6)^2=9$

10-8 Equations of Circle

18. center at (1, –2), passes through (3, –4)

SOLUTION:

Find the distance between the points to determine the radius.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{(1-3)^2 + (-2-(-4))^2} \quad (x_1, y_1) = (1, -2) \text{ and } (x_2, y_2) = (3, -4)$$

$$= \sqrt{4+4} \text{ or } \sqrt{8} \quad \text{Simplify.}$$

Write the equation of the circle using h = 1, k = –2, and $r = \sqrt{8}$.

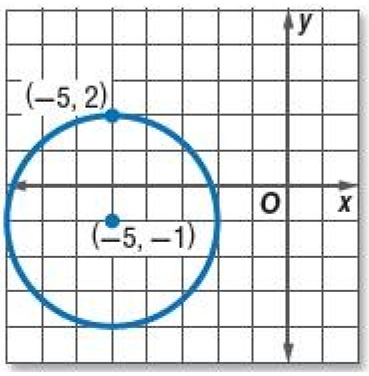
$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Equation of a circle}$$

$$(x-1)^2 + (y-(-2))^2 = \left(\sqrt{8}\right)^2 \quad h = 1, k = -2, r = \sqrt{8}$$

$$(x-1)^2 + (y+2)^2 = 8 \quad \text{Simplify.}$$

ANSWER:

$(x-1)^2 + (y+2)^2 = 8$

19. 

SOLUTION:

The center is (–5, –1) and the radius is 3.

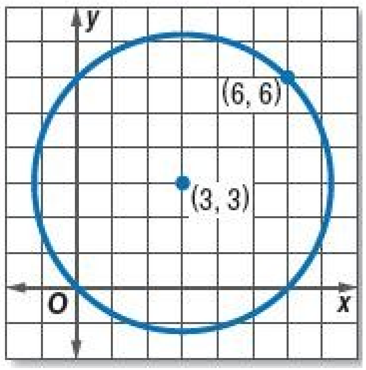
$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Equation of a circle}$$

$$(x-(-5))^2 + (y-(-1))^2 = 3^2 \quad h = -5, k = -1, r = 3$$

$$(x+5)^2 + (y+1)^2 = 9 \quad \text{Simplify.}$$

ANSWER:

$(x+5)^2 + (y+1)^2 = 9$

20. 

SOLUTION:

Find the distance between the points to determine the radius.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(6-3)^2 + (6-3)^2} \quad (x_1, y_1) = (6, 6) \text{ and } (x_2, y_2) = (3, 3)$$
$$= \sqrt{9+9} \text{ or } \sqrt{18} \quad \text{Simplify.}$$

Write the equation of the circle using $h = 3$, $k = 3$, and $r = \sqrt{18}$.

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Equation of a circle}$$
$$(x-3)^2 + (y-3)^2 = \left(\sqrt{18}\right)^2 \quad h = 3, k = 3, r = \sqrt{18}$$
$$(x-3)^2 + (y-3)^2 = 18 \quad \text{Simplify.}$$

ANSWER:

$(x-3)^2 + (y-3)^2 = 18$

21. **WEATHER** A Doppler radar screen shows concentric rings around a storm. If the center of the radar screen is the origin and each ring is 15 miles farther from the center, what is the equation of the third ring?

SOLUTION:

The radius of the third ring would be 15 + 15 + 15 or 45.

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Equation of a circle}$$
$$(x-0)^2 + (y-0)^2 = 45^2 \quad (h, k) = (0, 0), r = 45$$
$$x^2 + y^2 = 2025 \quad \text{Simplify.}$$

Therefore, the equation of the third ring is $x^2 + y^2 = 2025$.

ANSWER:

$x^2 + y^2 = 2025$

22. **GARDENING** A sprinkler waters a circular area that has a diameter of 10 feet. The sprinkler is located 20 feet north of the house. If the house is located at the origin, what is the equation for the circle of area that is watered?

SOLUTION:

Find the equation of a circle with a center of (0, 20) and a radius of $\frac{1}{2}(10)$ or 5 feet.

$$\begin{aligned} (x-h)^2+(y-k)^2 &= r^2 && \text{Equation of a circle} \\ (x-0)^2+(y-20)^2 &= 5^2 && (h,k)=(0,20),\ r=5 \\ x^2+(y-20)^2 &= 25 && \text{Simplify.} \end{aligned}$$

Therefore, the equation for the circle of the area that is watered is $x^2 + (y - 20)^2 = 25$.

ANSWER:

$x^2+(y-20)^2=25$

For each circle with the given equation, state the coordinates of the center and the measure of the radius. Then graph the equation.

23. $x^2 + y^2 = 36$

SOLUTION:

The standard form of the equation of a circle with center at (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.

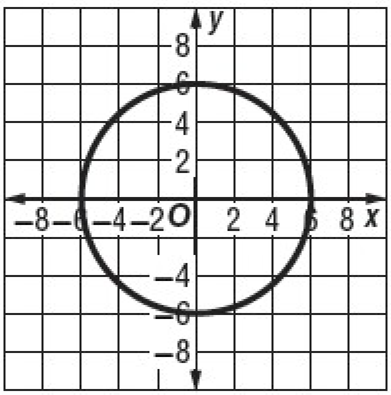
Rewrite $x^2 + y^2 = 36$ to find the center and the radius.

$$(x-0)^2 + (y-0)^2 = 6^2$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

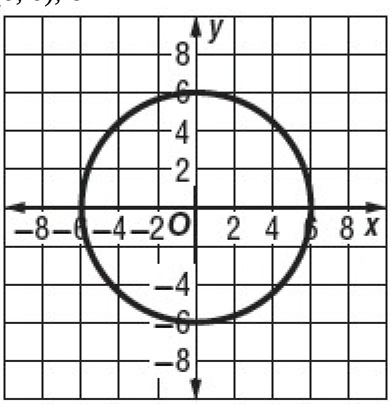
$$(x-h)^2 + (y-k)^2 = r^2$$

So, $h = 0$, $k = 0$, and $r = 6$. The center is at (0, 0), and the radius is 6.



ANSWER:

(0, 0); 6



24. $x^2 + y^2 - 4x - 2y = -1$

SOLUTION:

The standard form of the equation of a circle with center at (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.

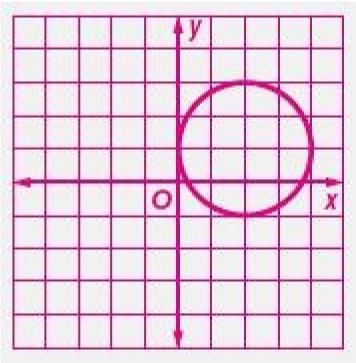
Rewrite $x^2 + y^2 - 4x - 2y = -1$ to find the center and the radius.

$$
\begin{aligned}
x^2 + y^2 - 4x - 2y &= -1 && \text{Original equation} \\
x^2 - 4x + y^2 - 2y &= -1 && \text{Isolate and group like terms.} \\
x^2 - 4x + 4 + y^2 - 2y + 1 &= -1 + 4 + 1 && \text{Complete the squares.} \\
(x-2)^2 + (y-1)^2 &= 4 && \text{Factor and simplify.} \\
(x-2)^2 + (y-1)^2 &= 2^2 && \text{Write 4 as } 2^2.
\end{aligned}
$$

So, $h = 2$, $k = 1$, and $r = 2$.

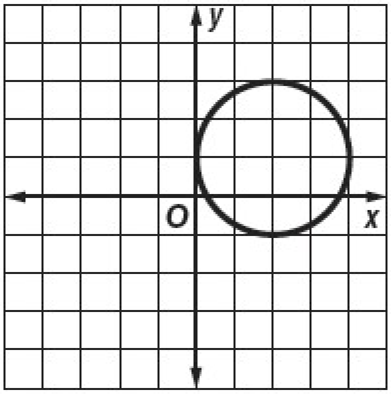
Therefore, the center and radius are (2, 1) and 2.

Plot the center and four points that are 2 units from this point. Sketch the circle through these four points.



