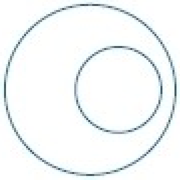
10-5 Tangents

1. Copy the figure shown, and draw the common tangents. If no common tangent exists, state *no common tangent*.



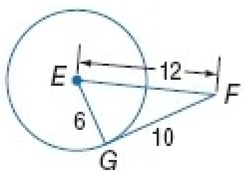
SOLUTION:

Every tangent drawn to the small circle will intersect the larger circle in two points. Every tangent drawn to the large circle will not intersect the small circle at any point. Since a tangent must intersect the circle at exactly one point, no common tangent exists for these two circles.

ANSWER:

no common tangent

Determine whether $\overline{FG}$ is tangent to $\odot E$. Justify your answer.

2. 

SOLUTION:

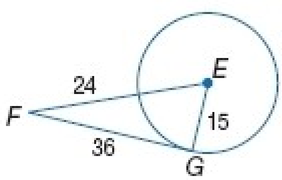
Test to see if ΔFEG is a right triangle.

$$10^2 + 6^2 \stackrel{?}{=} 12^2 \quad \text{Pythagorean Theorem}$$
$$136 \neq 144 \quad \text{Simplify.}$$

No; ΔFEG is not a right triangle, so $\overline{FG}$ is not tangent to circle E.

ANSWER:

No; $136 \neq 144$

3. 

SOLUTION:

Test to see if ΔEFG is a right triangle.

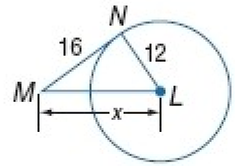
$$36^2 + 15^2 \stackrel{?}{=} 39^2 \quad \text{Pythagorean Theorem}$$
$$1521 = 1521 \checkmark \quad \text{Simplify.}$$

Yes; ΔEFG is a right triangle, so $\overline{FG}$ is tangent to circle E.

ANSWER:

yes; 1521 = 1521

Find *x*. Assume that segments that appear to be tangent are tangent.

4. 

SOLUTION:

By Theorem 10.10, $MN \perp NL$. So, ΔMNL is a right triangle.

$ML^2 = MN^2 + NL^2$ Pythagorean Theorem

$x^2 = 16^2 + 12^2$ $MN = 16, NL = 12$

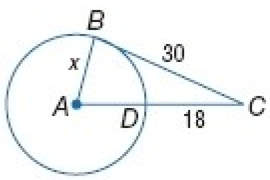
$x^2 = 256 + 144$ Multiply.

$x^2 = 400$ Simplify.

$x = 20$ Take the positive square root of each side.

ANSWER:

20

5. 

SOLUTION:

By Theorem 10.10, $BC \perp AB$. So, ΔABC is a right triangle.

$AB^2 + BC^2 = AC^2$ Pythagorean Theorem

$x^2 + 30^2 = (x + 18)^2$ $BC = 30$, $AC = AD + DC$ or $x + 18$

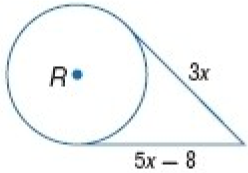
$x^2 + 900 = x^2 + 36x + 324$ Multiply.

$576 = 36x$ Subtract 324 and x^2 from each side.

$16 = x$ Divide each side by 36.

ANSWER:

16

6. 

SOLUTION:

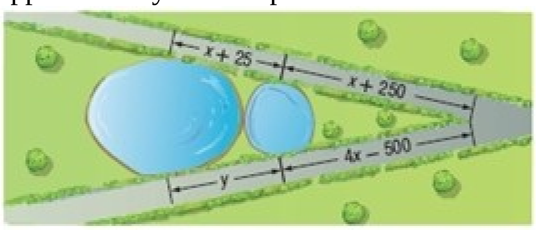
If two segments from the same exterior point are tangent to a circle, then they are congruent.

$$5x - 8 = 3x \quad \text{Tangent segments from same point are congruent.}$$
$$2x = 8 \quad \text{Add 8 and subtract } 3x \text{ from each side.}$$
$$x = 4 \quad \text{Divide each side by 2.}$$

ANSWER:

4

7. **LANDSCAPE ARCHITECT** A landscape architect is paving the two walking paths that are tangent to two approximately circular ponds as shown. The lengths given are in feet. Find the values of x and y.



SOLUTION:

If two segments from the same exterior point are tangent to a circle, then they are congruent. To find the value of x, use the lengths of the two sidewalk segments that are tangent to the smaller pond.

$$4x - 500 = x + 250$$
$$3x = 750$$
$$x = 250$$

To find the value of y, use the total lengths of the two sidewalk segments that are tangent to the larger pond and substitute 250 for the value of x.

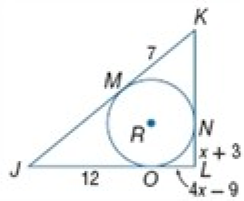
$$y + 4x - 500 = x + 250 + x + 25$$
$$y + 4(250) - 500 = 250 + 250 + 250 + 25$$
$$y = 275$$

Therefore, $x = 250$ and $y = 275$.

ANSWER:

$x = 250$; $y = 275$

8. **CCSS SENSE-MAKING** Triangle JKL is circumscribed about $\odot R$.



a. Find *x*.

b. Find the perimeter of ΔJKL.

