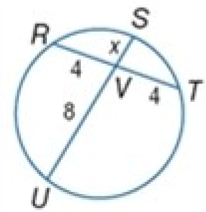
Find *x*. Assume that segments that appear to be tangent are tangent.

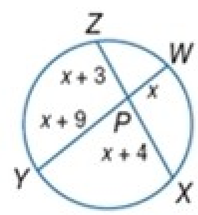
1. 

SOLUTION:

$$\begin{aligned} UV \cdot VS &= RV \cdot VU && \text{Theorem 10.15} \\ 8 \cdot x &= 4 \cdot 4 && \text{Substitution} \\ 8x &= 16 && \text{Multiply.} \\ x &= 2 && \text{Divide each side by 8.} \end{aligned}$$

ANSWER:

2

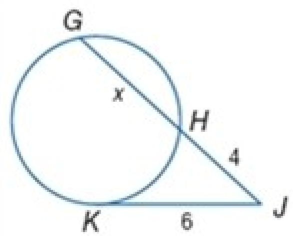
2. 

SOLUTION:

$$\begin{aligned} PW \cdot YP &= ZP \cdot PX && \text{Theorem 10.15} \\ x(x+9) &= (x+3)(x+4) && \text{Substitution} \\ x^2+9x &= x^2+7x+12 && \text{Multiply.} \\ 2x &= 12 && \text{Subtract } x^2 \text{ and } 7x \text{ from each side.} \\ x &= 6 && \text{Divide each side by 2.} \end{aligned}$$

ANSWER:

6

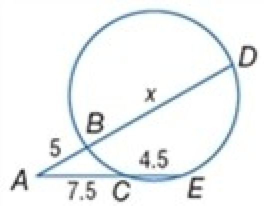
3. 

SOLUTION:

$$\begin{aligned} JH \cdot JG &= JK^2 && \text{Theorem 10.17} \\ 4(4+x) &= 6^2 && \text{Substitution} \\ 16+4x &= 36 && \text{Multiply.} \\ 4x &= 20 && \text{Subtract 16 from each side.} \\ x &= 5 && \text{Divide each side by 4.} \end{aligned}$$

ANSWER:

5

4. 

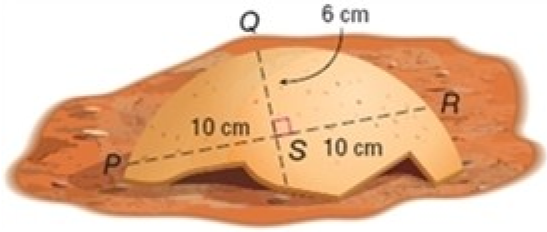
SOLUTION:

$$\begin{aligned} AB \cdot AD &= AC \cdot AE && \text{Theorem 10.16} \\ 5(5+x) &= 7.5(7.5+4.5) && \text{Substitution} \\ 25+5x &= 90 && \text{Multiply.} \\ 5x &= 65 && \text{Subtract 25 from each side.} \\ x &= 13 && \text{Divide each side by 5.} \end{aligned}$$

ANSWER:

13

5. **SCIENCE** A piece of broken pottery found at an archaeological site is shown. $\overline{QS}$ lies on a diameter of the circle. What was the circumference of the original pottery? Round to the nearest hundredth.



SOLUTION:

Let $\overline{QT}$ be the diameter of the circle. Then $\overline{QT}$ and $\overline{PR}$ are intersecting chords.
Find the length of the diameter by first finding the measure of *ST*.

$$\begin{aligned} QS \cdot ST &= PS \cdot SR && \text{Theorem 10.15} \\ 6 \cdot ST &= 10 \cdot 10 && \text{Substitution} \\ 6 \cdot ST &= 100 && \text{Multiply.} \\ ST &= 16\tfrac{2}{3} && \text{Divide each side by 6.} \end{aligned}$$

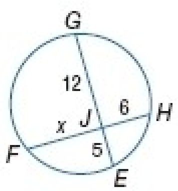
The diameter is equal to *QS* + *ST* or $22\frac{2}{3}$.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ &= \pi\left(22\tfrac{2}{3}\right) && \text{Substitution} \\ &\approx 71.21 && \text{Use a calculator.} \end{aligned}$$

ANSWER:

71.21 cm

Find *x* to the nearest tenth. Assume that segments that appear to be tangent are tangent.

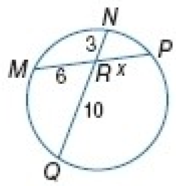
6. 

SOLUTION:

$$\begin{aligned} HJ \cdot JF &= GJ \cdot JE && \text{Theorem 10.15} \\ 6 \cdot x &= 12 \cdot 5 && \text{Substitution} \\ 6x &= 60 && \text{Multiply.} \\ x &= 10 && \text{Divide each side by 6.} \end{aligned}$$

ANSWER:

10



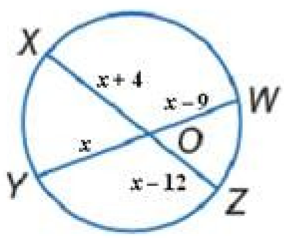
7.

SOLUTION:

$MR \cdot RP = QR \cdot RN$	Theorem 10.15
$6 \cdot x = 10 \cdot 3$	Substitution
$6x = 30$	Multiply.
$x = 5$	Divide each side by 6.

ANSWER:

5



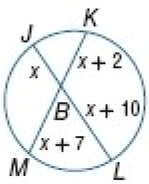
8.

