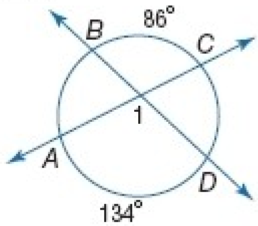
10-6 Secants, Tangents, and Angle Measures

Find each measure. Assume that segments that appear to be tangent are tangent.

1. $m\angle 1$



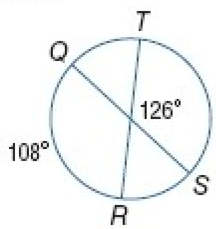
SOLUTION:

$$m\angle 1 = \frac{1}{2}[m(\text{arc } AD) + m(\text{arc } BC)] \quad \text{Theorem 10.12}$$
$$= \frac{1}{2}[134 + 86] \quad \text{Substitution}$$
$$= \frac{1}{2}(220) \text{ or } 110 \quad \text{Simplify.}$$

ANSWER:

110

2. $m\widehat{TS}$



SOLUTION:

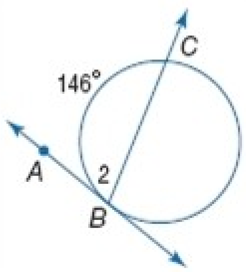
$$126 = \frac{1}{2}[m(\text{arc } TS) + m(\text{arc } QR)] \quad \text{Theorem 10.12}$$
$$252 = m(\text{arc } TS) + 86 \quad \text{Substitution and multiply each side by 2.}$$
$$144 = m(\text{arc } TS) \quad \text{Subtract 86 from each side.}$$

So, the measure of arc *TS* is 144.

ANSWER:

144

3. $m\angle 2$



SOLUTION:

$m\angle 2 = \frac{1}{2}[m(\text{arc}BC)]$ Theorem10.13

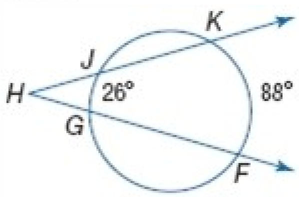
$= \frac{1}{2}[146]$ Substitution

$= 73$ Simplify.

ANSWER:

73

4. $m\angle H$



SOLUTION:

$m\angle H = \frac{1}{2}[m(\text{arc}KF) - m(\text{arc}JG)]$ Theorem10.14

$= \frac{1}{2}[88 - 26]$ Substitution

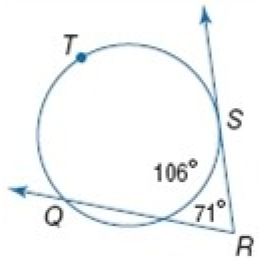
$= \frac{1}{2}(62)$ Simplify.

$= 31$ Multiply.

ANSWER:

31

5. $m\widehat{QTS}$



SOLUTION:

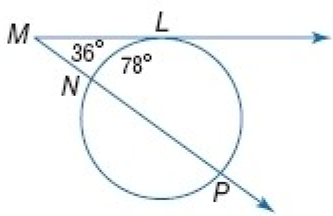
$$\begin{aligned} m\angle R &= \frac{1}{2}[m(\text{arc}\,QTS) - 106] && \text{Theorem 10.14} \\ 71 &= \frac{1}{2}[m(\text{arc}\,QTS) - 106] && \text{Substitution} \\ 142 &= m(\text{arc}\,QTS) - 106 && \text{Multiply each side by 2.} \\ 248 &= m(\text{arc}\,QTS) && \text{Add 106 to each side.} \end{aligned}$$

So, the measure of arc *QTS* is 248.

ANSWER:

248

6. $m\widehat{LP}$



SOLUTION:

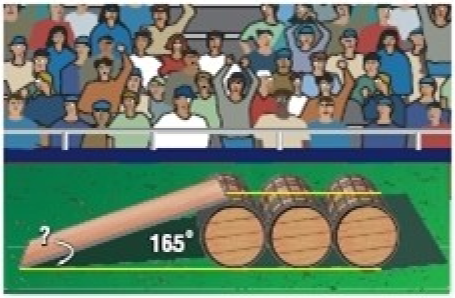
$$\begin{aligned} m\angle M &= \frac{1}{2}[m(\text{arc}\,LP) - m(\text{arc}\,NL)] && \text{Theorem 10.14} \\ 36 &= \frac{1}{2}[m(\text{arc}\,LP) - 78] && \text{Substitution} \\ 72 &= m(\text{arc}\,LP) - 78 && \text{Multiply each side by 2.} \\ 150 &= m(\text{arc}\,LP) && \text{Add 78 to both sides.} \end{aligned}$$

So, the measure of arc *LP* is 150.

ANSWER:

150

7. **STUNTS** A ramp is attached to the first of several barrels that have been strapped together for a circus motorcycle stunt as shown. What is the measure of the angle the ramp makes with the ground?



SOLUTION:

Let *x* be the measure of the angle the ramp makes with the ground which is formed by two intersecting tangents to the circle formed by the barrel. One arc has a measure of 165. The other arc is the major arc with the same endpoints, so its measure is 360 – 165 or 195.

$x = \frac{1}{2}(195 - 165)$ Theorem 10.14

$= \frac{1}{2}(30)$ Substitution

$= 15$ Multiply.

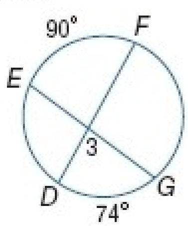
Therefore, the measure of the angle the ramp makes with the ground is 15.

ANSWER:

15

Find each measure. Assume that segments that appear to be tangent are tangent.

8. $m\angle 3$



SOLUTION:

$m\angle 3 = \frac{1}{2}[m(\text{arc}EF) + m(\text{arc}DG)]$ Theorem 10.12

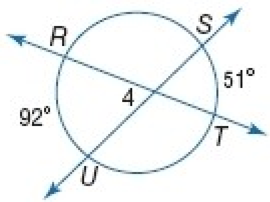
$= \frac{1}{2}[90 + 74]$ Substitution

$= \frac{1}{2}(164) \text{ or } 82$ Simplify.

ANSWER:

82

9. $m\angle 4$



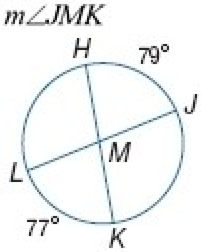
SOLUTION:

$$m\angle 4 = \frac{1}{2}[m(\text{arc}RU) + m(\text{arc}ST)] \quad \text{Theorem 10.12}$$
$$= \frac{1}{2}[92 + 51] \quad \text{Substitution}$$
$$= \frac{1}{2}(141) \text{ or } 71.5 \quad \text{Simplify.}$$

ANSWER:

71.5

10. $m\angle JMK$



SOLUTION:

$$m\angle JMH = \frac{1}{2}[m(\text{arc } LK) + m(\text{arc } JH)] \quad \text{Theorem 10.12}$$
$$= \frac{1}{2}[77 + 79] \quad \text{Substitution}$$
$$= \frac{1}{2}(156) \text{ or } 78 \quad \text{Simplify.}$$

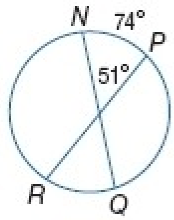
$\angle JMK$ and $\angle HMJ$ form a linear pair.

$$m\angle JMK + m\angle HMJ = 180 \quad \text{Sum of the angles in a linear pair is 180.}$$
$$m\angle JMK + 78 = 180 \quad \text{Substitution}$$
$$m\angle JMK = 102 \quad \text{Use a calculator.}$$

ANSWER:

102

11. $m\widehat{RQ}$



SOLUTION:

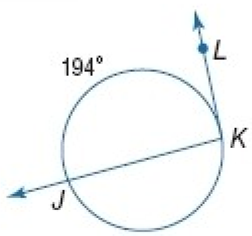
$$
\begin{aligned}
51 &= \frac{1}{2}[m(\text{arc}NP) + m(\text{arc}RQ)] && \text{Theorem 10.12}\\
51 &= \frac{1}{2}[74 + m(\text{arc}RQ)] && \text{Substitution}\\
102 &= 74 + m(\text{arc}RQ) && \text{Multiply each side by 2.}\\
28 &= m(\text{arc}RQ) && \text{Subtract 74 from each side.}
\end{aligned}
$$

Therefore, the measure of arc *RQ* is 28.