SOLUTION:

a. If two segments from the same exterior point are tangent to a circle, then they are congruent.

$$\begin{aligned} LO &= LN && \text{Tangents from same exterior point are congruent.} \\ 4x-9 &= x+3 && \text{Substitution} \\ 3x &= 12 && \text{Add 9 and subtract from each side.} \\ x &= 4 && \text{Divide each side by 3.} \end{aligned}$$

b. Since two tangent segments from the same exterior point are congruent, *JM* = *JO* = 12, *KN* = *KM* = 7, and *LO* = *LN* = 4 + 3 or 7. The sides of the triangle will have lengths of *JK* = 12 + 7 or 19, *KL* = 7 + 7 or 14, and *JL* = 12 + 7 or 19. To find the perimeter of a triangle, add the lengths of its sides.

$$\begin{aligned} P(\text{triangle } JKL) &= JK + KL + JL && \text{Perimeter of triangle is sum of the sides.} \\ &= 19 + 14 + 19 && \text{Substitution.} \\ &= 52 && \text{Simplify.} \end{aligned}$$

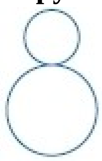
Therefore, the perimeter of triangle *JKL* is 52 units.

ANSWER:

a. 4

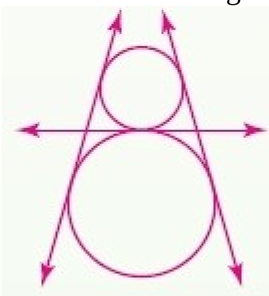
b. 52 units

Copy each figure and draw the common tangents. If no common tangent exists, state *no common tangent*.

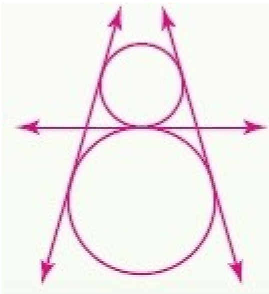
9. 

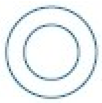
SOLUTION:

Three common tangents can be drawn.



ANSWER:



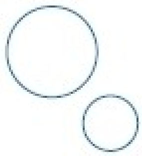
10. 

SOLUTION:

Every tangent drawn to the small circle will intersect the larger circle in two points. Every tangent drawn to the large circle will not intersect the small circle at any point. Since a tangent must intersect the circle at exactly one point, no common tangent exists for these two circles.

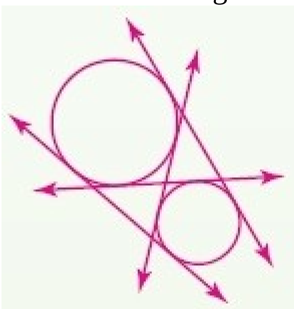
ANSWER:

no common tangent

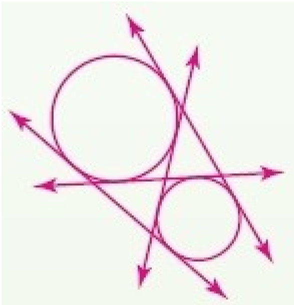
11. 

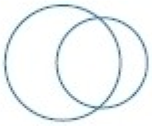
SOLUTION:

Four common tangents can be drawn to these two circles.



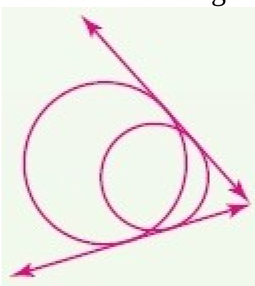
ANSWER:



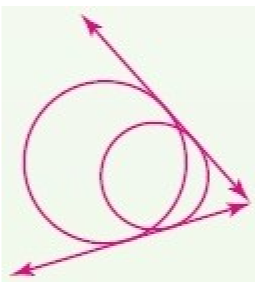
12. 

SOLUTION:

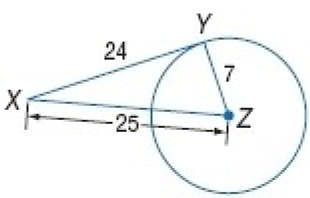
Two common tangents can be drawn to these two circles.



ANSWER:



Determine whether each $\overline{XY}$ is tangent to the given circle. Justify your answer.

13. 

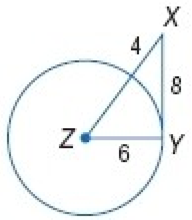
SOLUTION:

Yes;

$$24^2 + 7^2 \stackrel{?}{=} 25^2$$
$$625 = 625$$

ANSWER:

Yes; 625 = 625

14. 

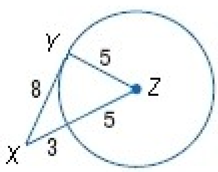
SOLUTION:

Yes;

$$8^2 + 6^2 \stackrel{?}{=} (6+4)^2$$
$$100 = 100$$

ANSWER:

Yes; 100 = 100

15. 

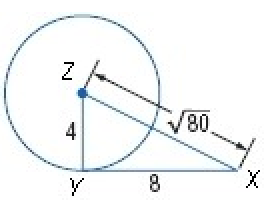
SOLUTION:

No;

$$8^2 + 5^2 \stackrel{?}{=} 8^2$$
$$89 \neq 64$$

ANSWER:

No; $89 \neq 64$

16. 

SOLUTION:

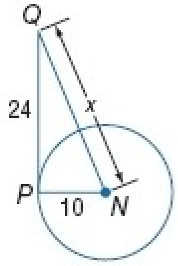
yes;

$$4^2 + 8^2 \stackrel{?}{=} (\sqrt{80})^2$$
$$80 = 80$$

ANSWER:

Yes; 80 = 80

Find *x*. Assume that segments that appear to be tangent are tangent. Round to the nearest tenth if necessary.