ANSWER:

(2, 1); 2



10-8 Equations of Circle

25. $x^2 + y^2 + 8x - 4y = -4$

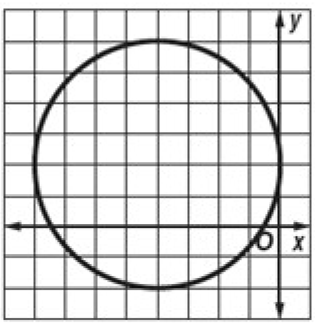
SOLUTION:

The standard form of the equation of a circle with center at (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.
Rewrite $x^2 + y^2 + 8x - 4y = -4$ to find the center and the radius.

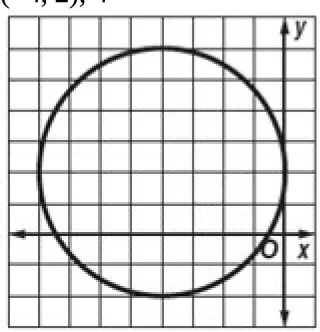
$$
\begin{aligned}
x^2 + y^2 + 8x - 4y &= -4 && \text{Original equation}\\
x^2 + 8x + y^2 - 4y &= -4 && \text{Isolate and group like terms.}\\
x^2 + 8x + 16 + y^2 - 4y + 4 &= -4 + 16 + 4 && \text{Complete the squares.}\\
(x+4)^2 + (y-2)^2 &= 16 && \text{Factor and simplify.}\\
(x+4)^2 + (y-2)^2 &= 4^2 && \text{Write 16 as } 4^2.
\end{aligned}
$$

So, h = –4, k = 2, and r = 4. The center is at (–4, 2), and the radius is 4.

Plot the center and four points that are 4 units from this point. Sketch the circle through these four points.



ANSWER:
(–4, 2); 4



26. $x^2 + y^2 - 16x = 0$

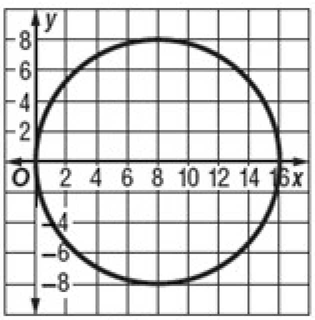
SOLUTION:

The standard form of the equation of a circle with center at (h, k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.
Rewrite $x^2 + y^2 - 16x = 0$ to find the center and the radius.

$$\begin{aligned} x^2 + y^2 - 16x &= 0 && \text{Original equation} \\ x^2 - 16x + y^2 &= 0 && \text{Isolate and group like terms.} \\ x^2 - 16x + 64 + y^2 &= 0 + 64 && \text{Complete the squares.} \\ (x-8)^2 + (y-0)^2 &= 64 && \text{Factor and simplify.} \\ (x-8)^2 + (y-0)^2 &= 8^2 && \text{Write 64 as } 8^2. \end{aligned}$$

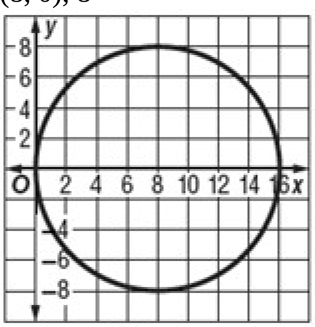
So, $h = 8$, $k = 0$, and $r = 8$. The center is at (8, 0), and the radius is 8.

Plot the center and four points that are 8 units from this point. Sketch the circle through these four points.



ANSWER:

(8, 0); 8



Write an equation of a circle that contains each set of points. Then graph the circle.

27. $A(1, 6)$, $B(5, 6)$, $C(5, 0)$

SOLUTION:

Step1: You are given three points that lie on a circle. Graph triangle *ABC* and construct the perpendicular bisectors of two sides to locate the center of the circle. Find the radius and then use the center and radius to write an equation. Construct the perpendicular bisectors of two sides. The center appears to be at (3, 3).

Step 2: Find the distance between the center and one of the points on the circle to determine the radius.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{(1-3)^2 + (6-3)^2} \quad (x_1, y_1) = (1, 6) \text{ and } (x_2, y_2) = (3, 3)$$

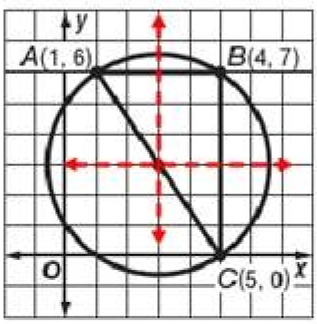
$$= \sqrt{4+9} \text{ or } \sqrt{13} \quad \text{Simplify.}$$

Step 3: Write the equation of the circle using $h = 3$, $k = 3$, and $r = \sqrt{13}$.

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Equation of a circle}$$

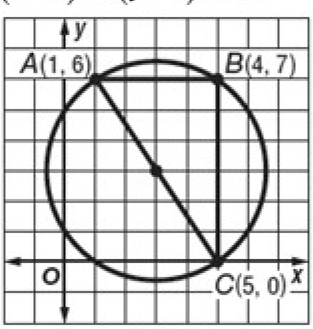
$$(x-3)^2 + (y-3)^2 = \left(\sqrt{13}\right)^2 \quad (h, k) = (3, 3), r = \sqrt{13}$$

$$(x-3)^2 + (y-3)^2 = 13 \quad \text{Simplify.}$$



ANSWER:

$(x-3)^2 + (y-3)^2 = 13$



10-8 Equations of Circle

28. $F(3, -3)$, $G(3, 1)$, $H(7, 1)$

SOLUTION:

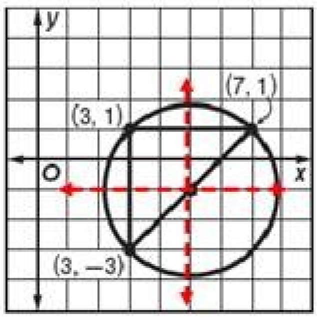
Step 1: You are given three points that lie on a circle. Graph triangle *FGH* and construct the perpendicular bisectors of two sides to locate the center of the circle. Find the radius and then use the center and radius to write an equation. Construct the perpendicular bisectors of two sides. The center appears to be at $(5, -1)$.

Step 2: Find the distance between the center and one of the points on the circle to determine the radius.

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(3-5)^2 + (1-(-1))^2} && (x_1, y_1) = (3, 1) \text{ and } (x_2, y_2) = (5, -1) \\ &= \sqrt{4+4} \text{ or } \sqrt{8} && \text{Simplify.} \end{aligned}$$

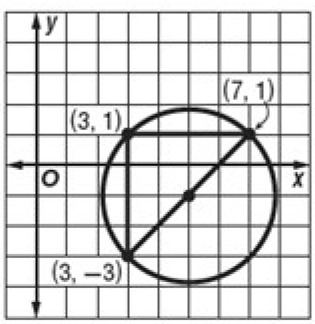
Step 3: Write the equation of the circle using $h = 5$, $k = -1$, and $r = \sqrt{8}$.

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 && \text{Equation of a circle} \\ (x-5)^2 + (y-(-1))^2 &= \left(\sqrt{8}\right)^2 && (h, k) = (5, -1), r = \sqrt{8} \\ (x-5)^2 + (y+1)^2 &= 8 && \text{Simplify.} \end{aligned}$$



ANSWER:

$(x-5)^2 + (y+1)^2 = 8$



10-8 Equations of Circle

Find the point(s) of intersection, if any, between each circle and line with the equations given.

29. $x^2+y^2=5$

$y=\frac{1}{2}x$

SOLUTION:

Use substitution to find the coordinates of the point(s) of intersection algebraically.

$$\begin{aligned} x^2+y^2 &= 5 && \text{Equation of circle} \\ x^2+\left(\tfrac{1}{2}x\right)^2 &= 5 && \text{Since } y=\tfrac{1}{2}x, \text{ substitute } \tfrac{1}{2}x \text{ for } y. \\ x^2+\tfrac{1}{4}x^2 &= 5 && \text{Multiply.} \\ \tfrac{5}{4}x^2 &= 5 && \text{Simplify.} \\ x^2 &= 4 && \text{Multiply each side by } \tfrac{4}{5}. \\ x &= \pm 2 && \text{Take the square root of each side.} \end{aligned}$$

So, $x = 2$ or -2. Use the equation $y = \frac{1}{2}x$ to find the corresponding y-values.