SOLUTION:

$YO \cdot OW = XO \cdot OZ$	Theorem 10.15
$x(x-9) = (x+4)(x-12)$	Substitution
$x^2-9x = x^2-8x-48$	Multiply.
$-x = -48$	Add $-x^2$ and $8x$ to each side.
$x = 48$	Divide each side by -1.

ANSWER:

48

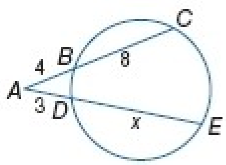
9. 

SOLUTION:

$$\begin{aligned} JB \cdot BL &= MB \cdot BK && \text{Theorem 10.15} \\ x(x+10) &= (x+7)(x+2) && \text{Substitution} \\ x^2+10x &= x^2+9x+14 && \text{Multiply.} \\ x &= 14 && \text{Subtract } x^2 \text{ and } 9x \text{ from each side.} \end{aligned}$$

ANSWER:

14

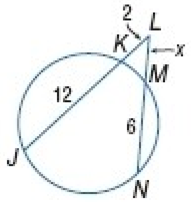
10. 

SOLUTION:

$$\begin{aligned} AB \cdot AC &= AD \cdot AE && \text{Theorem 10.15} \\ 4(4+8) &= 3(3+x) && \text{Substitution} \\ 48 &= 9+3x && \text{Multiply.} \\ 39 &= 3x && \text{Subtract 9 from each side.} \\ x &= 13 && \text{Divide each side by 3.} \end{aligned}$$

ANSWER:

13

11. 

SOLUTION:

$LK \cdot LJ = LM \cdot LN$ Theorem 10.16

$2(2+12) = x(x+6)$ Substitution

$28 = x^2 + 6x$ Multiply.

$0 = x^2 + 6x - 28$ Subtract 28 from each side.

Solve for *x* by using the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-28)}}{2(1)}$ Substitution

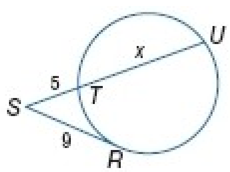
$x = \frac{-6 \pm \sqrt{148}}{2}$ Simplify.

$x \approx 3.1 \text{ or } -9.1$ Use a calculator.

Since lengths cannot be negative, the value of *x* is about 3.1.

ANSWER:

3.1

12. 

SOLUTION:

$ST \cdot SU = SR^2$ Theorem 10.17

$5(5+x) = 9^2$ Substitution

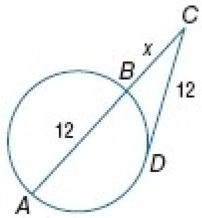
$25 + 5x = 81$ Multiply.

$5x = 56$ Subtract 25 from each side.

$x = 11.2$ Divide each side by 5.

ANSWER:

11.2

13. 

SOLUTION:

$$CB \cdot CA = CD^2 \quad \text{Theorem 10.17}$$
$$x(x+12) = 12^2 \quad \text{Substitution}$$
$$x^2+12x = 144 \quad \text{Multiply.}$$
$$x^2+12x-144 = 0 \quad \text{Subtract 144 from each side.}$$

Solve for x using the Quadratic Formula.

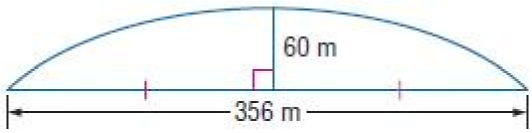
$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \quad \text{Quadratic Formula}$$
$$x = \frac{-12 \pm \sqrt{12^2-4(1)(-144)}}{2(1)} \quad \text{Substitution}$$
$$x = \frac{-12 \pm \sqrt{720}}{2} \quad \text{Simplify.}$$
$$x \approx 7.4 \text{ or } -19.4 \quad \text{Use a calculator.}$$

Since lengths cannot be negative, the value of x is about 7.4.

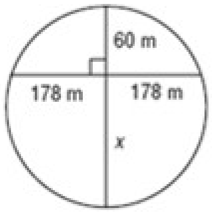
ANSWER:

7.4

14. **BRIDGES** What is the diameter of the circle containing the arc of the Sydney Harbour Bridge? Round to the nearest tenth.
Refer to the photo on Page 754.



SOLUTION:



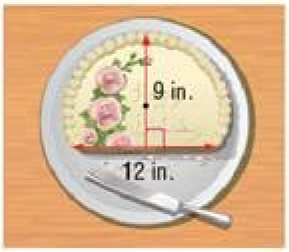
Since the height is perpendicular and bisects the chord, it must be part of the diameter. To find the diameter of the circle, first use Theorem 10.15 to find the value of *x*.

$$\begin{aligned} 60 \cdot x &= 178 \cdot 178 && \text{Theorem 10.15} \\ 60x &= 31{,}684 && \text{Multiply.} \\ x &= 528.1 && \text{Divide each side by 60.} \end{aligned}$$

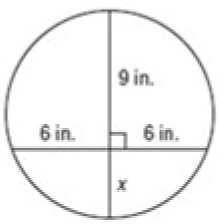
Add the two segments to find the diameter.
Therefore, the diameter of the circle is 528.01+ 60 or about 588.1 meters.

ANSWER:
588.1 m

15. **CAKES** Sierra is serving cake at a party. If the dimensions of the remaining cake are shown below, what was the original diameter of the cake?



SOLUTION:



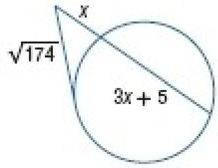
Since one of the perpendicular segments contains the center of the cake, it will bisect the chord. To find the diameter of the cake, first use Theorem 10.15 to find x.

$$\begin{aligned} 9 \cdot x &= 6 \cdot 6 && \text{Theorem 10.15} \\ 9x &= 36 && \text{Multiply.} \\ x &= 4 && \text{Divide each side by 9.} \end{aligned}$$

Add the two segments to find the diameter.
The diameter of the circle is 9 + 4 or 13 inches.