17. 

SOLUTION:

By Theorem 10.10, $PN \perp PQ$. So, ΔPQN is a right triangle.
Use the Pythagorean Theorem.

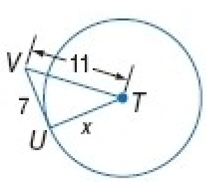
$PN^2 + PQ^2 = QN^2$

Substitute.

$$10^2 + 24^2 = x^2$$
$$100 + 576 = x^2$$
$$676 = x^2$$
$$\Rightarrow x = 26$$

ANSWER:

26

18. 

SOLUTION:

By Theorem 10.10, $UV \perp UT$. So, ΔUVT is a right triangle.
Use the Pythagorean Theorem.

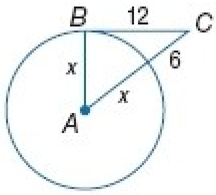
$UT^2 + UV^2 = VT^2$

Substitute.

$$x^2 + 7^2 = 11^2$$
$$x^2 + 49 = 121$$
$$x^2 = 72$$
$$\Rightarrow x \approx 8.5$$

ANSWER:

8.5

19. 

SOLUTION:

By Theorem 10.10, $AB \perp BC$. So, ΔABC is a right triangle.
Use the Pythagorean Theorem.

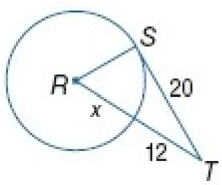
$AB^2 + BC^2 = AC^2$

Substitute.

$$x^2 + 12^2 = (x+6)^2$$
$$x^2 + 144 = x^2 + 36 + 12x$$
$$12x = 108$$
$$x = 9$$

ANSWER:

9

20. 

SOLUTION:

By Theorem 10.10, $RS \perp ST$. So, ΔRST is a right triangle.
Use the Pythagorean Theorem.

$RS^2 + ST^2 = RT^2$

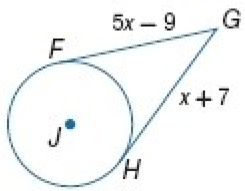
Substitute.

$$x^2 + 20^2 = (x+12)^2$$
$$x^2 + 400 = x^2 + 24x + 144$$
$$24x = 256$$
$$x \approx 10.7$$

ANSWER:

10.7

10-5 Tangents

21. 

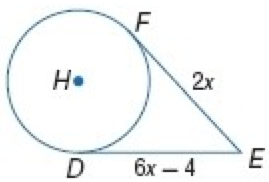
SOLUTION:

If two segments from the same exterior point are tangent to a circle, then they are congruent.

$$5x - 9 = x + 7$$
$$4x = 16$$
$$x = 4$$

ANSWER:

4

22. 

SOLUTION:

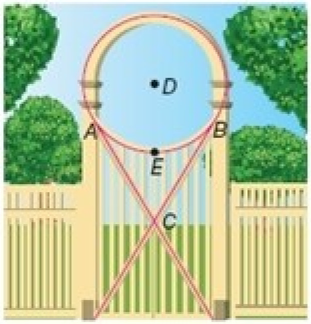
If two segments from the same exterior point are tangent to a circle, then they are congruent.

$$6x - 4 = 2x$$
$$4x = 4$$
$$x = 1$$

ANSWER:

1

23. **ARBORS** In the arbor shown, $\overline{AC}$ and $\overline{BC}$ are tangents to $\odot D$, and points *D*, *E*, and *C* are collinear. The radius of the circle is 26 inches and *EC* = 20 inches. Find each measure to the nearest hundredth.



a. *AC*
b. *BC*

SOLUTION:

a. Draw triangle *DAC*.

By Theorem 10.10, $AD \perp AC$. So, ΔADC is a right triangle. Since the radius of is 26, *AD* = 26 inches. By the Segment Addition Postulate, *DC* = *DE* + *EC*. So, *DC* = 26 + 20 or 46 inches. Use the Pythagorean Theorem to find *AC*.

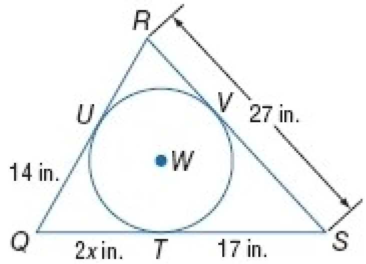
Therefore, the measure of *AC* is about 37.95 inches.

b. If two segments from the same exterior point are tangent to a circle, then they are congruent. and are both tangent to from point *C*.
So, $AC = BC \approx 37.95$ in.

ANSWER:

a. 37.95 in.
b. 37.95 in.

CCSS SENSE-MAKING Find the value of *x*. Then find the perimeter.

24. 

SOLUTION:

Find the missing measures.

Since ΔQRS is circumscribed about $\odot W$, $\overline{RU}$ and $\overline{RV}$ are tangent to $\odot W$, as are $\overline{QU}$, $\overline{QT}$, $\overline{VS}$, and $\overline{ST}$.

Therefore, $\overline{RU} \cong \overline{RV}$, $\overline{QU} \cong \overline{QT}$, and $\overline{VS} \cong \overline{ST}$.

So, $QU = QT$.