When $x = 2$, $y = \frac{1}{2}(2)$ or 1.

When $x = -2$, $y = \frac{1}{2}(-2)$ or -1.

Therefore, the points of intersection are $(-2, -1)$ and $(2, 1)$.

ANSWER:

$(-2, -1)$, $(2, 1)$

30. $x^2+y^2=2$

$y=-x+2$

SOLUTION:

Use substitution to find the coordinates of the point(s) of intersection algebraically.

$$\begin{aligned} x^2+y^2 &= 2 && \text{Equation of circle} \\ x^2+(-x+2)^2 &= 2 && \text{Since } y=-x+2, \text{ substitute } -x+2 \text{ for } y. \\ x^2+x^2-4x+4 &= 2 && \text{Multiply.} \\ 2x^2-4x+4 &= 2 && \text{Simplify.} \\ 2x^2-4x+2 &= 0 && \text{Subtract 2 from each side.} \\ 2(x-1)(x-1) &= 0 && \text{Factor.} \\ x &= 1 && \text{Zero Product Property} \end{aligned}$$

Use the equation $y = -x + 2$ to find the corresponding y-value. When $x = 1$, $y = -1 + 2$ or 1.
Therefore, the point of intersection is $(1, 1)$.

ANSWER:

$(1, 1)$

31. $x^2+(y+2)^2=8$

$y=x-2$

SOLUTION:

Use substitution to find the coordinates of the point(s) of intersection algebraically.

$$\begin{aligned} x^2+(y+2)^2 &= 8 && \text{Equation of circle} \\ x^2+(x-2+2)^2 &= 8 && \text{Since } y=x-2\text{, substitute } x-2 \text{ for } y. \\ x^2+x^2 &= 8 && \text{Simplify.} \\ 2x^2 &= 8 && \text{Simplify.} \\ x^2 &= 4 && \text{Divide each side by 2.} \\ x &= \pm 2 && \text{Take the square root of each side.} \end{aligned}$$

Use the equation $y = x - 2$ to find the corresponding y-values.
When $x = -2$, $y = -2 - 2$ or -4.
When $x = 2$, $y = 2 - 2$ or 0.
Therefore, the points of intersection are (–2, –4) and (2, 0).

ANSWER:

(–2, –4), (2, 0)

32. $(x+3)^2+y^2=25$

$y=-3x$

SOLUTION:

Use substitution to find the coordinates of the point(s) of intersection algebraically.

$$
\begin{aligned}
(x+3)^2+y^2 &= 25 && \text{Equation of circle}\\
(x+3)^2+(-3x)^2 &= 25 && \text{Since } y=-3x, \text{ substitute } -3x \text{ for } y.\\
x^2+6x+9+9x^2 &= 25 && \text{Multiply.}\\
10x^2+6x-16 &= 0 && \text{Subtract 25 from each side and simplify.}\\
2(5x+8)(x-1) &= 0 && \text{Factor.}\\
x &= -\tfrac{8}{5} \text{ or } 1 && \text{Zero Product Property}
\end{aligned}
$$

So, $x=-1\frac{3}{5}$ or 1. Use the equation $y=-3x$ to find the corresponding y-values.

When $x=-1\frac{3}{5}$, $y=-3\left(-1\frac{3}{5}\right)$ or $4\frac{4}{5}$.

When $x=1$, $y=-3(1)$ or -3.

Therefore, the points of intersection are $\left(-1\frac{3}{5}, 4\frac{4}{5}\right)$ and $(1, -3)$.

ANSWER:

$\left(-1\frac{3}{5}, 4\frac{4}{5}\right)$, $(1, -3)$

10-8 Equations of Circle

33. $x^2 + y^2 = 5$

$y = 3x$

SOLUTION:

Use substitution to find the coordinates of the point(s) of intersection algebraically.

$x^2 + y^2 = 5$	Equation of circle
$x^2 + (3x)^2 = 5$	Since $y = 3x$, substitute $3x$ for y.
$x^2 + 9x^2 = 5$	Multiply.
$10x^2 = 5$	Simplify.
$x^2 = \frac{1}{2}$	Divide each side by 10.
$x = \pm\sqrt{\frac{1}{2}}$ or $\pm\frac{\sqrt{2}}{2}$	Take the square root of each side and simplify.

Use the equation $y = 3x$ to find the corresponding y-values.

When $x = \frac{\sqrt{2}}{2}, y = 3\left(\frac{\sqrt{2}}{2}\right)$ or $\frac{3\sqrt{2}}{2}$.

When $x = -\frac{\sqrt{2}}{2}, y = 3\left(-\frac{\sqrt{2}}{2}\right)$ or $\frac{-3\sqrt{2}}{2}$

Therefore, the points of intersection are $\left(\frac{\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$ and $\left(-\frac{\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$.

ANSWER:

$\left(\frac{\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$

10-8 Equations of Circle

34. $(x-1)^2+(y-3)^2=4$

$y=-x$

SOLUTION:

Use substitution to find the coordinates of the point(s) of intersection algebraically.

$$\begin{aligned}(x-1)^2+(y-3)^2 &= 4 && \text{Equation of circle}\\ (x-1)^2+(-x-3)^2 &= 4 && \text{Since } y=-x\text{, substitute } -x \text{ for } y.\\ x^2-2x+1+x^2+6x+9 &= 4 && \text{Multiply.}\\ 2x^2+4x+6 &= 0 && \text{Subtract 4 from each side and simplify.}\\ x^2+2x+3 &= 0 && \text{Divide each side by 2.}\end{aligned}$$

Use the quadratic formula to find the solution(s) for x.

$$\begin{aligned}x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} && \text{Quadratic Formula}\\ x &= \frac{-2\pm\sqrt{2^2-4(1)(3)}}{2(1)} && a=1, b=2, c=3\\ x &= \frac{-2\pm\sqrt{-8}}{2} && \text{Simplify.}\end{aligned}$$

Since the solution contains the square root of a negative number, no real solution for x exists. Therefore, there are no points of intersection.

ANSWER:

no points of intersection

Write the equation of each circle.

35. a circle with a diameter having endpoints at (0, 4) and (6, –4)

SOLUTION:

The center of the circle is the midpoint of the diameter.

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \quad \text{Midpoint Formula}$$

$$(h, k) = \left(\frac{0+6}{2}, \frac{4+(-4)}{2}\right) \quad (x_1, y_1) = (0, 4), (x_2, y_2) = (6, -4)$$

$$(h, k) = (3, 0) \quad \text{Simplify.}$$

Find the distance between the center and one endpoint of the diameter to determine the radius.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{(3-0)^2 + (0-4)^2} \quad (x_1, y_1) = (3, 0) \text{ and } (x_2, y_2) = (0, 4)$$

$$= \sqrt{9+16} \text{ or } 5 \quad \text{Simplify.}$$

Write the equation of the circle using h = 3, k = 0, and r = 5.

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Equation of a circle}$$

$$(x-3)^2 + (y-0)^2 = 5^2 \quad h = 3, k = 0, r = 5$$

$$(x-3)^2 + y^2 = 25 \quad \text{Simplify.}$$

ANSWER:

$(x-3)^2 + y^2 = 25$

36. a circle with d = 22 and a center translated 13 units left and 6 units up from the origin

SOLUTION:

After the translation, the center of the circle is (0 – 13, 0 + 6) or (–13, 6).

Write the equation using h = –13, k = 6, and $r = \frac{1}{2}(22)$ or 11.

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Equation of a circle}$$

$$(x-(-13))^2 + (y-6)^2 = 11^2 \quad h = -13, k = 6, r = 11$$

$$(x+13)^2 + (y-6)^2 = 121 \quad \text{Simplify.}$$

ANSWER:

$(x+13)^2 + (y-6)^2 = 121$

37. **CCSS MODELING** Different-sized engines will launch model rockets to different altitudes. The higher a rocket goes, the larger the circle of possible landing sites becomes. Under normal wind conditions, the landing radius is three times the altitude of the rocket.

a. Write the equation of the landing circle for a rocket that travels 300 feet in the air.

b. What would be the radius of the landing circle for a rocket that travels 1000 feet in the air? Assume the center of the circle is at the origin.