ANSWER:
13 in.

CCSS STRUCTURE Find each variable to the nearest tenth. Assume that segments that appear to be tangent are tangent.

16. 

SOLUTION:

$$x \cdot (x + 3x + 5) = \sqrt{174}^2 \quad \text{Theorem 10.17}$$
$$x(4x + 5) = 174 \quad \text{Simplify.}$$
$$4x^2 + 5x - 174 = 0 \quad \text{Subtract 174 from each side.}$$

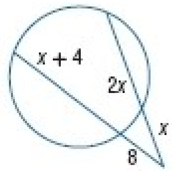
Solve for *x* using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula}$$
$$x = \frac{-5 \pm \sqrt{5^2 - 4(4)(-174)}}{2(4)} \quad \text{Substitution}$$
$$x = \frac{-5 \pm \sqrt{2809}}{8} \quad \text{Simplify.}$$
$$x = 6 \text{ or about } -7.3 \quad \text{Use a calculator.}$$

Since lengths cannot be negative, the value of *x* is 6.

ANSWER:

6

17. 

SOLUTION:

$$
\begin{aligned}
x(x+2x) &= 8(8+x+4) && \text{Theorem 10.17} \\
x(3x) &= 8(12+x) && \text{Simplify.} \\
3x^2 &= 96+8x && \text{Multiply.}
\end{aligned}
$$

$3x^2-8x-96=0$ Subtract $8x$ and 96 from each side.

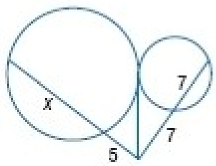
Solve for x using the Quadratic Formula.

$$
\begin{aligned}
x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} && \text{Quadratic Formula} \\
x &= \frac{-(-8)\pm\sqrt{(-8)^2-4(3)(-96)}}{2(3)} && \text{Substitution} \\
x &= \frac{8\pm\sqrt{1216}}{6} && \text{Simplify.} \\
x &\approx 7.1 \text{ or } -4.5 && \text{Use a calculator.}
\end{aligned}
$$

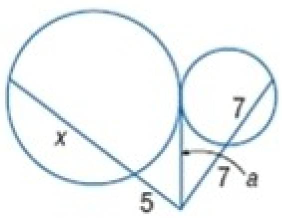
Since lengths cannot be negative, the value of x is about 7.1.

ANSWER:

7.1

18. 

SOLUTION:



First, find the length, a, of the tangent segment to both circles using the intersecting secant and tangent segments to the smaller circle.

$a^2 = 7(7+7)$ Theorem 10.17

$a^2 = 98$ Simplify.

$a = \sqrt{98}$ Take the positive square root of each side.

Find x using the intersecting secant and tangent segments to the larger circle.

$5(5+x) = \sqrt{98}^2$ Theorem 10.17

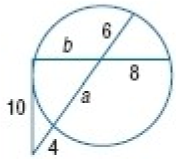
$25+5x = 98$ Multiply.

$5x = 73$ Subtract 25 from each side.

$x = 14.6$ Divide each side by 5.

ANSWER:

14.6

19. 

SOLUTION:

Use the intersecting tangent and secant segments to find *a*.

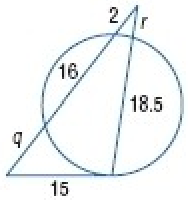
$$\begin{aligned} 4(4+a+6) &= 10^2 && \text{Theorem 10.17} \\ 4a+40 &= 100 && \text{Simplify.} \\ 4a &= 60 && \text{Subtract 40 from each side.} \\ 4a &= 60 && \text{Divide each side by 4.} \end{aligned}$$

Next, use the intersecting chords to find *b*.

$$\begin{aligned} 8 \cdot b &= 15(6) && \text{Theorem 10.15} \\ 8b &= 90 && \text{Multiply.} \\ b &\approx 11.3 && \text{Divide each side by 8.} \end{aligned}$$

ANSWER:

$a = 15$; $b \approx 11.3$

20. 

SOLUTION:

Use the intersecting tangent and secant segments to find *q*.

$$\begin{aligned} q(q+16) &= 15^2 && \text{Theorem 10.17} \\ q^2+16q &= 225 && \text{Multiply.} \\ q^2+16q-225 &= 0 && \text{Subtract 225 from each side.} \\ (q+25)(q-9) &= 0 && \text{Factor.} \\ q &= -25 \text{ or } 9 && \text{Zero Product Property} \end{aligned}$$

Since lengths cannot be negative, the value of *q* is 9.

Use the intersecting secant segments to find *r*.

$$\begin{aligned} r(r+18.5) &= 2(2+16) && \text{Theorem 10.16} \\ r^2+18.5r &= 36 && \text{Multiply.} \\ r^2+18.5r-36 &= 0 && \text{Subtract 36 from each side.} \end{aligned}$$

Solve for *r* using the Quadratic Formula.

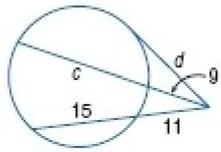
$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} && \text{Quadratic Formula} \\ r &= \frac{-18.5\pm\sqrt{18.5^2-4(1)(-36)}}{2(1)} && \text{Substitution} \\ r &= \frac{-18.5\pm\sqrt{486.25}}{2} && \text{Simplify.} \\ r &\approx 1.8 \text{ or } -20.3 && \text{Use a calculator.} \end{aligned}$$

Since lengths cannot be negative, the value of *r* is about 1.8.