$2x = 14$

$x = 7$

$TS = VS = 17$

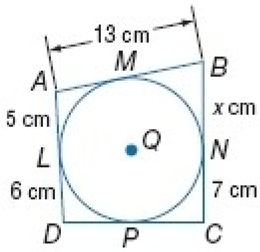
$RU = RV = 27 - 17 = 10$

To find the perimeter of a triangle, add the lengths of its sides.

$$\begin{aligned}\text{Perimeter of triangle } QRS &= QR + RS + SQ \\ &= QU + UR + RV + VS + ST + TQ \\ &= 14 + 10 + 10 + 17 + 17 + 14 \\ &= 82 \text{ in}\end{aligned}$$

ANSWER:

7; 82 in.

25. 

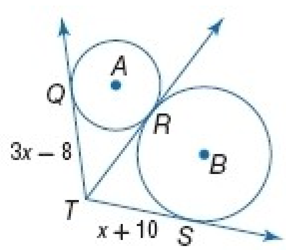
SOLUTION:

Since quadrilateral *ABCD* is circumscribed about $\odot Q$, $\overline{MB}$ and $\overline{NB}$ are tangent to $\odot Q$, as are $\overline{AM}$, $\overline{AL}$, $\overline{DL}$, $\overline{DP}$, $\overline{CP}$, and $\overline{CN}$. Therefore, $\overline{MB} \cong \overline{NB}$, $\overline{AM} \cong \overline{AL}$, $\overline{LD} \cong \overline{DP}$, and $\overline{PC} \cong \overline{NC}$.
So, $AM = AL$.
$AM = 5$
$MB = 13 - 5 = 8$
$x = BN = MB = 8$
$LD = DP = 6$
$PC = NC = 7$
To find the perimeter of a triangle, add the lengths of its sides.

$$\begin{aligned}\text{Perimeter of quadrilateral } ABCD &= AB + BC + CD + DA \\ &= AM + MB + BN + NC + CP + PD + DL + LA \\ &= 5 + 8 + 8 + 7 + 7 + 6 + 6 + 5 \\ &= 52 \text{ cm}\end{aligned}$$

ANSWER:
8; 52 cm

Find *x* to the nearest hundredth. Assume that segments that appear to be tangent are tangent.

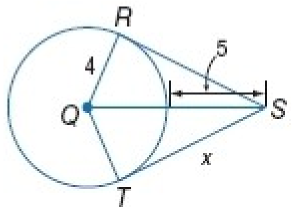
26. 

SOLUTION:

If two segments from the same exterior point are tangent to a circle, then they are congruent. Here, *TS* = *TR* and *TR* = *TQ*.
By the Transitive Property, *TS* = *TQ*.

$$\begin{aligned}x + 10 &= 3x - 8 \\ 2x &= 18 \\ x &= 9\end{aligned}$$

ANSWER:
9

27. 

SOLUTION:

QS = 4 + 5 = 9

By Theorem 10.10, $RS \perp QR$. So, ΔQRS is a right triangle.

Use the Pythagorean Theorem.

$QR^2 + RS^2 = QS^2$

Substitute.

$4^2 + RS^2 = 9^2$

$16 + RS^2 = 81$

$RS^2 = 65$

$RS \approx 8.06$

If two segments from the same exterior point are tangent to a circle, then they are congruent.

So, $RS = x \approx 8.06$.

ANSWER:

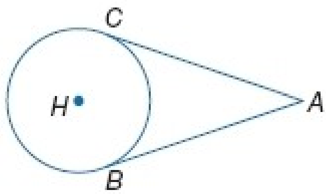
8.06

Write the specified type of proof.

28. two-column proof of Theorem 10.11

Given: $\overline{AC}$ is tangent to $\odot H$ at *C*. $\overline{AB}$ is tangent to $\odot H$ at *B*.

Prove: $\overline{AC} \cong \overline{AB}$



SOLUTION:

Proof:

Statements (Reasons)

1. $\overline{AC}$ is tangent to $\odot H$ at *C*; $\overline{AC}$ is tangent to $\odot H$ at *B*. (Given)
2. Draw $\overline{AH}$, $\overline{BH}$, and $\overline{CH}$. (Through any two points, there is one line.)
3. $\overline{AC} \perp \overline{CH}$, $\overline{AB} \perp \overline{BH}$ (Line tangent to a circle is $\perp$ to the radius at the pt. of tangency.)
4. $\angle ACH$ and $\angle ABH$ are right angles. (Def. of $\perp$ lines)
5. $\overline{CH} \cong \overline{BH}$ (All radii of a circle are $\cong$.)
6. $\overline{AH} \cong \overline{AH}$ (Reflexive Prop.)
7. $\triangle ACH \cong \triangle ABH$ (HL)
8. $\overline{AC} \cong \overline{AB}$ (CPCTC)

ANSWER:

Proof:

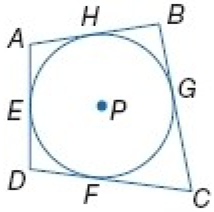
Statements (Reasons)

1. $\overline{AC}$ is tangent to $\odot H$ at *C*; $\overline{AC}$ is tangent to $\odot H$ at *B*. (Given)
2. Draw $\overline{AH}$, $\overline{BH}$, and $\overline{CH}$. (Through any two points, there is one line.)
3. $\overline{AC} \perp \overline{CH}$, $\overline{AB} \perp \overline{BH}$ (Line tangent to a circle is $\perp$ to the radius at the pt. of tangency.)
4. $\angle ACH$ and $\angle ABH$ are right angles. (Def. of $\perp$ lines)
5. $\overline{CH} \cong \overline{BH}$ (All radii of a circle are $\cong$.)
6. $\overline{AH} \cong \overline{AH}$ (Reflexive Prop.)
7. $\triangle ACH \cong \triangle ABH$ (HL)
8. $\overline{AC} \cong \overline{AB}$ (CPCTC)

29. two-column proof

Given: Quadrilateral *ABCD* is circumscribed about $\odot P$.