ANSWER:

28

12. $m\angle K$



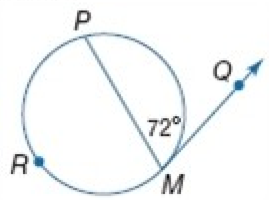
SOLUTION:

$$
\begin{aligned}
m\angle K &= \frac{1}{2}(194) && \text{Theorem 10.13}\\
&= 97 && \text{Multiply.}
\end{aligned}
$$

ANSWER:

97

13. $m\widehat{PM}$



SOLUTION:

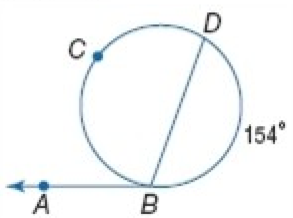
$$m\angle M = \frac{1}{2}[m(\text{arc}PM)] \quad \text{Theorem 10.13}$$
$$72 = \frac{1}{2}[m(\text{arc}PM)] \quad \text{Substitution}$$
$$144 = m(\text{arc}PM) \quad \text{Multiply each side by 2.}$$

So, the measure of arc *PM* is 144.

ANSWER:

144

14. $m\angle ABD$



SOLUTION:

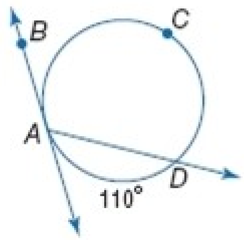
Arc *BD* and arc *BCD* are a minor and major arc that share the same endpoints.

$$m(\text{arc}BCD) = 360 - m(\text{arc}BD) \quad \text{Measure of major arc equals 360 minus the minor arc.}$$
$$= 360 - 154 \quad \text{Substitution}$$
$$= 206 \quad \text{Simplify.}$$
$$m\angle ABD = \frac{1}{2}[m(\text{arc}BCD)] \quad \text{Theorem 10.13}$$
$$= \frac{1}{2}[206] \quad \text{Substitution}$$
$$= 103 \quad \text{Simplify.}$$

ANSWER:

103

15. $m\angle DAB$



SOLUTION:

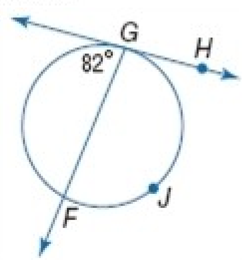
By Theorem 10.13, $m\angle DAB = \frac{1}{2}\left(m\widehat{ACD}\right)$.

$$\begin{aligned} m\angle DAB &= \frac{1}{2}(360 - 110) \\ &= \frac{1}{2}(250) \\ &= 125 \end{aligned}$$

ANSWER:

125

16. $m\widehat{GJF}$



SOLUTION:

By Theorem 10.13, $82 = \frac{1}{2}\left(m\widehat{GF}\right)$.

Solve for $m\widehat{GF}$.

$m\widehat{GF} = 164$

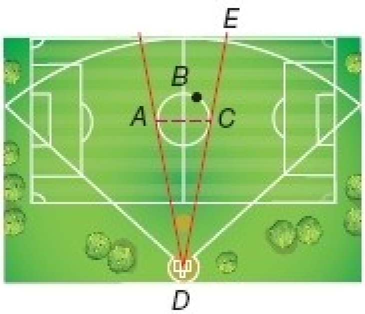
We know that $m\widehat{GJF} = 360 - m\widehat{GF}$.
Substitute.

$$\begin{aligned} m\widehat{GJF} &= 360 - 164 \\ &= 196 \end{aligned}$$

ANSWER:

196

17. **SPORTS** The multi-sport field shown includes a softball field and a soccer field. If $m\widehat{ABC} = 200$, find each measure.



a. $m\angle ACE$
b. $m\angle ADC$

SOLUTION:

a. By Theorem 10.13, $m\angle ACE = \frac{1}{2}\left(m\widehat{ABC}\right)$.

Substitute.

$$m\angle ACE = \frac{1}{2}(200)$$

Simplify.

$$m\angle ACE = 100$$

b. By Theorem 10.14, $m\angle ADC = \frac{1}{2}\left(m\widehat{ABC} - m\widehat{AC}\right)$.

Substitute.

$$m\angle ADC = \frac{1}{2}(200 - 160)$$

Simplify.

$$m\angle ADC = \frac{1}{2}(40)$$

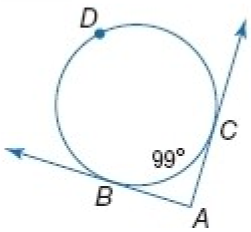
$$m\angle ADC = 20$$

ANSWER:

a. 100
b. 20

CCSS STRUCTURE Find each measure.

18. $m\angle A$



SOLUTION:

By Theorem 10.14, $m\angle A = \frac{1}{2}\left(m\widehat{BDC} - m\widehat{BC}\right)$.

Substitute.

$$m\angle A = \frac{1}{2}\left((360-99)-99\right)$$

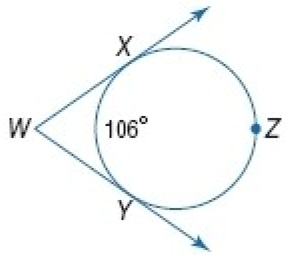
Simplify.

$$m\angle A = \frac{1}{2}(261-99)$$
$$= \frac{1}{2}(162)$$
$$= 81$$

ANSWER:

81

19. $m\angle W$



SOLUTION:

By Theorem 10.14, $m\angle W = \frac{1}{2}\left(m\widehat{XZY} - m\widehat{XY}\right)$.

Substitute.

$$m\angle W = \frac{1}{2}\left((360 - 106) - 106\right)$$

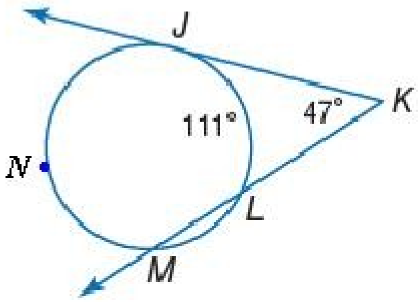
Simplify.

$$\begin{aligned} m\angle W &= \frac{1}{2}(254 - 106) \\ &= \frac{1}{2}(148) \\ &= 74 \end{aligned}$$

ANSWER:

74

20. $m(\text{arc } JNM)$



SOLUTION:

$m\angle K = \frac{1}{2}[m(\text{arc } JNM) - m(\text{arc } JL)]$ Theorem 10.14

$47 = \frac{1}{2}[m(\text{arc } JNM) - 111]$ Substitution

$94 = m(\text{arc } JNM) - 111$ Multiply each side by 2.