Therefore, the values of the variables are $q = 9$ and $r \approx 1.8$.

ANSWER:

$q = 9$; $r \approx 1.8$

21. 

SOLUTION:

Use the intersecting secant segments to find *c*.

$$\begin{aligned} 9(9+c) &= 11(11+15) && \text{Theorem 10.16} \\ 81+9c &= 286 && \text{Simplify.} \\ 9c &= 205 && \text{Subtract 81 from each side.} \\ c &\approx 22.8 && \text{Divide each side by 9.} \end{aligned}$$

Use the intersecting secant and tangent segments to find *d*.

$$\begin{aligned} d^2 &= 9(9+22.8) && \text{Theorem 10.15} \\ d^2 &= 286.2 && \text{Multiply.} \\ d &\approx 16.9 && \text{Take the positive square root of each side.} \end{aligned}$$

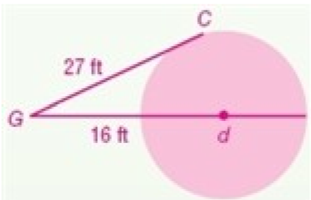
Therefore, the values of the variables are $c \approx 22.8$ and $d \approx 16.9$.

ANSWER:

$c \approx 22.8$; $d \approx 16.9$

22. **INDIRECT MEASUREMENT** Gwendolyn is standing 16 feet from a giant sequoia tree and Chet is standing next to the tree, as shown. The distance between Gwendolyn and Chet is 27 feet. Draw a diagram of this situation, and then find the diameter of the tree.

SOLUTION:



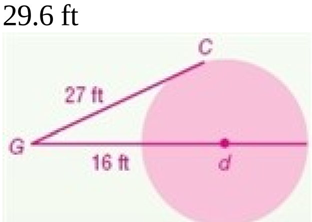
Let d be the diameter of the tree.
Use the intersecting tangent and secant segments to find d.

$$\begin{aligned} 16(16+d) &= 27^2 && \text{Theorem 10.17} \\ 256+16d &= 729 && \text{Multiply.} \\ 16d &= 473 && \text{Subtract 256 from each side.} \\ d &\approx 29.6 && \text{Divide each side by 16.} \end{aligned}$$

So, the diameter is about 29.6 ft.

ANSWER:

29.6 ft

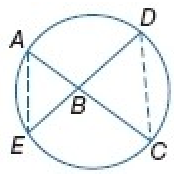


PROOF Prove each theorem.

23. two-column proof of Theorem 10.15

Given: $\overline{AC}$ and $\overline{DE}$ intersect at *B*.

Prove: $AB \cdot BC = EB \cdot BD$



SOLUTION:

Proof:

Statements (Reasons)

1. $\overline{AC}$ and $\overline{DE}$ intersect at *B*. (Given)
2. $\angle A \cong \angle D$, $\angle E \cong \angle C$ (Inscribed $\angle s$ that intercept the same arc are $\cong$.)
3. $\Delta ABE \sim \Delta DBC$ (AA Similarity)
4. $\frac{AB}{BD} = \frac{EB}{BC}$ (Definition of similar Δs)
5. $AB \cdot BC = EB \cdot BD$ (Cross products)

ANSWER:

Proof:

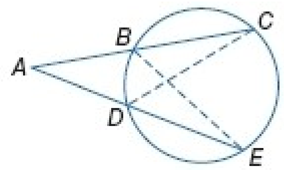
Statements (Reasons)

1. $\overline{AC}$ and $\overline{DE}$ intersect at *B*. (Given)
2. $\angle A \cong \angle D$, $\angle E \cong \angle C$ (Inscribed $\angle s$ that intercept the same arc are $\cong$.)
3. $\Delta ABE \sim \Delta DBC$ (AA Similarity)
4. $\frac{AB}{BD} = \frac{EB}{BC}$ (Definition of similar Δs)
5. $AB \cdot BC = EB \cdot BD$ (Cross products)

24. paragraph proof of Theorem 10.16

Given: Secants $\overline{AC}$ and $\overline{AE}$

Prove: $AB \cdot AC = AD \cdot AE$



SOLUTION:

Proof:

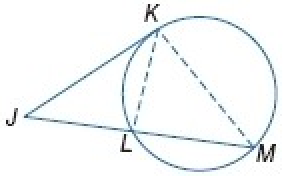
$\overline{AC}$ and $\overline{AE}$ are secant segments. By the Reflexive Property, $\angle BAD \cong \angle DAB$. Inscribed angles that intercept the same arc are congruent. So, $\angle ACD \cong \angle AEB$. By AA Similarity, $\Delta AEB \sim \Delta ACD.$ By the definition of similar triangles, $\frac{AB}{AD} = \frac{AE}{AC}.$ Since the cross products of a proportion are equal, $AB \cdot AC = AD \cdot AE.$

ANSWER:

Proof:

$\overline{AC}$ and $\overline{AE}$ are secant segments. By the Reflexive Property, $\angle BAD \cong \angle DAB$. Inscribed angles that intercept the same arc are congruent. So, $\angle ACD \cong \angle AEB$. By AA Similarity, $\Delta AEB \sim \Delta ACD.$ By the definition of similar triangles, $\frac{AB}{AD} = \frac{AE}{AC}.$ Since the cross products of a proportion are equal, $AB \cdot AC = AD \cdot AE.$