Prove: *AB* + *CD* = *AD* + *BC*



SOLUTION:

Statements (Reasons)

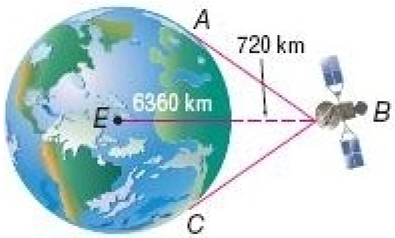
1. Quadrilateral *ABCD* is circumscribed about $\odot P$. (Given)
2. Sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are tangent to $\odot P$ at points *H*, *G*, *F*, and *E*, respectively. (Def. of circumscribed)
3. $\overline{EA} \cong \overline{AH}$; $\overline{HB} \cong \overline{BG}$; $\overline{GC} \cong \overline{CF}$; $\overline{FD} \cong \overline{DE}$ (Two segments tangent to a circle from the same exterior point are $\cong$.)
4. *AB* = *AH* + *HB*, *BC* = *BG* + *GC*, *CD* = *CF* + *FD*, *DA* = *DE* + *EA* (Segment Addition)
5. *AB* + *CD* = *AH* + *HB* + *CF* + *FD*; *DA* + *BC* = *DE* + *EA* + *BG* + *GC* (Substitution)
6. *AB* + *CD* = *AH* + *BG* +*GC* + *FD*; *DA* + *BC* = *FD* + *AH* + *BG* + *GC* (Substitution)
7. *AB* + *CD* = *FD* + *AH* + *BG* + *GC* (Comm. Prop. of Add.)
8. *AB* + *CD* = *DA* + *BC* (Substitution)

ANSWER:

Statements (Reasons)

1. Quadrilateral *ABCD* is circumscribed about $\odot P$. (Given)
2. Sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are tangent to $\odot P$ at points *H*, *G*, *F*, and *E*, respectively. (Def. of circumscribed)
3. $\overline{EA} \cong \overline{AH}$; $\overline{HB} \cong \overline{BG}$; $\overline{GC} \cong \overline{CF}$; $\overline{FD} \cong \overline{DE}$ (Two segments tangent to a circle from the same exterior point are $\cong$.)
4. *AB* = *AH* + *HB*, *BC* = *BG* + *GC*, *CD* = *CF* + *FD*, *DA* = *DE* + *EA* (Segment Addition)
5. *AB* + *CD* = *AH* + *HB* + *CF* + *FD*; *DA* + *BC* = *DE* + *EA* + *BG* + *GC* (Substitution)
6. *AB* + *CD* = *AH* + *BG* +*GC* + *FD*; *DA* + *BC* = *FD* + *AH* + *BG* + *GC* (Substitution)
7. *AB* + *CD* = *FD* + *AH* + *BG* + *GC* (Comm. Prop. of Add.)
8. *AB* + *CD* = *DA* + *BC* (Substitution)

30. **SATELLITES** A satellite is 720 kilometers above Earth, which has a radius of 6360 kilometers. The region of Earth that is visible from the satellite is between the tangent lines $\overline{BA}$ and $\overline{BC}$. What is *BA*? Round to the nearest hundredth.



SOLUTION:

EB = 6360 + 720 = 7080

By Theorem 10.10, $AE \perp AB$. So, ΔAEB is a right triangle.

Use the Pythagorean Theorem.

$AE^2 + AB^2 = EB^2$

Substitute.

$$6360^2 + AB^2 = 7080^2$$
$$40449600 + AB^2 = 50126400$$
$$AB^2 = 9676800$$
$$\Rightarrow AB \approx 3110.76 \text{ km}$$

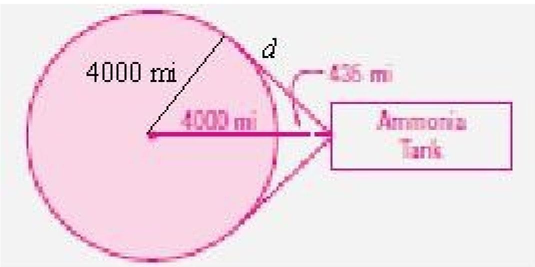
ANSWER:

3110.76 km

31. **SPACE TRASH** Orbital debris refers to materials from space missions that still orbit Earth. In 2007, a 1400-pound ammonia tank was discarded from a space mission. Suppose the tank has an altitude of 435 miles. What is the distance from the tank to the farthest point on the Earth's surface from which the tank is visible? Assume that the radius of Earth is 4000 miles. Round to the nearest mile, and include a diagram of this situation with your answer.

SOLUTION:

Draw a circle representing the earth and choose an exterior point of the circle to represent the position of the tank. The furthest point from which the tank can be seen from the Earth's surface would be the point of tangency of a tangent segment drawn from the tank to the circle. Draw the radius to the point of tangency to create a right triangle by Theorem 10.10. The length of the radius is 4000 miles and the distance to the tank is 4000 + 435 or 4435 miles.