SOLUTION:

a.

$$x^2 + y^2 = (3(300))^2$$

$$x^2 + y^2 = 810,000$$

b. The radius of the landing circle for a rocket is 3(1000) ft or 3000 ft.

ANSWER:

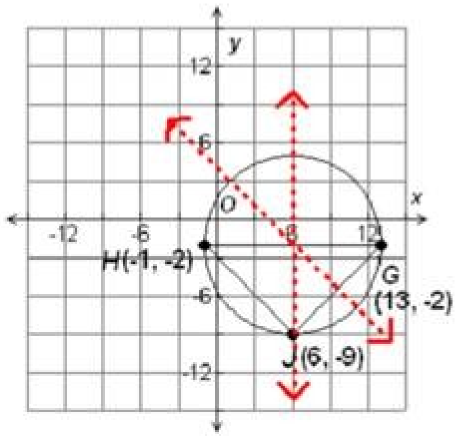
a. $x^2 + y^2 = 810,000$

b. 3000 ft

38. **SKYDIVING** Three of the skydivers in the circular formation shown have approximate coordinates of $G(13, -2)$, $H(-1, -2)$, and $J(6, -9)$.
a. What are the approximate coordinates of the center skydiver?
b. If each unit represents 1 foot, what is the diameter of the skydiving formation?

SOLUTION:

a. Use the three points to graph ΔGHJ and construct the perpendicular bisectors of two sides to locate the center of the circle.



From the figure, the center of the circle is (6, –2). Therefore, the coordinates of the center skydiver is (6, –2).
b. Find the distance between the center and one of the points on the circle to determine the radius.

$$\begin{aligned} r &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(6-6)^2 + (-2-(-9))^2} && (x_1, y_1) = (6, -2) \text{ and } (x_2, y_2) = (6, -9) \\ &= \sqrt{0 + 49} \text{ or } 7 && \text{Simplify.} \end{aligned}$$

The radius has a measure of 7, so the diameter is $2r$ or 14.
Therefore, the diameter of the skydiving formation is 14 feet.

ANSWER:

a. (6, –2)
b. 14 ft

10-8 Equations of Circle

39. **DELIVERY** Pizza and Subs offers free delivery within 6 miles of the restaurant. The restaurant is located 4 miles west and 5 miles north of Consuela's house.

a. Write and graph an equation to represent this situation if Consuela's house is at the origin of the coordinate system.

b. Can Consuela get free delivery if she orders pizza from Pizza and Subs? Explain.

SOLUTION:

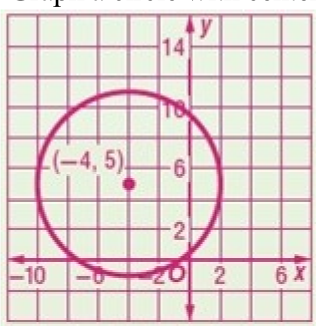
a. The restaurant is the center of the circle and is located at (0 – 5, 0 + 4) or (–5, 4).
Write the equation of a circle with h = –5, k = 4, and r = 6.

$$(x-h)^2+(y-k)^2=r^2 \quad \text{Equation of a circle}$$

$$(x-(-5))^2+(y-4)^2=6^2 \quad h=-5, k=4, r=6$$

$$(x+5)^2+(y-4)^2=36 \quad \text{Simplify.}$$

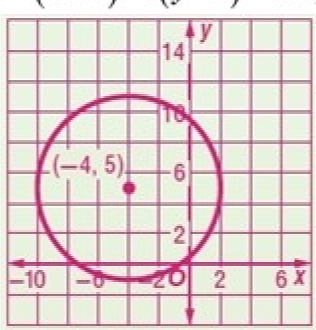
Graph a circle with center at (–5, 4) and a radius of 6 on a coordinate grid.



b. All homes within the circle get free delivery. Consuela's home at (0, 0) is located outside the circle, so she cannot get free delivery.

ANSWER:

a. $(x+4)^2+(y-5)^2=36$;



b. All homes within the circle get free delivery. Consuela's home at (0, 0) is located outside the circle, so she cannot get free delivery.

40. **INTERSECTIONS OF CIRCLES** Graph $x^2+y^2=4$ and $(x-2)^2+y^2=4$ on the same coordinate plane.

a. Estimate the point(s) of intersection between the two circles.

b. Solve $x^2+y^2=4$ for y.

c. Substitute the value you found in part **b** into $(x-2)^2+y^2=4$ and solve for x.

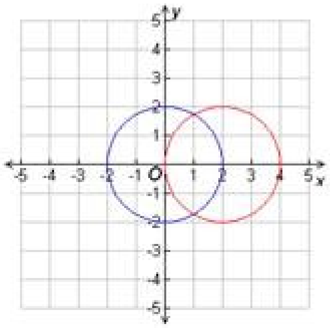
d. Substitute the value you found in part **c** into $x^2+y^2=4$ and solve for y.

e. Use your answers to parts **c** and **d** to write the coordinates of the points of

intersection. Compare these coordinates to your estimate from part **a**.
f. Verify that the point(s) you found in part **d** lie on both circles.

SOLUTION:

a.



The points of intersection for the two circles appear to be about (1, 1.7) and (1, –1.7).

b.

$x^2 + y^2 = 4$ Original equation

$y^2 = 4 - x^2$ Subtract x^2 from each side.

$y = \pm\sqrt{4 - x^2}$ Take the square root of each side.

c.

$(x-2)^2 + y^2 = 4$ Equation of second circle

$(x-2)^2 + \left(\pm\sqrt{4 - x^2}\right)^2 = 4$ Since $y = \pm\sqrt{4 - x^2}$, substitute $\pm\sqrt{4 - x^2}$ for y.

$x^2 - 4x + 4 + 4 - x^2 = 4$ Multiply.

$-4x + 8 = 4$ Simplify.

$-4x = -4$ Subtract 8 from each side.

$x = 1$ Divide each side by -4.

d.

$x^2 + y^2 = 4$ Equation of first circle

$(1)^2 + y^2 = 4$ Substitute $x = 1$.

$1 + y^2 = 4$ Simplify.

$y^2 = 3$ Subtract 1 from each side.

$y = \pm\sqrt{3}$ Take the square root of each side.

e. The coordinates of the points of intersection are the solutions to the equations of the two circles. Since x = 1 and y = $\pm\sqrt{3}$, the points of intersection are $\left(1, \sqrt{3}\right)$ and $\left(1, -\sqrt{3}\right)$; Since $\sqrt{3}$ is about 1.73, the coordinates are approximately the same.

f. Verify that each point is on the circles by substituting it into both equations.

For $\left(1, \sqrt{3}\right)$: For $\left(1, -\sqrt{3}\right)$:

$$\begin{aligned} x^2+y^2 &= 4 \\ 1^2+\left(\sqrt{3}\right)^2 &\stackrel{?}{=} 4 \\ 1+3 &\stackrel{?}{=} 4 \checkmark \\ (x-2)^2+y^2 &= 4 \\ (1-2)^2+\left(\sqrt{3}\right)^2 &\stackrel{?}{=} 4 \\ (-1)^2+3 &\stackrel{?}{=} 4 \\ 1+3 &\stackrel{?}{=} 4 \checkmark \end{aligned}$$

$$\begin{aligned} x^2+y^2 &= 4 \\ 1^2+\left(-\sqrt{3}\right)^2 &\stackrel{?}{=} 4 \\ 1+3 &\stackrel{?}{=} 4 \checkmark \\ (x-2)^2+y^2 &= 4 \\ (1-2)^2+\left(-\sqrt{3}\right)^2 &\stackrel{?}{=} 4 \\ (-1)^2+3 &\stackrel{?}{=} 4 \\ 1+3 &\stackrel{?}{=} 4 \checkmark \end{aligned}$$

ANSWER:

a. ≈(1, 1.7) and (1, -1.7)
b. $y = \pm\sqrt{4-x^2}$
c. $x = 1$
d. $y = \pm\sqrt{3}$
e. $(1, \sqrt{3})$, $(1, -\sqrt{3})$; The coordinates are approximately the same.
f. Verify $(1, \sqrt{3})$.

$$\begin{aligned} &1^2+\left(\sqrt{3}\right)^2=4 \\ &1+3=4 \checkmark \\ &(1-2)^2+\left(\sqrt{3}\right)^2=4 \\ &(-1)^2+3=4 \\ &1+3=4 \checkmark \end{aligned}$$

Verify $(1, -\sqrt{3})$.

$$\begin{aligned} &1^2+\left(-\sqrt{3}\right)^2=4 \\ &1+3=4 \checkmark \\ &(1-2)^2+\left(-\sqrt{3}\right)^2=4 \\ &(-1)^2+3=4 \\ &1+3=4 \checkmark \end{aligned}$$

10-8 Equations of Circle

41. Prove or disprove that the point (1, $2\sqrt{2}$) lies on a circle centered at the origin and containing the point (0, -3).