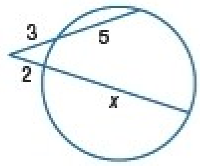
25. two-column proof of Theorem 10.17
Given: tangent $\overline{JK}$, secant $\overline{JM}$
Prove: $JK^2 = JL \cdot JM$



SOLUTION:
Proof:
Statements (Reasons)
1. tangent $\overline{JK}$ and secant $\overline{JM}$ (Given)
2. $m\angle KML = \frac{1}{2}m\widehat{KL}$ (The measure of an inscribed $\angle$ equals half the measure of its intercept arc.)
3. $m\angle JKL = \frac{1}{2}m\widehat{KL}$ (The measure of an $\angle$ formed by a secant and a tangent = half the measure of its intercepted arc.)
4. $m\angle KML = m\angle JKL$ (Substitution)
5. $\angle KML \cong \angle JKL$ (Definition of $\cong \angle s$)
6. $\angle J \cong \angle J$ (Reflexive Property)
7. $\Delta JMK \sim \Delta JKL$ (AA Similarity)
8. $\frac{JK}{JL} = \frac{JM}{JK}$ (Definition of $\sim \Delta s$)
9. $JK^2 = JL \cdot JM$ (Cross products)

ANSWER:
Proof:
Statements (Reasons)
1. tangent $\overline{JK}$ and secant $\overline{JM}$ (Given)
2. $m\angle KML = \frac{1}{2}m\widehat{KL}$ (The measure of an inscribed $\angle$ equals half the measure of its intercept arc.)
3. $m\angle JKL = \frac{1}{2}m\widehat{KL}$ (The measure of an $\angle$ formed by a secant and a tangent = half the measure of its intercepted arc.)
4. $m\angle KML = m\angle JKL$ (Substitution)
5. $\angle KML \cong \angle JKL$ (Definition of $\cong \angle s$)
6. $\angle J \cong \angle J$ (Reflexive Property)
7. $\Delta JMK \sim \Delta JKL$ (AA Similarity)
8. $\frac{JK}{JL} = \frac{JM}{JK}$ (Definition of $\sim \Delta s$)
9. $JK^2 = JL \cdot JM$ (Cross products)

26. **CRITIQUE** Tiffany and Jun are finding the value of x in the figure at the right. Tiffany wrote $3(5) = 2x$, and Jun wrote $3(8) = 2(2 + x)$. Is either of them correct? Explain your reasoning.



SOLUTION:

Jun; the segments intersect outside of the circle, so the correct equation involves the products of the measures of a secant and its external secant segment.

ANSWER:

Jun; the segments intersect outside of the circle, so the correct equation involves the products of the measures of a secant and its external secant segment.

27. **WRITING IN MATH** Compare and contrast the methods for finding measures of segments when two secants intersect in the exterior of a circle and when a secant and a tangent intersect in the exterior of a circle.

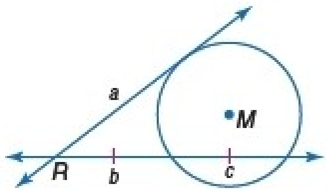
SOLUTION:

Sample answer: When two secants intersect in the exterior of a circle, the product of the measures of one secant segment and its external segment is equal to the product of the measures of the other secant segment and its external segment. When a secant and a tangent intersect at an exterior point, the product of the measures of the secant segment and its external segment is equal to the square of the measure of the tangent segment, because for the tangent the measures of the external segment and the whole segment are the same.

ANSWER:

Sample answer: When two secants intersect in the exterior of a circle, the product of the measures of one secant segment and its external segment is equal to the product of the measures of the other secant segment and its external segment. When a secant and a tangent intersect at an exterior point, the product of the measures of the secant segment and its external segment is equal to the square of the measure of the tangent segment, because for the tangent the measures of the external segment and the whole segment are the same.

28. **CHALLENGE** In the figure, a line tangent to circle M and a secant line intersect at R. Find a. Show the steps that you used.



SOLUTION:

Use the intersecting tangent and secant segments to find the value of a.

$$
\begin{aligned}
a^2 &= b(b+c) && \text{Theorem 10.17}\\
a^2 &= b(b+b) && c=b\\
a^2 &= 2b^2 && \text{Simplify.}\\
a &= \pm\sqrt{2b^2} && \text{Take the square root of each side.}\\
a &= b\sqrt{2} && \text{Simplify.}
\end{aligned}
$$

ANSWER:

$b = c$

$a^2 = b(b+c)$

$a^2 = b(b+b)$

$a^2 = b(2b)$

$a^2 = 2b^2$

$a = \pm\sqrt{2b^2}$

$a = b\sqrt{2}$ Segments do not have negative length.

29. **REASONING** When two chords intersect at the center of a circle, are the measures of the intercepting arcs *sometimes, always,* or *never* equal to each other?

SOLUTION:

Since the chords intersect at the center, the measure of each intercepted arc is equal to the measure of its related central angle. Since vertical angles are congruent, the opposite arcs will be congruent. All four arcs will only be congruent when all four central angles are congruent. This will only happen when the chords are perpendicular and the angles are right angles
Therefore, the measures of the intercepted arcs are sometimes equal to each other.

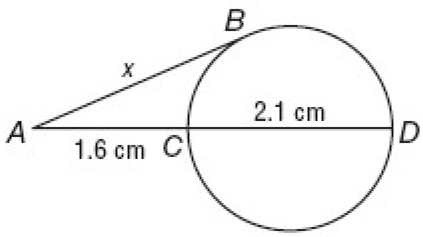
ANSWER:

Sometimes; they are equal when the chords are perpendicular.