Let *d* be the length of one of the tangent segments from the tank to the Earth's surface.
Use the Pythagorean Theorem to find *d*

$$4000^2 + d^2 = 4435^2 \quad \text{Pythagorean Theorem}$$

$$16,000,000 + d^2 = 19,669,225 \quad \text{Simplify.}$$

$$d^2 = 3,669,225 \quad \text{Subtract } 16,000,000 \text{ from each side.}$$

$$d \approx 1916 \quad \text{Take the positive square root of each side.}$$

Therefore, the distance from the tank to the furthest point on the Earth's surface from which the tank is visible is about 1916 miles.

ANSWER:

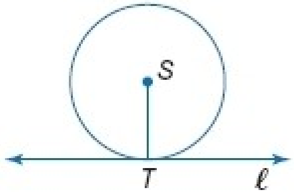
1916 mi;

32. **PROOF** Write an indirect proof to show that if a line is tangent to a circle, then it is perpendicular to a radius of the circle. (Part 1 of Theorem 10.10)

Given: ℓ is tangent to $\odot S$ at *T*; $\overline{ST}$ is a radius of $\odot S$.

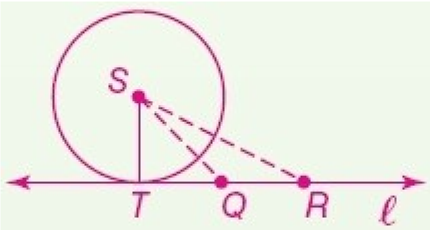
Prove: $\ell \perp \overline{ST}$.

(*Hint*: Assume ℓ is *not* $\perp$ to $\overline{ST}$.)



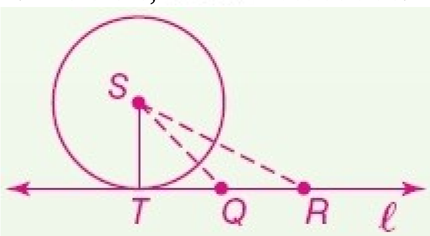
SOLUTION:

Proof: Assume that ℓ is not $\perp$ to $\overline{ST}$. If ℓ is not $\perp$ to $\overline{ST}$, some other segment $\overline{SQ}$ must be $\perp$ to ℓ. Also, there is a point *R* on $\overleftrightarrow{TR}$ as shown in the diagram such that $\overline{QT} \cong \overline{QR}$. $\angle SQT$ and $\angle SQR$ are right angles by the definition of perpendicular. $\angle SQT \cong \angle SQR$ and $\overline{SQ} \cong \overline{SQ}$. $\Delta SQT \cong \Delta SQR$ by SAS, so $\overline{ST} \cong \overline{SR}$ by CPCTC. Thus, both *T* and *R* are on $\odot S$. For two points of ℓ to also be on $\odot S$ contradicts the given fact that ℓ is tangent to $\odot S$ at *T*. Therefore, $\ell \perp \overline{ST}$ must be true.



ANSWER:

Proof: Assume that ℓ is not $\perp$ to $\overline{ST}$. If ℓ is not $\perp$ to $\overline{ST}$, some other segment $\overline{SQ}$ must be $\perp$ to ℓ. Also, there is a point *R* on $\overleftrightarrow{TR}$ as shown in the diagram such that $\overline{QT} \cong \overline{QR}$. $\angle SQT$ and $\angle SQR$ are right angles by the definition of perpendicular. $\angle SQT \cong \angle SQR$ and $\overline{SQ} \cong \overline{SQ}$. $\Delta SQT \cong \Delta SQR$ by SAS, so $\overline{ST} \cong \overline{SR}$ by CPCTC. Thus, both *T* and *R* are on $\odot S$. For two points of ℓ to also be on $\odot S$ contradicts the given fact that ℓ is tangent to $\odot S$ at *T*. Therefore, $\ell \perp \overline{ST}$ must be true.

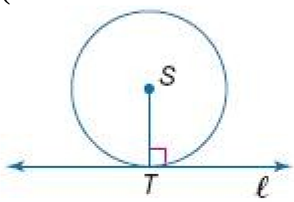


33. **PROOF** Write an indirect proof to show that if a line is perpendicular to the radius of a circle at its endpoint, then the line is a tangent of the circle. (Part 2 of Theorem 10.10)

Given: $\ell \perp \overline{ST}$; $\overline{ST}$ is a radius of $\odot S$.

Prove: ℓ is tangent to $\odot S$.

(*Hint*: Assume ℓ is *not* tangent to $\odot S$.)



SOLUTION:

Proof: Assume that ℓ is not tangent to $\odot S$. Since ℓ intersects $\odot S$ at *T*, it must intersect the circle in another place. Call this point *Q*. Then *ST* = *SQ*. $\triangle STQ$ is isosceles, so $\angle T \cong \angle Q$. Since $\overline{ST} \perp \ell$, $\angle T$ and $\angle Q$ are right angles. This contradicts that a triangle can only have one right angle. Therefore, ℓ is tangent to $\odot S$.

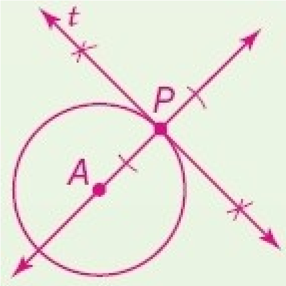
ANSWER:

Proof: Assume that ℓ is not tangent to $\odot S$. Since ℓ intersects $\odot S$ at *T*, it must intersect the circle in another place. Call this point *Q*. Then *ST* = *SQ*. $\triangle STQ$ is isosceles, so $\angle T \cong \angle Q$. Since $\overline{ST} \perp \ell$, $\angle T$ and $\angle Q$ are right angles. This contradicts that a triangle can only have one right angle. Therefore, ℓ is tangent to $\odot S$.