SOLUTION:

Find the distance between the two points to determine the radius of the circle.

$$\begin{aligned} r &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(0-0)^2+(-3-0)^2} && (x_1, y_1) = (0, 0) \text{ and } (x_2, y_2) = (0, -3) \\ &= \sqrt{0+9} \text{ or } 3 && \text{Simplify.} \end{aligned}$$

Write the equation using $(h, k) = (0, 0)$ and $r = 3$.

So, the equation of a circle centered at the origin and containing the point (0, –3) is $x^2+y^2=9$.

Substitute (1, $2\sqrt{2}$) for (x, y) and see if the resulting statement is true to determine if the point lies on the circle.

$$\begin{aligned} x^2+y^2 &= 9 \\ (1)^2+\left(2\sqrt{2}\right)^2 &\stackrel{?}{=} 9 \\ 1+8 &\stackrel{?}{=} 9 \\ 9 &\stackrel{?}{=} 9\ \checkmark \end{aligned}$$

Since the statement is true, the point (1, $2\sqrt{2}$) lies on the circle.

ANSWER:

The equation of a circle centered at the origin and containing the point (0, –3) is $x^2+y^2=9$. The point (1, $2\sqrt{2}$) lies on the circle, since evaluating $x^2+y^2=9$ for $x = 1$ and $y = 2\sqrt{2}$ results in a true equation.

$1^2+\left(2\sqrt{2}\right)^2=9$

$1+8=9$

$9=9\ \checkmark$

42. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a compound locus for a pair of points. A **compound locus** satisfies more than one distinct set of conditions.

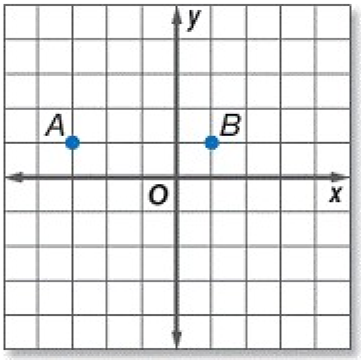
a. TABULAR Choose two points *A* and *B* in the coordinate plane. Locate 5 coordinates from the locus of points equidistant from *A* and *B*.

b. GRAPHICAL Represent this same locus of points by using a graph.

c. VERBAL Describe the locus of all points equidistant from a pair of points.

d. GRAPHICAL Using your graph from part b, determine and graph the locus of all points in a plane that are a distance of *AB* from *B*.

e. VERBAL Describe the locus of all points in a plane equidistant from a single point. Then describe the locus of all points that are both equidistant from *A* and *B* and are a distance of *AB* from *B*. Describe the graph of the compound locus.

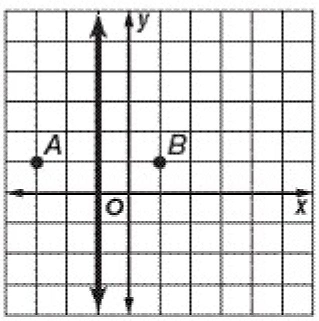


SOLUTION:

a. Sample answer:

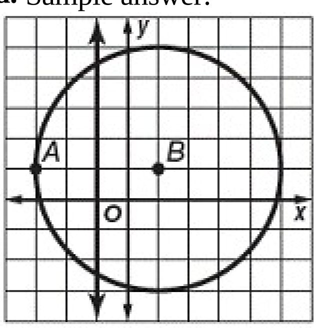
x	y
−1	−3
−1	−1
−1	0
−1	2
−1	4

b. Sample answer:



c. a line that is the perpendicular bisector of $\overline{AB}$

d. Sample answer:



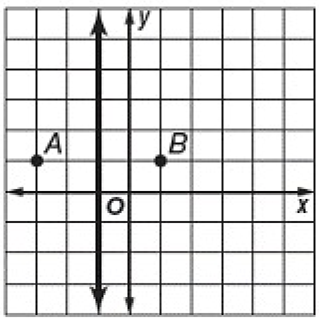
e. The locus of all points in a plane equidistant from a point is a circle. The locus of points that are both equidistant from *A* and *B* and are a distance of *AB* from *B* is the intersection of the locus of points equidistant from *A* and *B* and the locus of points that are a distance of *AB* from *B*. Graphically, the compound locus is represented as two points.

ANSWER:

a. Sample answer:

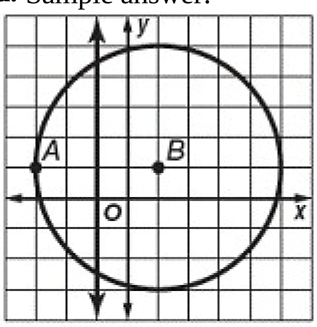
x	y
−1	−3
−1	−1
−1	0
−1	2
−1	4

b. Sample answer:



c. a line that is the perpendicular bisector of $\overline{AB}$

d. Sample answer:



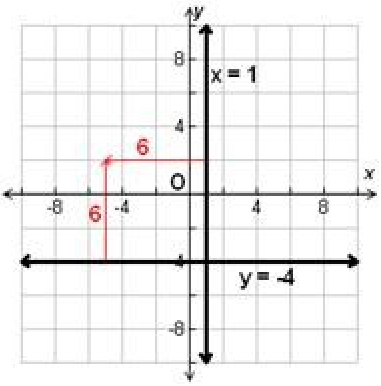
e. The locus of all points in a plane equidistant from a point is a circle. The locus of points that are both equidistant from *A* and *B* and are a distance of *AB* from *B* is the intersection of the locus of points equidistant from *A* and *B* and the locus of points that are a distance of *AB* from *B*. Graphically, the compound locus is represented as two points.

10-8 Equations of Circle

43. A circle with a diameter of 12 has its center in the second quadrant. The lines $y = -4$ and $x = 1$ are tangent to the circle. Write an equation of the circle.

SOLUTION:

To find the center of the circle, draw the two tangent lines on a coordinate grid and then find the point located in quadrant II that is 6 units away from each tangent line.(Remember, the distance between a point and a line is the length of the perpendicular segment from the point to the line.)



The center of the circle is (–5, 2).

Write the equation with $h = -5$, $k = 2$, and $r = \frac{1}{2}(12)$ or 6.

$$(x-h)^2+(y-k)^2 = r^2 \quad \text{Equation of a circle}$$

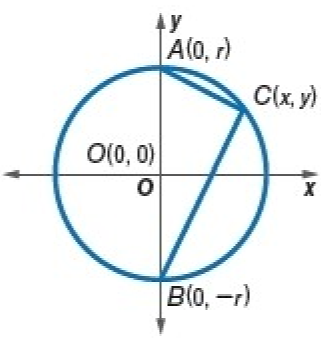
$$(x-(-5))^2+(y-2)^2 = 6^2 \quad h=-5, k=2, r=6$$

$$(x+5)^2+(y-2)^2 = 36 \quad \text{Simplify.}$$

ANSWER:

$(x+5)^2+(y-2)^2=36$

44. **CHALLENGE** Write a coordinate proof to show that if an inscribed angle intercepts the diameter of a circle, as shown, the angle is a right angle.