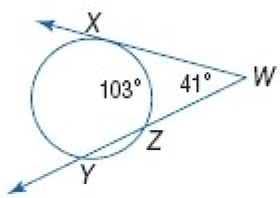
$205 = m(\text{arc } JNM)$ Add 111 to both sides.

So, the measure of arc *JNM* is 205.

ANSWER:

205

21. $m\widehat{XY}$



SOLUTION:

By Theorem 10.14, $m\angle W = \frac{1}{2}\left(m\widehat{XY} - 103\right)$.

Substitute.

$41 = \frac{1}{2}\left(m\widehat{XY} - 103\right)$

Simplify.

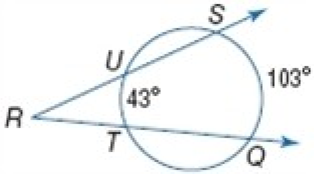
$82 = m\widehat{XY} - 103$

$m\widehat{XY} = 185$

ANSWER:

185

22. $m\angle R$



SOLUTION:

By Theorem 10.14, $m\angle R = \frac{1}{2}\left(m\widehat{SQ} - m\widehat{UT}\right)$.

Substitute.

$m\angle R = \frac{1}{2}(103 - 43)$

Simplify.

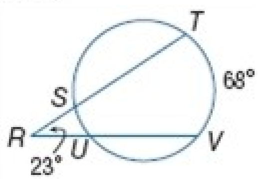
$m\angle R = \frac{1}{2}(60)$

$m\angle R = 30$

ANSWER:

30

23. $m\widehat{SU}$



SOLUTION:

By Theorem 10.14, $m\angle R = \frac{1}{2}\left(m\widehat{TV} - m\widehat{SU}\right)$.

Substitute.

$23 = \frac{1}{2}\left(68 - m\widehat{SU}\right)$

Solve for $m\widehat{SU}$.

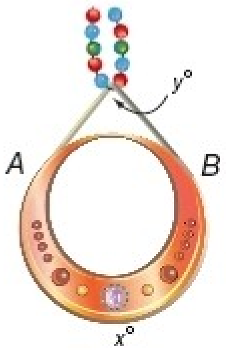
$46 = 68 - m\widehat{SU}$

$m\widehat{SU} = 22$

ANSWER:

22

24. **JEWELRY** In the circular necklace shown, A and B are tangent points. If $x = 260$, what is y?



SOLUTION:

By Theorem 10.14, $y = \frac{1}{2}\left(x - m\widehat{AB}\right)$.

Substitute.

$y = \frac{1}{2}(260 - 100)$

Simplify.

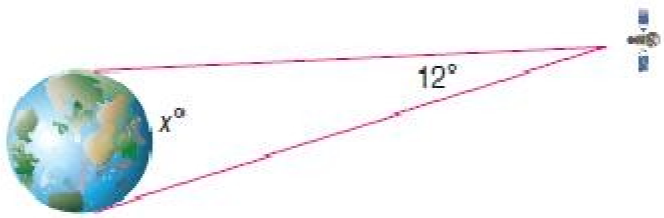
$y = \frac{1}{2}(160)$

$y = 80$

ANSWER:

80

25. **SPACE** A satellite orbits above Earth's equator. Find x, the measure of the planet's arc, that is visible to the satellite.



SOLUTION:

The measure of the visible arc is x and the measure of the arc that is not visible is $360 - x$. Use Theorem 10.14 to find the value of x.

$12 = \frac{1}{2}[(360 - x) - x]$ Theorem 10.14

$12 = \frac{1}{2}[360 - 2x]$ Simplify.

$12 = 180 - x$ Multiply.

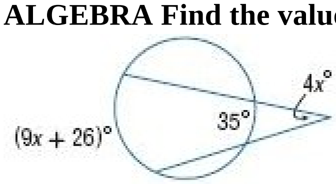
$x = 168$ Add x and subtract 12 from each side.

Therefore, the measure of the planet's arc that is visible to the satellite is 168.

ANSWER:

168

ALGEBRA Find the value of *x*.

26. 

SOLUTION:

By Theorem 10.14, $4x = \frac{1}{2}((9x + 26) - 35)$.

Solve for x.

$4x = \frac{1}{2}((9x + 26) - 35)$

$4x = \frac{1}{2}(9x + 26 - 35)$

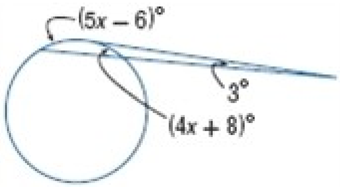
$4x = \frac{1}{2}(9x - 9)$

$8x = 9x - 9$

$x = 9$

ANSWER:

9

27. 

SOLUTION:

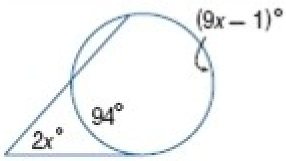
By Theorem 10.14, $3 = \frac{1}{2}((5x-6)-(4x+8))$.

Solve for x.

$$3 = \frac{1}{2}(5x-6-4x-8)$$
$$3 = \frac{1}{2}(x-14)$$
$$6 = x-14$$
$$x = 20$$

ANSWER:

20

28. 

SOLUTION:

By Theorem 10.14, $2x = \frac{1}{2}((9x-1)-(94))$.

Solve for x.

$$2x = \frac{1}{2}((9x-1)-(94))$$
$$2x = \frac{1}{2}(9x-1-94)$$
$$2x = \frac{1}{2}(9x-95)$$
$$4x = 9x-95$$
$$5x = 95$$
$$x = 19$$

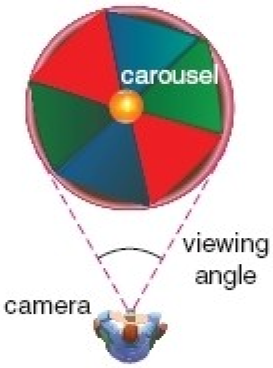
ANSWER:

19

29. **PHOTOGRAPHY** A photographer frames a carousel in his camera shot as shown so that the lines of sight form tangents to the carousel.

a. If the camera's viewing angle is $35°$, what is the arc measure of the carousel that appears in the shot?

b. If you want to capture an arc measure of $150°$ in the photograph, what viewing angle should be used?



SOLUTION:

a. Let *x* be the measure of the carousel that appears in the shot.

By Theorem 10.14, $35 = \frac{1}{2}\left(360 - x - (x)\right)$.

Solve for x.

$35 = \frac{1}{2}(360 - 2x)$
$70 = 360 - 2x$
$2x = 290$
$x = 145$

b. Let *x* be the measure of the camera's viewing angle.

By Theorem 10.14, $x = \frac{1}{2}\left((360 - 150) - (150)\right)$.

Solve for x.

$x = \frac{1}{2}(210 - 150)$
$= \frac{1}{2}(60)$
$= 30$

ANSWER:

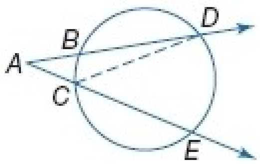
a. 145
b. 30

CCSS ARGUMENTS For each case of Theorem 10.14, write a two-column proof.