30. **OPEN ENDED** Investigate Theorem 10.17 by drawing and labeling a circle that has a secant and a tangent intersecting outside the circle. Measure and label the two parts of the secant segment to the nearest tenth of a centimeter. Use an equation to find the measure of the tangent segment. Verify your answer by measuring the segment.

SOLUTION:

Sample answer:



$x^2 = 1.6(1.6 + 2.1)$ Theorem 10.17

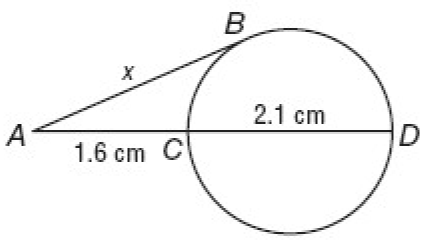
$x^2 = 5.92$ Simplify.

$x \approx 2.4$ Take the positive square root of each side.

Therefore, the measure of the tangent segment is about 2.4 centimeters.

ANSWER:

Sample answer:



$x \approx 2.4$ cm

31. **WRITING IN MATH** Describe the relationship among segments in a circle when two secants intersect inside a circle.

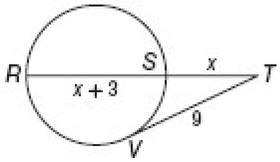
SOLUTION:

Sample answer: The product of the parts on one intersecting chord equals the product of the parts of the other chord.

ANSWER:

Sample answer: The product of the parts on one intersecting chord equals the product of the parts of the other chord.

32. $\overline{TV}$ is tangent to the circle, and R and S are points on the circle. What is the value of x to the nearest tenth?



A 7.6
B 6.4
C 5.7
D 4.8

SOLUTION:

$x(2x+3) = 9^2$ Theorem 10.17

$2x^2+3x = 81$ Multiply.

$2x^2+3x-81 = 0$ Subtract 81 from each side.

Solve for x using the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ Quadratic Formula

$x = \frac{-3 \pm \sqrt{3^2-4(2)(-81)}}{2(2)}$ Substitution

$x = \frac{-3 \pm \sqrt{657}}{4}$ Simplify.

$x \approx 5.7$ or -7.2 Use a calculator.

Since length can not be negative, the value of x is 5.7.
Therefore, the correct answer is C.

ANSWER:
C

33. **ALGEBRA** A department store has all of its jewelry discounted 40%. It is having a sale that says you receive an additional 20% off the already discounted price. How much will you pay for a ring with an original price of $200?
F $80
G $96
H $120
J $140

SOLUTION:

$$\begin{aligned}\text{Discount} &= 200 \times \frac{40}{100}\\ &= 80\end{aligned}$$

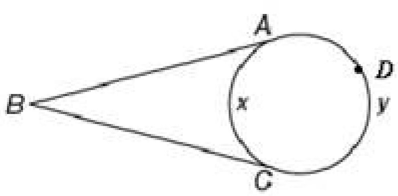
Discount price = 200 – 80 = 120

$$\begin{aligned}\text{Discount of discounted price} &= 120 \times \frac{20}{100}\\ &= 24\end{aligned}$$

Discount price = 120 – 24
= 96
So, the correct choice is G.

ANSWER:
G

34. **EXTENDED RESPONSE** The degree measures of minor arc AC and major arc ADC are x and y, respectively.
a. If $m\angle ABC = 70°$, write two equations relating x and y.
b. Find x and y.



SOLUTION:
a. Since the sum of the arcs of a circle is 360, $x + y = 360$,
and by Theorem 10.14, $y - x = 140$.
b.

$$\begin{aligned} x + y &= 360 && \text{Original equations}\\ \underline{y - x} &\underline{=} \underline{140} &&\\ 2y &= 500 && \text{Add the two equations.}\\ y &= 250 && \text{Divide each side by 2.}\\ x + 250 &= 360 && \text{Substitute into the first equation.}\\ y &= 110 && \text{Divide each side by 2.}\end{aligned}$$

Therefore, $x = 110°$ and $y = 250°$.

ANSWER:
a. $x + y = 360$ and $y - x = 140$
b. $x = 110°, y = 250°$

35. **SAT/ACT** During the first two weeks of summer vacation, Antonia earned \$100 per week. During the next six weeks, she earned \$150 per week. What was her average weekly pay?

A \$50
B \$112.50
C \$125
D \$135
E \$137.50

SOLUTION:

$$\text{Average} = \frac{2(100) + 6(150)}{8}$$

Simplify.

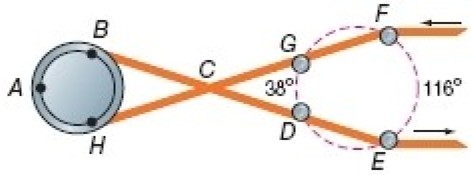
$$\text{Average} = 137.5$$

So, the correct choice is E.

ANSWER:

E

36. **WEAVING** Once yarn is woven from wool fibers, it is often dyed and then threaded along a path of pulleys to dry. One set of pulleys is shown. Note that the yarn appears to intersect itself at *C*, but in reality it does not. Use the information from the diagram to find $m\widehat{BH}$.



SOLUTION:

First, use intersecting secants to find $m\angle GCD$.

$$\begin{aligned} m\angle GCD &= \frac{1}{2}[m(\text{arc}FE) - m(\text{arc}GD)] && \text{Theorem 10.14} \\ &= \frac{1}{2}[116 - 38] && \text{Substitution} \\ &= \frac{1}{2}(78) && \text{Simplify.} \\ &= 39 && \text{Multiply.} \end{aligned}$$

By vertical angles, $m\angle BCH = m\angle GCD$. So, $m\angle BCH = 39$.