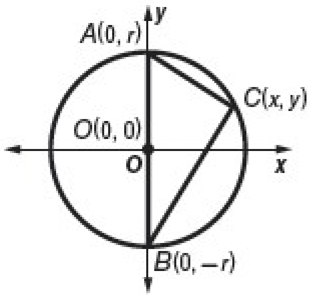


SOLUTION:

Given: $\overline{AB}$ is a diameter of $\odot O$, and C is a point on $\odot O$.

Prove: $\angle ACB$ is a right angle.

10-8 Equations of Circle



Proof: If (x, y) is a point on the circle with center (0, 0) and a radius of r, then $x^2 + y^2 = r^2$. Using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$,

$\overline{AC}$ has a slope of $\frac{y - r}{x - 0}$ or $\frac{y - r}{x}$, and $\overline{CB}$ has a slope of $\frac{y - (-r)}{x - 0}$ or $\frac{y + r}{x}$.

Two lines are perpendicular if and only if the product of their slopes is –1.

$$\frac{y-r}{x} \cdot \frac{y+r}{x} = \frac{y^2 - r^2}{x^2} \qquad \text{Multiply.}$$

$$= \frac{y^2 - (x^2 + y^2)}{x^2} \qquad r^2 = x^2 + y^2$$

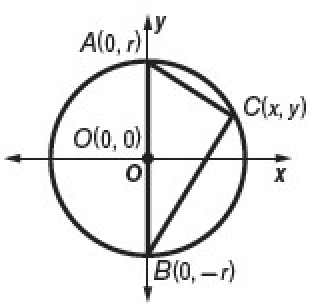
$$= \frac{y^2 - x^2 - y^2}{x^2} \qquad -x^2 + y^2 = -x^2 - y^2$$

$$= \frac{-x^2}{x^2} \text{ or } -1 \qquad \text{Simplify.}$$

Since the product of the slope of $\overline{AC}$ and $\overline{CB}$ is –1, $\overline{AC} \perp \overline{CB}$ and $\angle ACB$ is a right angle.

ANSWER:

Given: $\overline{AB}$ is a diameter of $\odot O$, and C is a point on $\odot O$.
Prove: $\angle ACB$ is a right angle.



Proof: If (x, y) is a point on the circle with center (0, 0) and a radius of r, then $x^2 + y^2 = r^2$.

$\overline{AC}$ has slope $\frac{y - r}{x}$, and $\overline{CB}$ has slope $\frac{y - (-r)}{x}$ or $\frac{y + r}{x}$.

Two lines are perpendicular if and only if the product of their slopes is –1.

$$\frac{y-r}{x} \cdot \frac{y+r}{x} = \frac{y^2-r^2}{x^2} \qquad \text{Multiply.}$$

$$= \frac{y^2-(x^2+y^2)}{x^2} \qquad r^2 = x^2 + y^2$$

$$= \frac{y^2-x^2-y^2}{x^2} \qquad -x^2 + y^2 = -x^2 - y^2$$

$$= \frac{-x^2}{x^2} \text{ or } -1 \qquad \text{Simplify.}$$

Since the product of the slope of $\overline{AC}$ and $\overline{CB}$ is –1, $\overline{AC} \perp \overline{CB}$ and $\angle ACB$ is a right angle.

45. **CCSS REASONING** A circle has the equation $(x-5)^2 + (y+7)^2 = 16$. If the center of the circle is shifted 3 units right and 9 units up, what would be the equation of the new circle? Explain your reasoning.

SOLUTION:

The circle with equation $(x - 5)^2 + (y + 7)^2 = 16$ has its center at (5, –7) and a radius of $\sqrt{16}$ or 4. After the circle is translated:

$(x, y) \quad \rightarrow \quad (x+3, y+9)$

$(5, -7) \quad \rightarrow \quad (5+3, -7+9)$ or $(8, 2)$

So, the center of the circle after being shifted 3 units right and 9 units up is (8, 2).
Write the equation using $h = 8$, $k = 2$, and $r = 4$.

$(x-h)^2 + (y-k)^2 = r^2$ Equation of a circle

$(x-8)^2 + (y-2)^2 = 4^2$ $h = 8, k = 2, r = 4$

$(x-8)^2 + (y-2)^2 = 16$ Simplify.

.

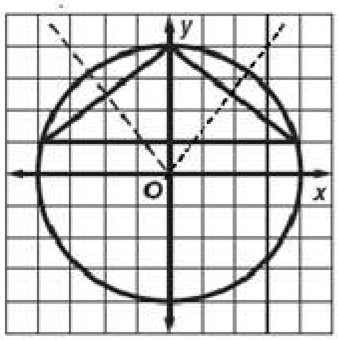
ANSWER:

$(x-8)^2 + (y-2)^2 = 16$; the first circle has its center at (5, –7). If the circle is shifted 3 units right and 9 units up, the new center is at (8, 2), so the new equation becomes $(x-8)^2 + (y-2)^2 = 16$.

46. **OPEN ENDED** Graph three noncollinear points on a coordinate plane. Draw a triangle by connecting the points. Then construct the circle that circumscribes it.

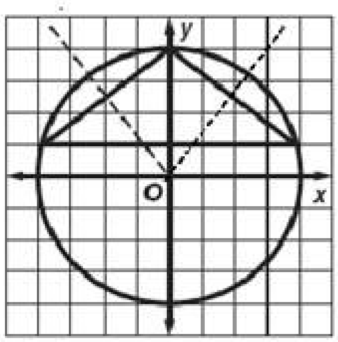
SOLUTION:

Sample answer:



ANSWER:

Sample answer:



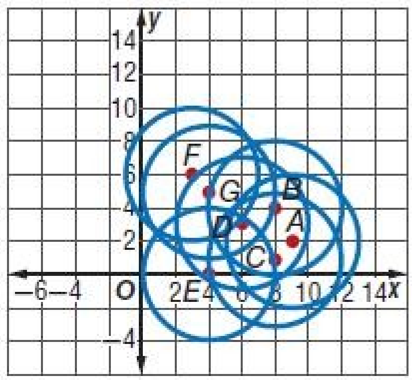
47. **DELIVERY** Pizza and Subs offers free delivery within 6 miles of the restaurant. The restaurant is located 4 miles west and 5 miles north of Consuela's house.

a. Write and graph an equation to represent this situation if Consuela's house is at the origin of the coordinate system.

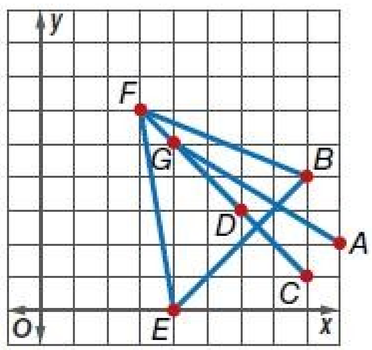
b. Can Consuela get free delivery if she orders pizza from Pizza and Subs? Explain.

SOLUTION:

Method 1: Draw a circle of radius 200 centered on each station.



Method 2: Draw line segments to connect pairs of stations that are more than 200 miles apart.



Since *BE* > 200, *BF* > 200, and *FE* > 200, stations *B*, *E*, and F can use the same frequency. Since *CG* > 200, stations *C* and *D* can use the same frequency, but cannot use the frequency of *B*, *E*, and *F*, since *BC* < 200. *A* and *D* must use other frequencies since *AD* < 200, *AB* < 200, and *DB* < 200. Therefore, the least number of frequencies that can be assigned are 4.

ANSWER:

a. 4

b–c. Method 1: Draw circles centered on each station that have a radius of 4 units. Stations outside of a circle can have the same frequency as the station at the center of the circle.