30. Case 1

Given: secants $\overrightarrow{AD}$ and $\overrightarrow{AE}$

Prove: $m\angle A = \frac{1}{2}\left(m\widehat{DE} - m\widehat{BC}\right)$



SOLUTION:

Statements (Reasons)

1. $\overrightarrow{AD}$ and $\overrightarrow{AE}$ are secants to the circle. (Given)
2. $m\angle DCE = \frac{1}{2}m\widehat{DE}$, $m\angle ADC = \frac{1}{2}m\widehat{BC}$ (The measure of an inscribed $\angle = \frac{1}{2}$ the measure of its intercepted arc.)
3. $m\angle DCE = m\angle ADC + m\angle A$ (Exterior $\angle s$ Theorem)
4. $\frac{1}{2}m\widehat{DE} = \frac{1}{2}m\widehat{BC} + m\angle A$ (Substitution)
5. $\frac{1}{2}m\widehat{DE} - \frac{1}{2}m\widehat{BC} = m\angle A$ (Subtraction Prop.)
6. $\frac{1}{2}\left(m\widehat{DE} - m\widehat{BC}\right) = m\angle A$ (Distributive Prop.)

ANSWER:

Statements (Reasons)

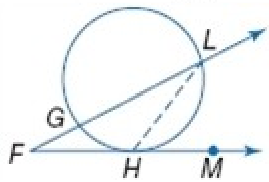
1. $\overrightarrow{AD}$ and $\overrightarrow{AE}$ are secants to the circle. (Given)
2. $m\angle DCE = \frac{1}{2}m\widehat{DE}$, $m\angle ADC = \frac{1}{2}m\widehat{BC}$ (The measure of an inscribed $\angle = \frac{1}{2}$ the measure of its intercepted arc.)
3. $m\angle DCE = m\angle ADC + m\angle A$ (Exterior $\angle s$ Theorem)
4. $\frac{1}{2}m\widehat{DE} = \frac{1}{2}m\widehat{BC} + m\angle A$ (Substitution)
5. $\frac{1}{2}m\widehat{DE} - \frac{1}{2}m\widehat{BC} = m\angle A$ (Subtraction Prop.)
6. $\frac{1}{2}\left(m\widehat{DE} - m\widehat{BC}\right) = m\angle A$ (Distributive Prop.)

10-6 Secants, Tangents, and Angle Measures

31. Case 2

Given: tangent $\overrightarrow{FM}$ and secant $\overrightarrow{FL}$

Prove: $m\angle F = \frac{1}{2}\left(m\widehat{LH} - m\widehat{GH}\right)$



SOLUTION:

Statements (Reasons)

1. $\overline{FM}$ is a tangent to the circle and $\overleftrightarrow{FL}$ is a secant to the circle. (Given)
2. $m\angle FLH = \frac{1}{2}m\widehat{HG}$, $m\angle LHM = \frac{1}{2}m\widehat{LH}$ (The meas. of an inscribed $\angle = \frac{1}{2}$ the measure of its intercepted arc.)
3. $m\angle LHM = m\angle FLH + m\angle F$ (Exterior $\angle s$ Theorem)
4. $\frac{1}{2}m\widehat{LH} = \frac{1}{2}m\widehat{HG} + m\angle F$ (Substitution)
5. $\frac{1}{2}m\widehat{LH} - \frac{1}{2}m\widehat{HG} = m\angle F$ (Subtraction Prop.)
6. $\frac{1}{2}\left(m\widehat{LH} - m\widehat{HG}\right) = m\angle F$ (Distributive Prop.)

ANSWER:

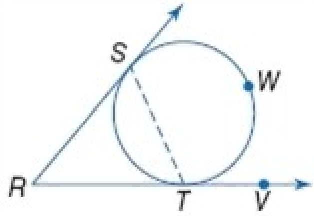
Statements (Reasons)

1. $\overline{FM}$ is a tangent to the circle and $\overleftrightarrow{FL}$ is a secant to the circle. (Given)
2. $m\angle FLH = \frac{1}{2}m\widehat{HG}$, $m\angle LHM = \frac{1}{2}m\widehat{LH}$ (The meas. of an inscribed $\angle = \frac{1}{2}$ the measure of its intercepted arc.)
3. $m\angle LHM = m\angle FLH + m\angle F$ (Exterior $\angle s$ Theorem)
4. $\frac{1}{2}m\widehat{LH} = \frac{1}{2}m\widehat{HG} + m\angle F$ (Substitution)
5. $\frac{1}{2}m\widehat{LH} - \frac{1}{2}m\widehat{HG} = m\angle F$ (Subtraction Prop.)
6. $\frac{1}{2}\left(m\widehat{LH} - m\widehat{HG}\right) = m\angle F$ (Distributive Prop.)

32. Case 3

Given: tangent $\overrightarrow{RS}$ and $\overrightarrow{RV}$

Prove: $m\angle R = \frac{1}{2}(m\widehat{SWT} - m\widehat{ST})$



SOLUTION:

Statements (Reasons)

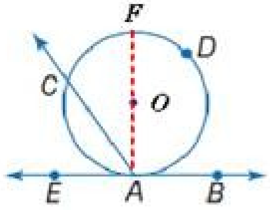
1. $\overrightarrow{RS}$ and $\overrightarrow{RV}$ are tangents to the circle. (Given)
2. $m\angle STV = \frac{1}{2}m\widehat{SWT}, m\angle RST = \frac{1}{2}m\widehat{ST}$ (The meas. of an secant-tangent $\angle = \frac{1}{2}$ the measure of its intercepted arc.)
3. $m\angle STV = m\angle RST + m\angle R$ (Exterior $\angle$s Theorem)
4. $\frac{1}{2}m\widehat{SWT} = \frac{1}{2}m\widehat{ST} + m\angle R$ (Substitution)
5. $\frac{1}{2}m\widehat{SWT} - \frac{1}{2}m\widehat{ST} = m\angle R$ (Subtraction Prop.)
6. $\frac{1}{2}(m\widehat{SWT} - m\widehat{ST}) = m\angle R$ (Distributive Prop.)

ANSWER:

Statements (Reasons)

1. $\overrightarrow{RS}$ and $\overrightarrow{RV}$ are tangents to the circle. (Given)
2. $m\angle STV = \frac{1}{2}m\widehat{SWT}, m\angle RST = \frac{1}{2}m\widehat{ST}$ (The meas. of an secant-tangent $\angle = \frac{1}{2}$ the measure of its intercepted arc.)
3. $m\angle STV = m\angle RST + m\angle R$ (Exterior $\angle$s Theorem)
4. $\frac{1}{2}m\widehat{SWT} = \frac{1}{2}m\widehat{ST} + m\angle R$ (Substitution)
5. $\frac{1}{2}m\widehat{SWT} - \frac{1}{2}m\widehat{ST} = m\angle R$ (Subtraction Prop.)
6. $\frac{1}{2}(m\widehat{SWT} - m\widehat{ST}) = m\angle R$ (Distributive Prop.)