Since major arc *BAH* has the same endpoints as minor arc *BH*, $m(\text{arc } BAH) = 360 - m(\text{arc } BH)$.

$$\begin{aligned} m\angle BCH &= \frac{1}{2}[m(\text{arc}BAH) - m(\text{arc}BH)] && \text{Theorem 10.15} \\ 39 &= \frac{1}{2}[360 - m(\text{arc}BH) - m(\text{arc}BH)] && \text{Substitution} \\ 78 &= 360 - 2m(\text{arc}BH) && \text{Multiply each side by 2 and simplify.} \\ 2m(\text{arc}BH) &= 282 && \text{Add } -78 \text{ and } 2m(\text{arc}BH) \text{ to each side.} \\ m(\text{arc}BH) &= 141 && \text{Divide each side by 2.} \end{aligned}$$

Therefore, the measure of arc *BH* is 141.

ANSWER:

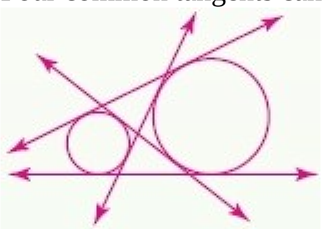
141

Copy the figure shown and draw the common tangents. If no common tangent exists, state *no common tangent*.

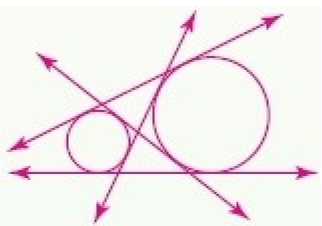
37. 

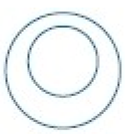
SOLUTION:

Four common tangents can be drawn to these two circles.



ANSWER:



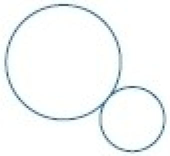
38. 

SOLUTION:

Every tangent drawn to the small circle will intersect the larger circle in two points. Every tangent drawn to the large circle will not intersect the small circle at any point. Since a tangent must intersect the circle at exactly one point, no common tangent exists for these two circles.

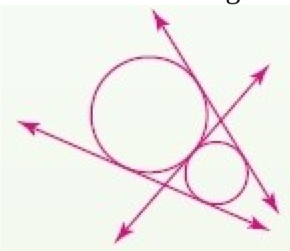
ANSWER:

no common tangent

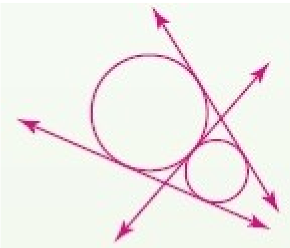
39. 

SOLUTION:

Three common tangents can be drawn to these two circles.



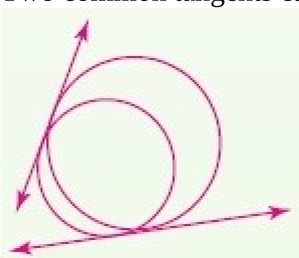
ANSWER:



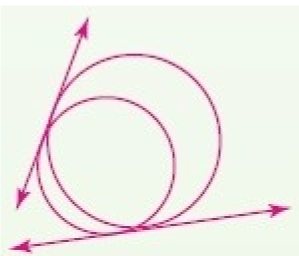
40. 

SOLUTION:

Two common tangents can be drawn to these two circles.



ANSWER:



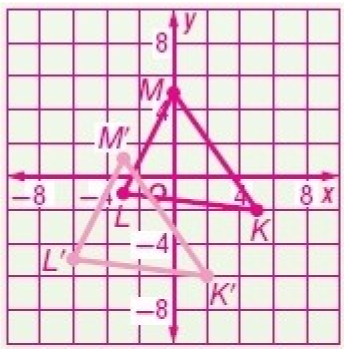
COORDINATE GEOMETRY Graph each figure and its image along the given vector.

41. ΔKLM with vertices $K(5, -2)$, $L(-3, -1)$, and $M(0, 5)$; $\langle -3, -4 \rangle$

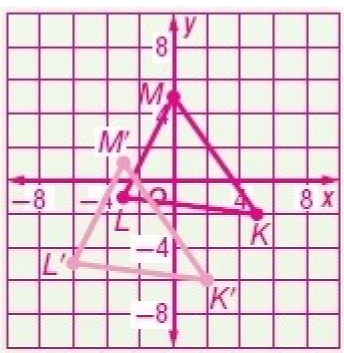
SOLUTION:

$$\begin{array}{lcl} (x, y) & \rightarrow & (x-3, y-4) \\ K(5, -2) & \rightarrow & K'(2, -6) \\ L(-3, -1) & \rightarrow & L'(-6, -5) \\ M(0, 5) & \rightarrow & M'(-3, 1) \end{array}$$

The coordinates of the vertices of the translated triangle are $K'(2, -6)$, $L'(-6, -5)$, and $M'(-3, 1)$.