34. **CCSS TOOLS** Construct a line tangent to a circle through a point on the circle. Use a compass to draw $\odot A$. Choose a point *P* on the circle and draw $\overleftrightarrow{AP}$. Then construct a segment through point *P* perpendicular to $\overleftrightarrow{AP}$. Label the tangent line *t*. Explain and justify each step.

SOLUTION:

Sample answer:



Step 1: Draw circle *A* and label a point *P* on the circle.

Step 2: Draw $\overleftrightarrow{AP}$. (Two points determine a line.)

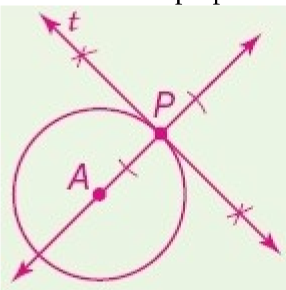
Step 3: Construct line *t* perpendicular to $\overleftrightarrow{AP}$ through point *P*. (The tangent is perpendicular to the radius at its endpoint.)

ANSWER:

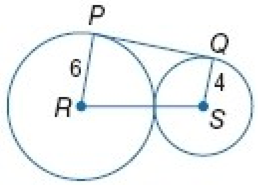
Sample answer:

a. Draw $\overleftrightarrow{AP}$. (Two points determine a line.)

b. Construct a perpendicular at *P*. (The tangent is perpendicular to the radius at its endpoint.)

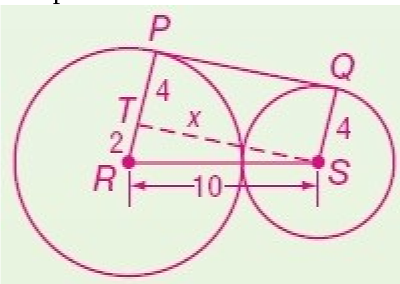


35. **CHALLENGE** $\overline{PQ}$ is tangent to circles *R* and *S*. Find *PQ*. Explain your reasoning.



SOLUTION:

Sample answer:



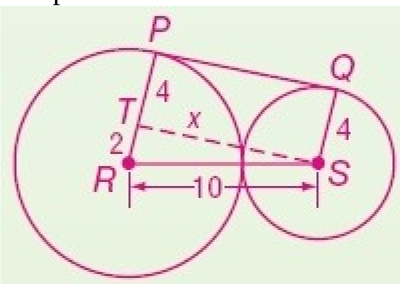
Draw $\overline{ST}$ perpendicular to $\overline{PR}$. (Through a point not on a line exactly one perpendicular can be drawn to another line.) Since $\angle STP$ is a right angle, *PQST* is a rectangle with *PT* = *QS* or 4 and *PQ* = *ST*. Triangle *RST* is a right triangle with *RT* = *PR* – *TR* or 2 and *RS* = *PR* + *QS* or 10. Let *x* = *TS* and use the Pythagorean Theorem to find the value of *x*.

$RT^2 + TS^2$	$= RS^2$	Pythagorean Theorem
$2^2 + x^2$	$= 10^2$	$RT = 2, RS = 10$
x^2	$= 96$	Simplify and subtract 4 from each side.
x	≈ 9.8	Take the positive square root of each side.

The measure of *ST* is about 9.8. Since *PQ* = *ST*, then the measure of *PQ* is also about 9.8.

ANSWER:

Sample answer:



Using the Pythagorean Theorem, $2^2 + x^2 = 10^2$, so $x \approx 9.8$. Since *PQST* is a rectangle, *PQ* = *x* = 9.8.

36. **WRITING IN MATH** Explain and justify each step in the construction on page 720.

SOLUTION:

(Step 1) A compass is used to draw circle *C* and a point *A* outside of circle *C*. Segment $\overline{CA}$ is drawn. (There is exactly one line through points *A* and *C*.)

(Step 2) A line ℓ is constructed bisecting line $\overline{CA}$. (Every segment has exactly one perpendicular bisector.) According to the definition of perpendicular bisector, point *X* is the midpoint of $\overline{AC}$.

(Step 3) A second circle, *X*, is then drawn with a radius $\overline{XC}$ which intersects circle *C* at points *D* and *E*. (Two circles can intersect at a maximum of two points.)

(Step 4) $\overleftrightarrow{AD}$ and $\overline{DC}$ are then drawn. (Through two points, there is exactly one line.) ΔADC is inscribed in a semicircle, so $\angle ADC$ is a right angle. (Theorem 10.8) $\overleftrightarrow{AD}$ is tangent to $\odot C$ at point *D* because it intersects the circle in exactly one point.(Definition of a tangent line.)

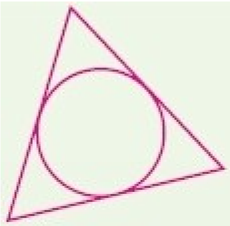
ANSWER:

First, a compass is used to draw circle *C* and a point *A* outside of circle *C*. Segment $\overline{CA}$ is drawn. There is exactly one line through points *A* and *C*. Next, a line ℓ is constructed bisecting $\overline{CA}$. According to the definition of a perpendicular bisector, line ℓ is exactly half way between point *C* and point *A*. A second circle, *X*, is then drawn with a radius $\overline{XC}$ which intersects circle *C* at points *D* and *E*. Two circles can intersect at a maximum of two points. $\overleftrightarrow{AD}$ and $\overline{DC}$ are then drawn, and ΔADC is inscribed in a semicircle. $\angle ADC$ is a right angle and $\overleftrightarrow{AD}$ is tangent to $\odot C$. $\overleftrightarrow{AD}$ is tangent to $\odot C$ at point *D* because it intersects the circle in exactly one point.