33. **PROOF** Write a paragraph proof of Theorem 10.13.



a. Given: $\overleftrightarrow{AB}$ is a tangent of $\odot O$.
$\overleftrightarrow{AC}$ is a secant of $\odot O$.
$\angle CAE$ is acute.
Prove: $m\angle CAE = \frac{1}{2}m(\text{arc } CA)$

b. Prove that if $\angle CAB$ is obtuse, $m\angle CAB = \frac{1}{2}m(\text{arc } CDA)$

SOLUTION:

a. Proof: By Theorem 10.10, $\overline{OA} \perp \overline{AB}$. So, $\angle FAE$ is a right $\angle$ with measure of 90 and arc *FCA* is a semicircle with measure of 180. Since $\angle CAE$ is acute, *C* is in the interior of $\angle FAE$, so by the Angle and Arc Addition Postulates, $m\angle FAE = m\angle FAC + m\angle CAE$ and $m(\text{arc } FCA) = m(\text{arc } FC) + m(\text{arc } CA)$. By substitution, $90 = m\angle FAC + m\angle CAE$ and $180 = m(\text{arc } FC) + m(\text{arc } CA)$. So, $90 = \frac{1}{2}m(\text{arc } FC) + \frac{1}{2}m(\text{arc } CA)$ by Division Prop., and $m\angle FAC + m\angle CAE = \frac{1}{2}m(\text{arc } FC) + \frac{1}{2}m(\text{arc } CA)$ by substitution. $m\angle FAC = \frac{1}{2}m(\text{arc } FC)$ since $\angle FAC$ is inscribed, so substitution yields $\frac{1}{2}m(\text{arc } FC) + m\angle CAE = \frac{1}{2}m(\text{arc } FC) + \frac{1}{2}m(\text{arc } CA)$. By Subtraction Prop., $m\angle CAE = \frac{1}{2}m(\text{arc } CA)$.

b. Proof: Using the Angle and Arc Addition Postulates, $m\angle CAB = m\angle CAF + m\angle FAB$ and $m(\text{arc } CDA) = m(\text{arc } CF) + m(\text{arc } FDA)$. Since $\overline{OA} \perp \overline{AB}$ and $\overline{FA}$ is a diameter, $\angle FAB$ is a right angle with a measure of 90 and arc *FDA* is a semicircle with a measure of 180. By substitution, $m\angle CAB = m\angle CAF + 90$ and $m(\text{arc } CDA) = m(\text{arc } CF) + 180$. Since $\angle CAF$ is inscribed, $m\angle CAF = \frac{1}{2}m(\text{arc } CF)$ and by substitution, $m\angle CAB = \frac{1}{2}m(\text{arc } CF) + 90$. Using the Division and Subtraction Properties on the Arc Addition equation yields $\frac{1}{2}m(\text{arc } CDA) - \frac{1}{2}m(\text{arc } CF) = 90$. By substituting for 90, $m\angle CAB = \frac{1}{2}m(\text{arc } CF) + \frac{1}{2}m(\text{arc } CDA) - \frac{1}{2}m(\text{arc } CF)$. Then by subtraction, $m\angle CAB = \frac{1}{2}m(\text{arc } CDA)$.

ANSWER:

a. Proof: By Theorem 10.10, $\overline{OA} \perp \overline{AB}$. So, $\angle FAE$ is a right $\angle$ with measure of 90 and arc *FCA* is a semicircle with measure of 180. Since $\angle CAE$ is acute, *C* is in the interior of $\angle FAE$, so by the Angle and Arc Addition Postulates, $m\angle FAE = m\angle FAC + m\angle CAE$ and $m(\text{arc } FCA) = m(\text{arc } FC) + m(\text{arc } CA)$. By substitution, $90 = m\angle FAC + m\angle CAE$ and $180 = m(\text{arc } FC) + m(\text{arc } CA)$. So, $90 = \frac{1}{2}m(\text{arc } FC) + \frac{1}{2}m(\text{arc } CA)$ by Division Prop., and $m\angle FAC + m\angle CAE = \frac{1}{2}m(\text{arc } FC) + \frac{1}{2}m(\text{arc } CA)$ by substitution. $m\angle FAC = \frac{1}{2}m(\text{arc } FC)$ since $\angle FAC$ is inscribed, so substitution yields $\frac{1}{2}m(\text{arc } FC) + m\angle CAE = \frac{1}{2}m(\text{arc } FC) + \frac{1}{2}m(\text{arc } CA)$. By Subtraction Prop., $m\angle CAE = \frac{1}{2}m(\text{arc } CA)$.

b. Proof: Using the Angle and Arc Addition Postulates, $m\angle CAB = m\angle CAF + m\angle FAB$ and $m(\text{arc } CDA) = m(\text{arc } CF) + m(\text{arc } FDA)$. Since $\overline{OA} \perp \overline{AB}$ and $\overline{FA}$ is a diameter, $\angle FAB$ is a right angle with a measure of 90 and arc *FDA* is a semicircle with a measure of 180. By substitution, $m\angle CAB = m\angle CAF + 90$ and $m(\text{arc } CDA) = m(\text{arc } CF)$

+ 180. Since $\angle CAF$ is inscribed, $m\angle CAF = \frac{1}{2}$ m(arc CF) and by substitution, $m\angle CAB = \frac{1}{2}$ m(arc CF) + 90. Using the Division and Subtraction Properties on the Arc Addition equation yields $\frac{1}{2}m$(arc CDA) – $\frac{1}{2}$ m(arc CF) = 90. By substituting for 90, $m\angle CAB = \frac{1}{2}$ m(arc CF) + $\frac{1}{2}m$(arc CDA) – $\frac{1}{2}$ m(arc CF). By subtraction, $m\angle CAB = \frac{1}{2}m$(arc CDA).

34. **OPTICAL ILLUSION** The design shown is an example of optical wallpaper. $\overline{BC}$ is a diameter of $\odot Q$. If $m\angle A$ = 26 and $m\widehat{CE}$ = 67, what is $m\widehat{DE}$?
Refer to the image on page 748.

SOLUTION:
First, find the measure of arc *BD*.

$$m\angle A = \frac{1}{2}[m(\text{arc}\,CE) - m(\text{arc}\,BD)] \quad \text{Theorem 10.14}$$
$$26 = \frac{1}{2}[67 - m(\text{arc}\,BD)] \quad \text{Substitution}$$
$$52 = 67 - m(\text{arc}\,BD) \quad \text{Multiply each side by 2.}$$
$$m(\text{arc}\,BD) = 15 \quad \text{Add } m(\text{arc}\,BD) \text{ and subtract 52 from each side.}$$

Since $\overline{BC}$ is a diameter, arc *BDE* is a semicircle and has a measure of 180.

$$m(arc\,CE) + m(arc\,DE) + m(arc\,BD) = m(\text{arc}\,BDE) \quad \text{Arc Addition Postulate}$$
$$67 + m(arc\,DE) + 15 = 180 \quad \text{Substitution}$$
$$m(arc\,DE) + 82 = 180 \quad \text{Simplify.}$$
$$m(arc\,DE) = 98 \quad \text{Subtract 82 from each side.}$$

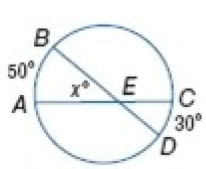
ANSWER:
98

35. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship between Theorems 10.12 and 10.6.
a. GEOMETRIC Copy the figure shown. Then draw three successive figures in which the position of point *D* moves closer to point *C*, but points *A*, *B*, and *C* remain fixed.

b. TABULAR Estimate the measure of $\widehat{CD}$ for each successive circle, recording the measures of $\widehat{AB}$ and $\widehat{CD}$ in a table. Then calculate and record the value of *x* for each circle.