Method 2: Use the Pythagorean theorem to identify stations that are more than 200 miles apart. Using Method 2, plot the points representing the stations on a graph. Stations that are more than 4 units apart on the graph will be more than 200 miles apart and will thus be able to use the same frequency. Assign station A to the first frequency. Station B is within 4 units of station A, so it must be assigned the second frequency. Station C is within 4 units of both stations A and B, so it must be assigned a third frequency. Station D is also within 4 units of stations A, B, and C, so it must be assigned a fourth frequency, Station E is $\sqrt{29}$ or about 5.4 units away from station A, so it can share the first frequency. Station F is $\sqrt{29}$ or about 5.4 units away from station B, so it can share the second frequency Station G is $\sqrt{32}$ or about 5.7 units away from station C, so it can share the third frequency. Therefore, the least number of frequencies that can be assigned is 4.

CHALLENGE Find the coordinates of point *P* on $\overrightarrow{AB}$ that partitions the segment into the given ratio *AP* to *PB*.

48. $A(0, 0)$, $B(3, 4)$, 2 to 3

SOLUTION:

Let $P = (x, y)$. The point is on line AB, so first find the slope and equation of line AB.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\ m &= \frac{4-0}{3-0} && (x_1, y_1) = (0, 0); (x_2, y_2) = (3, 4) \\ m &= \frac{4}{3} && \text{Simplify.} \end{aligned}$$

The y-intercept is 0, so the equation of the line is $y = \frac{4}{3}x$.

Next, find the length of segment AB.

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(3-0)^2 + (4-0)^2} && (x_1, y_1) = (0, 0) \text{ and } (x_2, y_2) = (3, 4) \\ &= \sqrt{9 + 16} \text{ or } 5 && \text{Simplify.} \end{aligned}$$

Since the ratio of AP to PB is 2 to 3, the ratio of AP to AB is 2 to 5. Find the length of segment AP.

$$\begin{aligned} \frac{AP}{AB} &= \frac{2}{5} \\ \frac{AP}{5} &= \frac{2}{5} \\ AP &= 2 \end{aligned}$$

The points 2 units away from (0, 0) are described by the circle with equation $x^2 + y^2 = 4$.

To determine point P, find the intersection of the circle and the line.

$$\begin{aligned} x^2 + y^2 &= 4 && \text{Equation of circle} \\ x^2 + \left(\frac{4}{3}x\right)^2 &= 4 && \text{Since } y = \tfrac{4}{3}x, \text{ substitute } \tfrac{4}{3}x \text{ for } y. \\ x^2 + \frac{16}{9}x^2 &= 4 && \text{Multiply.} \\ \frac{25}{9}x^2 &= 4 && \text{Simplify.} \\ x^2 &= \frac{36}{25} && \text{Multiply each side by } \tfrac{9}{25}. \\ x &= \pm\frac{6}{5} \text{ or } \pm 1.2 && \text{Take the square root of each side.} \end{aligned}$$

Since P is on directed line segment AB, the point must be in quadrant I, so $x = 1.2$. Use the equation of the line to find the y-value.

$$\begin{aligned} y &= \frac{4}{3}x \\ y &= \frac{4}{3}(1.2) \\ y &= 1.6 \end{aligned}$$

The point of intersection of the circle and the line in quadrant I is (1.2, 1.6).

Therefore, point P on $\overrightarrow{AB}$ that partitions the segment into a ratio of 2 to 3 has coordinates (1.2, 1.6).

ANSWER:

(1.2, 1.6)

49. $A(0, 0)$, $B(–8, 6)$, 4 to 1

SOLUTION:

Let $P = (x, y)$. The point is on line AB, so first find the slope and equation of line AB.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\ m &= \frac{6-0}{-8-0} && (x_1, y_1) = (0, 0);\ (x_2, y_2) = (-8, 6) \\ m &= \frac{6}{-8} \text{ or } -\frac{3}{4} && \text{Simplify.} \end{aligned}$$

The y-intercept is 0, so the equation of the line is $y = -\frac{3}{4}x$.

Next, find the length of segment AB.

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(-8-0)^2 + (6-0)^2} && (x_1, y_1) = (0, 0) \text{ and } (x_2, y_2) = (-8, 6) \\ &= \sqrt{64 + 36} \text{ or } 10 && \text{Simplify.} \end{aligned}$$

Since the ratio of AP to PB is 4 to 1, the ratio of AP to AB is 4 to 5. Find the length of segment AP.

$$\begin{aligned} \frac{AP}{AB} &= \frac{4}{5} \\ \frac{AP}{10} &= \frac{4}{5} \\ AP &= 8 \end{aligned}$$

The points 8 units away from (0, 0) are described by the circle with equation $x^2 + y^2 = 64$.

To determine point P, find the intersection of the circle and the line.

$$\begin{aligned} x^2 + y^2 &= 64 && \text{Equation of circle} \\ x^2 + \left(-\frac{3}{4}x\right)^2 &= 64 && \text{Since } y = -\frac{3}{4}x, \text{ substitute } -\frac{3}{4}x \text{ for } y. \\ x^2 + \frac{9}{16}x^2 &= 64 && \text{Multiply.} \\ \frac{25}{16}x^2 &= 64 && \text{Simplify.} \\ x^2 &= \frac{1024}{25} && \text{Multiply each side by } \frac{16}{25}. \\ x &= \pm\frac{32}{5} \text{ or } \pm 6.4 && \text{Take the square root of each side.} \end{aligned}$$

Since P is on directed line segment AB, the point must be in quadrant II, so $x = –6.4$. Use the equation of the line to find the y-value.

$$\begin{aligned} y &= -\frac{3}{4}x \\ y &= -\frac{3}{4}(-6.4) \\ y &= 4.8 \end{aligned}$$

The point of intersection of the circle and the line in quadrant II is (–6.4, 4.8).

Therefore, point P on $\overrightarrow{AB}$ that partitions the segment into a ratio of 4 to 1 has coordinates (–6.4, 4.8).

ANSWER:

(–6.4, 4.8)

50. **WRITING IN MATH** Describe how the equation for a circle changes if the circle is translated *a* units to the right and *b* units down.

SOLUTION:

Sample answer: The equation for a circle is $(x-h)^2+(y-k)^2=r^2$. When the circle is shifted *a* units to the right, the new *x*-coordinate of the center is $x + a$. When the circle is translated *b* units down, the new *y*-coordinate of the center is $y - b$. The new equation for the circle is $[x-(h+a)]^2+[y-(k-b)]^2=r^2$ or $(x-h-a)^2+(y-k+b)^2=r^2$.

ANSWER:

Sample answer: The equation for a circle is $(x-h)^2+(y-k)^2=r^2$. When the circle is shifted *a* units to the right, the new *x*-coordinate of the center is $x + a$. When the circle is translated *b* units down, the new *y*-coordinate of the center is $y - b$. The new equation for the circle is $[x-(h+a)]^2+[y-(k-b)]^2=r^2$ or $(x-h-a)^2+(y-k+b)^2=r^2$.

51. Which of the following is the equation of a circle with center (6, 5) that passes through (2, 8)?

A $(x-6)^2+(y-5)^2=5^2$

B $(x-5)^2+(y-6)^2=7^2$

C $(x+6)^2+(y+5)^2=5^2$

D $(x-2)^2+(y-8)^2=7^2$

SOLUTION:

Find the distance between the points to determine the radius.

$$\begin{aligned} r &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(6-2)^2+(5-8)^2} && (x_1, y_1) = (6, 5) \text{ and } (x_2, y_2) = (2, 8) \\ &= \sqrt{16+9} \text{ or } 5 && \text{Simplify.} \end{aligned}$$

Write the equation using $h = 6$, $k = 5$, and $r = 5$.

$$\begin{aligned} (x-h)^2+(y-k)^2 &= r^2 && \text{Equation of a circle} \\ (x-6)^2+(y-5)^2 &= 5^2 && h=6, k=5, r=5 \\ (x-6)^2+(y-5)^2 &= 25 && \text{Simplify.} \end{aligned}$$

So, the correct choice is A.

ANSWER:

A

52. **ALGEBRA** What are the solutions of $n^2 - 4n = 21$?
F 3, 7
G 3, –7
H –3, 7
J –3, –7

SOLUTION:

$$\begin{aligned} n^2 - 4n &= 21 && \text{Original equation} \\ n^2 - 4n - 21 &= 0 && \text{Subtract 21 from each side.} \\ (n-7)(n+3) &= 0 && \text{Factor.} \\ n &= 7 \text{ or } -3 && \text{Zero Product Property} \end{aligned}$$

The solutions of the equation are –3, 7.
So, the correct choice is H.