ANSWER:

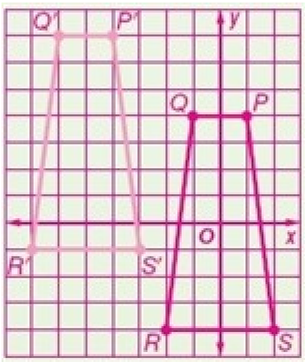


42. quadrilateral *PQRS* with vertices *P*(1, 4), *Q*(–1, 4), *R*(–2, –4), and *S*(2, –4); $\langle -5, 3 \rangle$

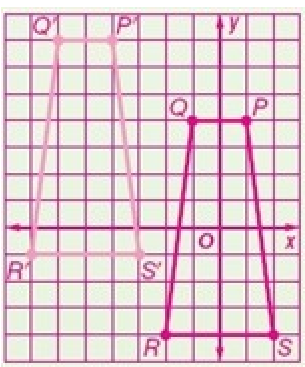
SOLUTION:

$$\begin{array}{lll} (x, y) & \to & (x - 5, y + 3) \\ P(1, 4) & \to & P'(-4, 7) \\ Q(-1, 4) & \to & Q'(-6, 7) \\ R(-2, -4) & \to & R'(-7, -1) \\ S(2, -4) & \to & S'(-3, -1) \end{array}$$

The coordinates of the vertices of the translated quadrilateral are *P'*(– 4, 7), *Q'*(–6, 7), *R'*(–7, –1), and *S'*(– 3, –1).



ANSWER:

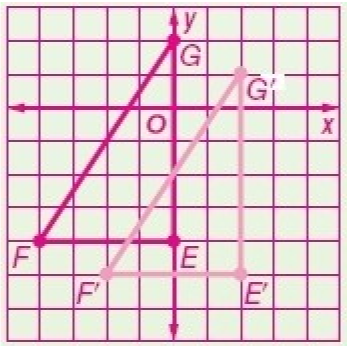


43. ΔEFG with vertices $E(0, –4)$, $F(–4, –4)$, and $G(0, 2)$; $\langle 2, -1 \rangle$

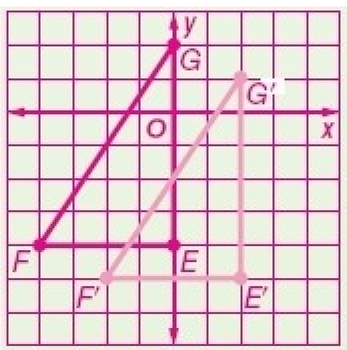
SOLUTION:

$$(x, y) \rightarrow (x + 2, y - 1)$$
$$E(0, -4) \rightarrow E'(2, -5)$$
$$F(-4, -4) \rightarrow F'(-2, -5)$$
$$G(0, 2) \rightarrow G'(2, 1)$$

The coordinates of the vertices of the translated triangle are $E'(2, –5)$, $F'(–2, –5)$, and $G'(2, 1)$.



ANSWER:



Write an equation in slope-intercept form of the line having the given slope and y-intercept.

44. m: 3, y-intercept: –4

SOLUTION:

The slope-intercept form of a line of slope m and y-intercept b is given by $y = mx + b$.
For this line, substitute $m = 3$ and $b = –4$.
So, the equation of the line is $y = 3x – 4$.

ANSWER:

$y = 3x – 4$

45. m: 2, (0, 8)

SOLUTION:

To find the slope-intercept form of the equation of this line use the point-slope form of a line, $y - y_1 = m(x - x_1)$ where m is the slope and (x_1, y_1) is a point on the line.

Here, $m = 2$ and $(x_1, y_1) = (0, 8)$.

$$\begin{aligned} (y - y_1) &= m(x - x_1) && \text{Point - slope form of line} \\ y - 8 &= 2(x - 0) && m = 2, (x_1, y_1) = (0, 8) \\ y - 8 &= 2x && \text{Multiply.} \\ y &= 2x + 8 && \text{Add 8 to each side.} \end{aligned}$$

ANSWER:

$y = 2x + 8$

46. $m: \frac{5}{8}, (0, -6)$

SOLUTION:

To find the slope-intercept form of the equation of this line use the point-slope form of a line, $y - y_1 = m(x - x_1)$ where m is the slope and (x_1, y_1) is a point on the line.

Here, $m = \frac{5}{8}$ and $(x_1, y_1) = (0, -6)$.

$$\begin{aligned} (y - y_1) &= m(x - x_1) && \text{Point - slope form of line} \\ y - (-6) &= \frac{5}{8}(x - 0) && m = \frac{5}{8}, (x_1, y_1) = (0, -6) \\ y + 6 &= \frac{5}{8}x && \text{Simplify.} \\ y &= \frac{5}{8}x - 6 && \text{Subtract 6 from each side.} \end{aligned}$$

ANSWER:

$y = \frac{5}{8}x - 6$

47. $m: \frac{2}{9}, y-\text{intercept}: \frac{1}{3}$

SOLUTION:

The slope-intercept form of a line of slope *m* and *y*-intercept *b* is given by $y = mx + b$. For this line, substitute $m = \frac{2}{9}$ and $b = \frac{1}{3}$.

So, the equation of the line is $y = \frac{2}{9}x + \frac{1}{3}$.

ANSWER:

$y = \frac{2}{9}x + \frac{1}{3}$

48. $m: -1, b: -3$

SOLUTION:

The slope-intercept form of a line of slope *m* and *y*-intercept *b* is given by $y = mx + b$.
For this line, substitute $m = -1$ and $b = -3$.
So, the equation of the line is $y = -1x - 3$ or $y = -x - 3$.

ANSWER:

$y = -x - 3$

49. $m: -\frac{1}{12}, b: 1$

SOLUTION:

The slope-intercept form of a line of slope *m* and *y*-intercept *b* is given by $y = mx + b$.

For this line, substitute $m = -\frac{1}{12}$ and $b = 1$.

So, the equation of the line is $y = -\frac{1}{12}x + 1$.

ANSWER:

$y = -\frac{1}{12}x + 1$