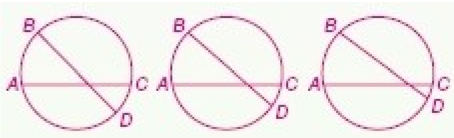
c. VERBAL Describe the relationship between $m\widehat{AB}$ and the value of *x* as $m\widehat{CD}$ approaches zero. What type of angle does $\angle AEB$ become when $m\widehat{CD} = 0$?

d. ANALYTICAL Write an algebraic proof to show the relationship between Theorems 10.12 and 10.6 described in part c.



SOLUTION:

a. Redraw the circle with points A, B, and C in the same place but place point D closer to C each time. Then draw chords $\overline{AC}$ and $\overline{BD}$.



b.

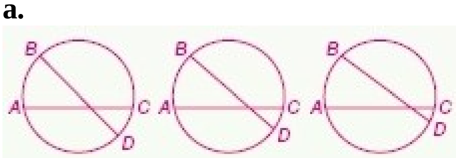
	Circle 1	Circle 2	Circle 3
$\widehat{CD}$	25	15	5
$\widehat{AB}$	50	50	50
x	37.5	32.5	27.5

c. As the measure of $\widehat{CD}$ gets closer to 0, the measure of x approaches half of $m\widehat{AB}$; $\angle AEB$ becomes an inscribed angle.

d. $x = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$; $x = \frac{1}{2}(m\widehat{AB} + 0)$; $x = \frac{1}{2}m\widehat{AB}$

ANSWER:

a.



b.

	Circle 1	Circle 2	Circle 3
$\widehat{CD}$	25	15	5
$\widehat{AB}$	50	50	50
x	37.5	32.5	27.5

c. As the measure of $\widehat{CD}$ gets closer to 0, the measure of x approaches half of $m\widehat{AB}$; $\angle AEB$ becomes an inscribed angle.

d. $x = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$; $x = \frac{1}{2}(m\widehat{AB} + 0)$; $x = \frac{1}{2}m\widehat{AB}$

36. **WRITING IN MATH** Explain how to find the measure of an angle formed by a secant and a tangent that intersect outside a circle.

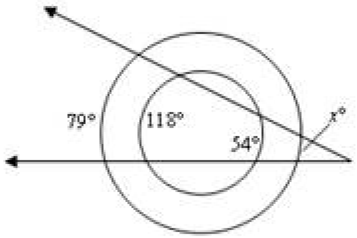
SOLUTION:

Find the difference of the two intercepted arcs and divide by 2.

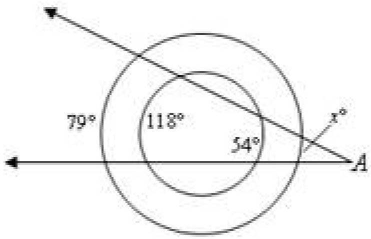
ANSWER:

Find the difference of the two intercepted arcs and divide by 2.

37. **CHALLENGE** The circles below are concentric. What is *x*?



SOLUTION:



Use the arcs intercepted on the smaller circle to find $m\angle A$.

$m\angle A = \frac{1}{2}[118 - 54]$ Theorem 10.14

$= \frac{1}{2}[64]$ Simplify.

$= 32$ Multiply.

Use the arcs intercepted on the larger circle and $m\angle A$ to find the value of x.

$32 = \frac{1}{2}[79 - x]$ Theorem 10.14

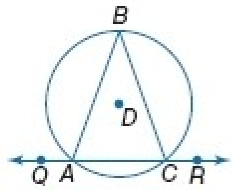
$64 = 79 - x$ Multiply each side by 2.

$x = 15$ Add x and subtract 64 from each side.

ANSWER:

15

38. **REASONS** Isosceles $\triangle ABC$ is inscribed in $\odot D$. What can you conclude about $m\widehat{AB}$ and $m\widehat{BC}$? Explain.



SOLUTION:

Sample answer: $m\widehat{AB} = m\widehat{BC}$; $m\angle BAC = m\angle BCA$ because the triangle is isosceles. Since $\angle BAC$ and $\angle BCA$ are inscribed angles, by theorem 10.6, $m(\text{arc } AB) = 2m\angle BCA$ and $m(\text{arc } BC) = 2m\angle BAC$. So, $m(\text{arc } AB) = m(\text{arc } BC)$.

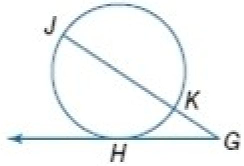
ANSWER:

Sample answer: $m\widehat{AB} = m\widehat{BC}$; $m\angle BAC = m\angle BCA$ because the triangle is isosceles. Since $\angle BAC$ and $\angle BCA$ are inscribed angles, by theorem 10.6, $m(\text{arc } AB) = 2m\angle BCA$ and $m(\text{arc } BC) = 2m\angle BAC$. So, $m(\text{arc } AB) = m(\text{arc } BC)$.

39. **CCSS ARGUMENTS** In the figure, $\overline{JK}$ is a diameter and $\overline{GH}$ is a tangent.

a. Describe the range of possible values for $m\angle G$. Explain.

b. If $m\angle G = 34$, find the measures of minor arcs HJ and KH. Explain.



SOLUTION:

a. $m\angle G \leq 90$; $m\angle G < 90$ for all values except when $\overrightarrow{JG} \perp \overrightarrow{GH}$ at G, then $m\angle G = 90$.

b. $m\widehat{KH} = 56$; $m\widehat{HJ} = 124$; Because a diameter is involved the intercepted arcs measure $(180 - x)$ and x degrees. Hence solving $\frac{180 - x - x}{2} = 34$ leads to the answer.

ANSWER:

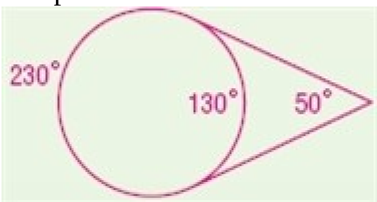
a. $m\angle G \leq 90$; $m\angle G < 90$ for all values except when $\overrightarrow{JG} \perp \overrightarrow{GH}$ at G, then $m\angle G = 90$.

b. $m\widehat{KH} = 56$; $m\widehat{HJ} = 124$; Because a diameter is involved the intercepted arcs measure $(180 - x)$ and x degrees. Hence solving $\frac{180 - x - x}{2} = 34$ leads to the answer.

40. **OPEN ENDED** Draw a circle and two tangents that intersect outside the circle. Use a protractor to measure the angle that is formed. Find the measures of the minor and major arcs formed. Explain your reasoning.

SOLUTION:

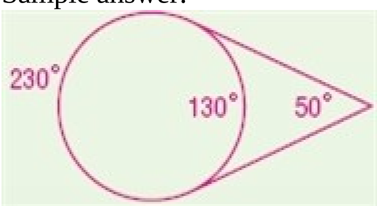
Sample answer:



By Theorem 10.13, $m\angle 1 = \frac{1}{2}(x - y)$. So, 50° = $\frac{1}{2}[(360 - x) - x]$. Therefore, x (minor arc) = 130, and y (major arc) = 360° – 130° or 230°.