ANSWER:
H

53. **SHORT RESPONSE** Solve: 5(x – 4) = 16.
Step 1: 5x - 4 = 16
Step 2: 5x = 20
Step 3: x = 4
Which is the first incorrect step in the solution shown above?

SOLUTION:
Step 1. The 5 has not been distributed correctly.

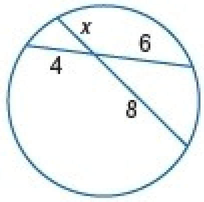
ANSWER:
Step 1

54. **SAT/ACT** The center of $\odot F$ has coordinates (–4, 0). If the circle has a radius of 4, which point lies on $\odot F$?
A (4, 0)
B (0, 4)
C (4, 3)
D (–4, 4)
E (0, 8)

SOLUTION:
The points on $\odot F$ have a distance of 4 units from the center. Only point D is 4 units away from (-4, 0). So, the correct answer is D.

ANSWER:
D

Find *x*.

55. 

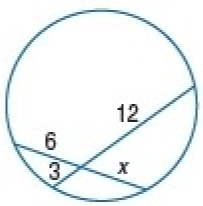
SOLUTION:

If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

$$\begin{aligned} 8 \cdot x &= 4 \cdot 6 && \text{Theorem 10.15} \\ 8x &= 24 && \text{Multiply.} \\ x &= 3 && \text{Divide each side by 8.} \end{aligned}$$

ANSWER:

3

56. 

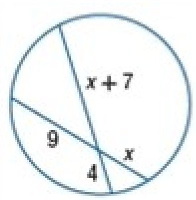
SOLUTION:

If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

$$\begin{aligned} 6 \cdot x &= 3 \cdot 12 && \text{Theorem 10.15} \\ 6x &= 36 && \text{Multiply.} \\ x &= 6 && \text{Divide each side by 6.} \end{aligned}$$

ANSWER:

6

57. 

SOLUTION:

If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

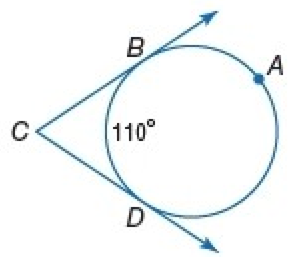
$$\begin{aligned} 9 \cdot x &= 4(x+7) && \text{Theorem 10.15} \\ 9x &= 4x + 28 && \text{Multiply.} \\ 5x &= 28 && \text{Subtract } 4x \text{ from each side.} \\ x &= 5.6 && \text{Divide each side by 5.} \end{aligned}$$

ANSWER:

5.6

Find each measure.

58. $m\angle C$



SOLUTION:

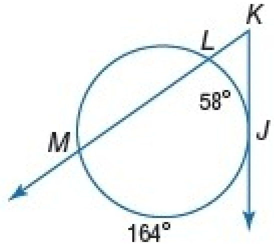
Major arc *BAD* shares the same endpoints as minor arc *BD*, so *m*(arc *BAD*) = 360 - *m*(arc *BD*) or 250.

$$\begin{aligned} m\angle C &= \frac{1}{2}[m(\text{arc}BAD) - m(\text{arc}BD)] && \text{Theorem 10.14} \\ &= \frac{1}{2}[250 - 110] && m(\text{arc}BAD) = 250, m(\text{arc}BD) = 110 \\ &= \frac{1}{2}(140) && \text{Simplify.} \\ &= 70 && \text{Multiply.} \end{aligned}$$

ANSWER:

70

59. $m\angle K$



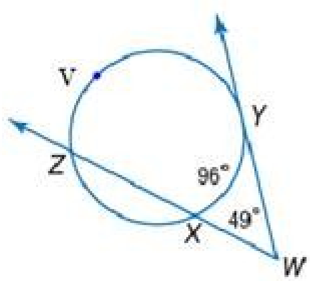
SOLUTION:

$$\begin{aligned} m\angle K &= \tfrac{1}{2}[m(\text{arc}\,MJ) - m(\text{arc}\,LJ)] && \text{Theorem 10.14} \\ &= \tfrac{1}{2}[164 - 58] && m(\text{arc}\,MJ) = 164, m(\text{arc}\,LJ) = 58 \\ &= \tfrac{1}{2}(106) && \text{Simplify.} \\ &= 53 && \text{Multiply.} \end{aligned}$$

ANSWER:

53

60. m(arc *YVZ*)



SOLUTION:

$$\begin{aligned} m\angle W &= \tfrac{1}{2}[m(\text{arc}\,YVZ) - m(\text{arc}\,XY)] && \text{Theorem 10.14} \\ 49 &= \tfrac{1}{2}[m(\text{arc}\,YVZ) - 96] && m\angle W = 49, m(\text{arc}\,XY) = 96 \\ 98 &= m(\text{arc}\,YVZ) - 96 && \text{Multiply each side by 2.} \\ 194 &= m(\text{arc}\,YVZ) && \text{Add 96 to each side.} \end{aligned}$$

Therefore, the measure of arc *YVZ* is 194.

ANSWER:

194

61. **STREETS** The neighborhood where Vincent lives has round-abouts where certain streets meet. If Vincent rides his bike once around the very edge of the grassy circle, how many feet will he have ridden?

SOLUTION:

You need to find the circumference of the round-about in feet. The diameter of the round-about is 3 yards, which is 3(3) or 9 feet.

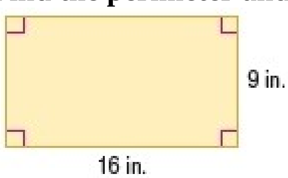
$$\begin{aligned} C &= \pi d \\ &= \pi(9) \\ &\approx 28.3 \end{aligned}$$

Vincent will have ridden his bike about 28.3 feet.

ANSWER:

28.3 ft

Find the perimeter and area of each figure.

62. 

SOLUTION:

$$\begin{aligned} P &= 2\ell + 2w && \text{Perimeter formula for rectangle} \\ &= 2(16) + 2(9) && \text{Substitution} \\ &= 32 + 18 && \text{Multiply.} \\ &= 50 && \text{Simplify.} \end{aligned}$$

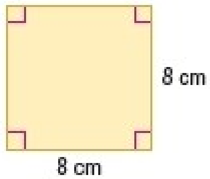
So, the perimeter of the rectangle is 50 inches.

$$\begin{aligned} A &= \ell w && \text{Area formula for rectangle} \\ &= (16)(9) && \text{Substitution} \\ &= 144 && \text{Multiply.} \end{aligned}$$

So, the area of the rectangle is 144 in.^{2}.

ANSWER:

50 in.; 144 in^2

63. 

SOLUTION:

$$\begin{aligned} P &= 4s && \text{Perimeter formula for square} \\ &= 4(8) && \text{Substitution} \\ &= 32 && \text{Multiply.} \end{aligned}$$

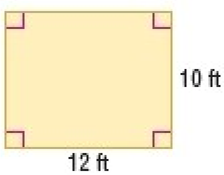
So, the perimeter of the figure is 32 centimeters.

$$\begin{aligned} A &= s^2 && \text{Area formula for square} \\ &= 8^2 && \text{Substitution} \\ &= 64 && \text{Simplify.} \end{aligned}$$

So, the area of the square is 64 cm^2.

ANSWER:

32 cm; 64 cm^2

64. 

SOLUTION:

$$\begin{aligned} P &= 2\ell + 2w && \text{Perimeter formula for rectangle} \\ &= 2(12) + 2(10) && \text{Substitution} \\ &= 24 + 20 && \text{Multiply.} \\ &= 44 && \text{Simplify.} \end{aligned}$$

So, the perimeter of the rectangle is 44 feet.

$$\begin{aligned} A &= \ell w && \text{Area formula for rectangle} \\ &= (12)(10) && \text{Substitution} \\ &= 120 && \text{Simplify.} \end{aligned}$$

So, the area of the rectangle is 120 ft^2.

ANSWER:

44 ft; 120 ft^2