ANSWER:

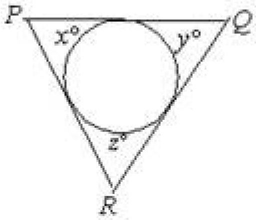
Sample answer:



By Theorem 10.13, $m\angle 1 = \frac{1}{2}(x - y)$. So, 50 = $\frac{1}{2}[(360 - x) - x]$. Therefore, x (minor arc) = 130, and y (major arc) = 360 – 130 or 230.

41. **WRITING IN MATH** A circle is inscribed within ΔPQR. If $m\angle P = 50$ and $m\angle Q = 60$, describe how to find the measures of the three minor arcs formed by the points of tangency.

SOLUTION:



Sample answer: Use Theorem 10.14 to find each minor arc.

$m\angle P = \frac{1}{2}[(360 - x) - x]$ Theorem 10.14

$50 = \frac{1}{2}[360 - 2x]$ Substitute and simplify.

$50 = 180 - x$ Multiply.

$x = 130$ Add x and subtract 50 from each side.

$m\angle Q = \frac{1}{2}[(360 - y) - y]$ Theorem 10.14

$60 = \frac{1}{2}[360 - 2y]$ Substitute and simplify.

$60 = 180 - y$ Multiply.

$y = 120$ Add y and subtract 60 from each side.

The sum of the angles of a triangle is 180, so $m\angle R = 180 - (50 + 60)$ or 70.

$m\angle R = \frac{1}{2}[(360 - z) - z]$ Theorem 10.14

$70 = \frac{1}{2}[360 - 2z]$ Substitute and simplify.

$70 = 180 - z$ Multiply.

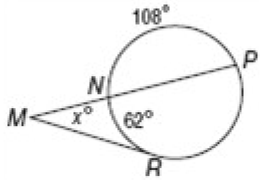
$z = 110$ Add z and subtract 70 from each side.

Therefore, the measures of the three minor arcs are 130, 120, and 110.

ANSWER:

Sample answer: Using Theorem 10.14, 60° = 1/2((360 – x) – x) or 120°; repeat for 50° to get 130°. The third arc can be found by adding 50° and 60° and subtracting from 360 to get 110°

42. What is the value of *x* if $m\widehat{NR} = 62$ and $m\widehat{NP} = 108$?



A 23°
B 31°
C 64°
D 128°

SOLUTION:

By Theorem 10.14, $x = \frac{1}{2}\left(m\widehat{PR} - m\widehat{NR}\right)$.

Substitute.

$$x = \frac{1}{2}\left((360 - (62 + 108)) - 62\right)$$

Simplify.

$$\begin{aligned} x &= \frac{1}{2}(360 - (62 + 108) - 62) \\ &= \frac{1}{2}(360 - 170 - 62) \\ &= \frac{1}{2}(360 - 170 - 62) \\ &= \frac{1}{2}(128) \\ &= 64 \end{aligned}$$

So, the correct choice is C.

ANSWER:

C

43. **ALGEBRA** Points $A(-4, 8)$ and $B(6, 2)$ are both on circle C, and $\overline{AB}$ is a diameter. What are the coordinates of C?
F (2, 10)
G (10, –6)
H (5, –3)
J (1, 5)

SOLUTION:

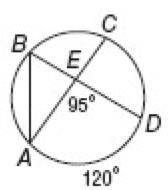
Here, C is the midpoint of $\overline{AB}$.
Use the midpoint formula, $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, to find the coordinates of C.

$$\left(\frac{-4+6}{2}, \frac{8+2}{2}\right) = \left(\frac{2}{2}, \frac{10}{2}\right) = (1,5)$$

So, the correct choice is J.

ANSWER:
J

44. **SHORT RESPONSE** If $m\angle AED = 95$ and $m\widehat{AD} = 120,$ what is $m\angle BAC$?

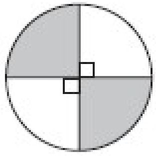


SOLUTION:

Since $m\widehat{AD} = 120$, $m\angle ABD = \frac{1}{2}(120) = 60$.
We know that vertical angles are congruent.
So, $m\angle BEC = 95$.
Since $m\angle BEC = m\angle AED = 95$, $m\angle BEA = m\angle CED = 85$.
We know that the sum of the measures of all interior angles of a triangle is 180.
$m\angle BEA + m\angle BAE + m\angle ABE = 180$
Substitute.
$$85 + m\angle BAE + 60 = 180$$
$$m\angle BAE = 35$$
Since $m\angle BAE = 35$, $m\angle BAC = 35$.

ANSWER:
35

45. **SAT/ACT** If the circumference of the circle below is 16π, what is the total area of the shaded regions?



A 64π units^2
B 32π units^2
C 12π units^2
D 8π units^2
D 2π units^2

SOLUTION:

First, use the circumference to find the radius of the circle.

$$\begin{aligned} C &= 2\pi r && \text{Circumference Formula} \\ 16\pi &= 2\pi r && \text{Substitution} \\ \frac{16\pi}{2\pi} &= r && \text{Divide each side by } 2\pi. \\ r &= 8 && \text{Simplify.} \end{aligned}$$

The shaded regions of the circle comprise half of the circle, so its area is half the area of the circle.

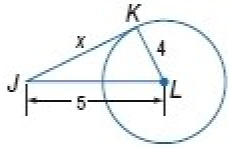
$$\begin{aligned} A_{\text{shaded}} &= \frac{1}{2}\pi r^2 && \text{Area Formula} \\ &= \frac{1}{2}[\pi(8^2)] && \text{Substitution} \\ &= \frac{1}{2}[64\pi] && \text{Simplify.} \\ &= 32\pi && \text{Multiply.} \end{aligned}$$

The area of the shaded regions is 32π.
So, the correct choice is B.

ANSWER:

B

Find *x*. Assume that segments that appear to be tangent are tangent.

46. 

SOLUTION:

By theorem 10.10, $JK \perp KL$. So, ΔJKL is a right triangle.
Use the Pythagorean Theorem.

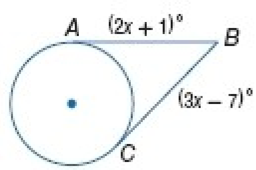
$JK^2 + KL^2 = JL^2$

Substitute.

$$x^2 + 4^2 = 5^2$$
$$25 - 16 = x^2$$
$$9 = x^2$$
$$\Rightarrow x = 3$$

ANSWER:

3

47. 

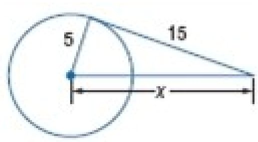
SOLUTION:

If two segments from the same exterior point are tangent to a circle, then they are congruent.

$$3x - 7 = 2x + 1$$
$$x = 8$$

ANSWER:

8

48. 

SOLUTION:

Use Theorem 10.10 and the Pythagorean Theorem.

$5^2 + 15^2 = x^2$

Solve for *x*.

$5^2 + 15^2 = x^2$

$25 + 225 = x^2$

$250 = x^2$

$\Rightarrow x = 5\sqrt{10}$

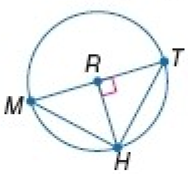
ANSWER:

$5\sqrt{10}$

49. **PROOF** Write a two-column proof.

Given: $\widehat{MHT}$ is a semicircle; $\overline{RH} \perp \overline{TM}$.

Prove: $\frac{TR}{RH} = \frac{TH}{HM}$



SOLUTION:

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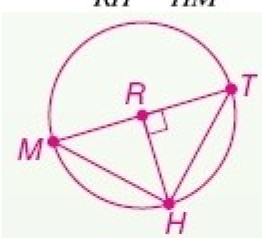
$\overline{RH} \perp \overline{TM}$.

Prove: $\frac{TR}{RH} = \frac{TH}{HM}$



Proof:

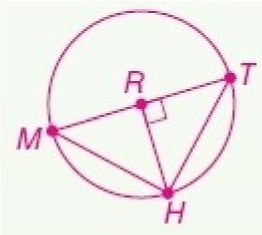
Statements (Reasons)

1. *MHT* is a semicircle; $\overline{RH} \perp \overline{TM}$. (Given)
2. $\angle THM$ is a right angle. (If an inscribed angle intercepts a semicircle, the angle is a right angle.)
3. $\angle TRH$ is a right angle (Def. of $\perp$ lines)
4. $\angle THM \cong \angle TRH$ (All rt. angles are $\cong$.)
5. $\angle T \cong \angle T$ (Reflexive Prop.)
6. $\Delta TRH \sim \Delta THM$ (AA Sim.)

7. $\frac{TR}{RH} = \frac{TH}{HM}$ (Def. of $\sim \Delta s$)

ANSWER:

Given: $\widehat{MHT}$ is a semicircle.
$\overline{RH} \perp \overline{TM}$.
Prove: $\frac{TR}{RH} = \frac{TH}{HM}$



Proof:
Statements (Reasons)
1. MHT is a semicircle; $\overline{RH} \perp \overline{TM}$. (Given)
2. $\angle THM$ is a right angle. (If an inscribed angle intercepts a semicircle, the angle is a right angle.)
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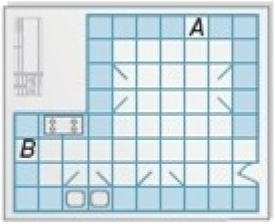
50. **REMODELING** The diagram at the right shows the floor plan of Trent's kitchen. Each square on the diagram represents a 3-foot by 3-foot area. While remodeling his kitchen, Trent moved his refrigerator from square A to square B. Describe one possible combination of transformations that could be used to make this move.



SOLUTION:

The move is a translation 15 feet out from the wall and 21 feet to the left, then a rotation of 90° counterclockwise.

ANSWER:

The move is a translation 15 feet out from the wall and 21 feet to the left, then a rotation of 90° counterclockwise.

COORDINATE GEOMETRY Find the measure of each angle to the nearest tenth of a degree by using the Distance Formula and an inverse trigonometric ratio.

51. $\angle C$ in triangle *BCD* with vertices *B*(–1, –5), *C*(–6, –5), and *D*(–1, 2)

SOLUTION:

Use the Distance Formula to find the length of each side.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BC = \sqrt{(-6-(-1))^2 + (-5-(-5))^2} = \sqrt{25} = 5$$

$$CD = \sqrt{(-1-(-6))^2 + (2-(-5))^2} = \sqrt{74}$$

$$DB = \sqrt{(-1-(-1))^2 + (2-(-5))^2} = \sqrt{49} = 7$$

Since $5^2 + 7^2 = \sqrt{74}^2$, triangle *BCD* is a right triangle.

So, *CD* is the length of the hypotenuse and *BC* is the length of the leg adjacent to $\angle C$.

Write an equation using the cosine ratio.

$$\cos C = \frac{BC}{CD} = \frac{5}{\sqrt{74}}$$

If $\cos C = \frac{5}{\sqrt{74}}$, then $C = \cos^{-1}\left(\frac{5}{\sqrt{74}}\right)$.

Use a calculator.

$m\angle C \approx 54.5°$

ANSWER:

54.5

52. $\angle X$ in right triangle *XYZ* with vertices *X*(2, 2), *Y*(2, –2), and *Z*(7, –2)

SOLUTION:

Use the Distance Formula to find the length of each side.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$XY = \sqrt{(2-2)^2 + (-2-2)^2} = \sqrt{16} = 4$$

$$YZ = \sqrt{(7-2)^2 + (-2-(-2))^2} = \sqrt{25} = 5$$

$$ZX = \sqrt{(7-2)^2 + (-2-2)^2} = \sqrt{41}$$

In right triangle *XYZ*, *ZX* is the length of the hypotenuse and *YZ* is the length of the leg opposite $\angle X$.

Write an equation using the sine ratio.

$$\sin X = \frac{YZ}{ZX} = \frac{5}{\sqrt{41}}$$

If $\sin X = \frac{5}{\sqrt{41}}$, then $X = \sin^{-1}\left(\frac{5}{\sqrt{41}}\right)$.

Use a calculator.

$m\angle X \approx 51.3°$

ANSWER:

51.3

Solve each equation.

53. $x^2 + 13x = -36$

SOLUTION:

$$x^2 + 13x = -36 \quad \text{Original equation}$$
$$x^2 + 13x + 36 = 0 \quad \text{Add 36 to each side.}$$
$$(x+9)(x+4) = 0 \quad \text{Factor.}$$
$$x = -9 \text{ or } -4 \quad \text{Zero Product Property}$$

Therefore, the solution is –9, –4.

ANSWER:

–4, –9

54. $x^2 - 6x = -9$

SOLUTION:

$$x^2 - 6x = -9 \quad \text{Original equation}$$
$$x^2 - 6x + 9 = 0 \quad \text{Add 9 to each side.}$$
$$(x-3)(x-3) = 0 \quad \text{Factor.}$$
$$x = 3 \quad \text{Zero Product Property}$$

Therefore, the solution is 3.

ANSWER:

3

55. $3x^2 + 15x = 0$

SOLUTION:

$$3x^2 + 15x = 0 \quad \text{Original equation}$$
$$3x(x+5) = 0 \quad \text{Factor.}$$
$$x = 0 \text{ or } -5 \quad \text{Zero Product Property}$$

Therefore, the solution is –5, 0.

ANSWER:

0, –5

56. $28 = x^2 + 3x$

SOLUTION:

$$\begin{aligned} 28 &= x^2 + 3x && \text{Original equation} \\ 0 &= x^2 + 3x - 28 && \text{Subtract 28to each side.} \\ 0 &= (x+7)(x-4) && \text{Factor.} \\ x &= -7 \text{ or } 4 && \text{Zero Product Property} \end{aligned}$$

Therefore, the solution is –7, 4.

ANSWER:

–7, 4

57. $x^2 + 12x + 36 = 0$

SOLUTION:

$$\begin{aligned} x^2 + 12x + 36 &= 0 && \text{Original equation} \\ (x+6)(x+6) &= 0 && \text{Factor.} \\ x &= -6 && \text{Zero Product Property} \end{aligned}$$

Therefore, the solution is –6.

ANSWER:

–6

58. $x^2 + 5x = -\frac{25}{4}$

SOLUTION:

$$\begin{aligned} x^2 + 5x &= -\frac{25}{4} && \text{Original equation} \\ x^2 + 5x + \frac{25}{4} &= 0 && \text{Add } \frac{25}{4} \text{ to each side.} \\ (x+\frac{5}{2})(x+\frac{5}{2}) &= 0 && \text{Factor.} \\ x &= -\frac{5}{2} && \text{Zero Product Property} \end{aligned}$$

Therefore, the solution is $-\frac{5}{2}$.

ANSWER:

$-\frac{5}{2}$