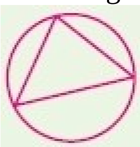
37. **OPEN ENDED** Draw a circumscribed triangle and an inscribed triangle.

SOLUTION:

Sample answer:
Circumscribed
Use a compass to draw a circle. Using a straightedge, draw 3 intersecting tangent lines to the circle. The triangle formed by the 3 tangents will be a circumscribed triangle.

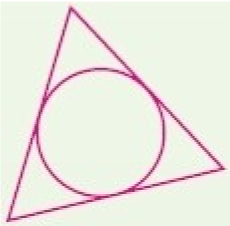


Inscribed
Use a compass to draw a circle. Choose any 3 points on the circle. Construct 3 line segments connecting the points. The triangle formed by the 3 line segments will be an inscribed triangle.

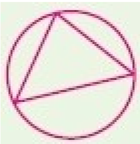


ANSWER:

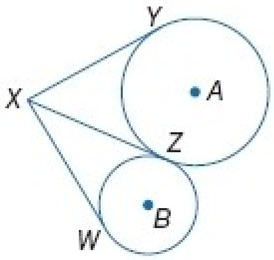
Sample answer:
Circumscribed



Inscribed



38. **REASONING** In the figure, $\overline{XY}$ and $\overline{XZ}$ are tangent to $\odot A$. $\overline{XZ}$ and $\overline{XW}$ are tangent to $\odot B$. Explain how segments $\overline{XY}$, $\overline{XZ}$, and $\overline{XW}$ can all be congruent if the circles have different radii.



SOLUTION:

By Theorem 10.11, if two segments from the same exterior point are tangent to a circle, then they are congruent. So, $\overline{XY} \cong \overline{XZ}$ and $\overline{XZ} \cong \overline{XW}$. By the transitive property, $\overline{XY} \cong \overline{XW}$. Thus, even though $\odot A$ and $\odot B$ have different radii, $\overline{XY} \cong \overline{XZ} \cong \overline{XW}$.

ANSWER:

By Theorem 10.11, if two segments from the same exterior point are tangent to a circle, then they are congruent. So, $\overline{XY} \cong \overline{XZ}$ and $\overline{XZ} \cong \overline{XW}$. Thus, $\overline{XY} \cong \overline{XZ} \cong \overline{XW}$.

39. **WRITING IN MATH** Is it possible to draw a tangent from a point that is located anywhere outside, on, or inside a circle? Explain.

SOLUTION:

From a point outside the circle, two tangents can be drawn.(Draw a line connecting the center of the circle and the exterior point. Draw a line through the exterior point that is tangent to the circle and draw a radius to the point of tangency creating a right triangle. On the other side of the line through the center a congruent triangle can be created. This would create a second line tangent to the circle.) From a point on the circle, one tangent can be drawn. (The tangent line is perpendicular to the radius at the point of tangency and through a point on a line only one line can be drawn perpendicular to the given line.) From a point inside the circle, no tangents can be drawn because a line would intersect the circle in two points. (Every interior point of the circle be a point on some chord of the circle.)

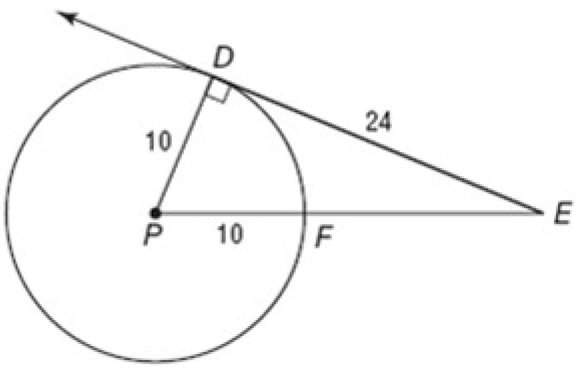
ANSWER:

No; sample answer: Two tangents can be drawn from a point outside a circle and one tangent can be drawn from a point on a circle. However, no tangents can be drawn from a point inside the circle because a line would intersect the circle in two points.

40. $\odot P$ has a radius of 10 centimeters, and $\overline{ED}$ is tangent to the circle at point *D*. *F* lies both on $\odot P$ and on segment $\overline{EP}$. If *ED* = 24 centimeters, what is the length of $\overline{EF}$?

A 10 cm
B 16 cm
C 21.8 cm
D 26 cm

SOLUTION:



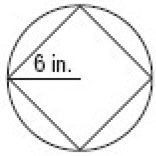
Apply Theorem 10.10 and the Pythagorean Theorem.

$$DE^2 + PD^2 = PE^2$$
$$24^2 + 10^2 = PE^2$$
$$100 + 576 = PE^2$$
$$676 = PE^2$$
$$\Rightarrow PE = 26$$

So, *EF* = 26 – 10 = 16.
The correct choice is B.

ANSWER:
B

41. **SHORT RESPONSE** A square is inscribed in a circle having a radius of 6 inches. Find the length of each side of the square.



SOLUTION:

Use the Pythagorean Theorem.

$6^2 + 6^2 = s^2$

$36 + 36 = s^2$

$72 = s^2$

$\Rightarrow s = 6\sqrt{2}$ or about 8.5 in.

ANSWER:

$6\sqrt{2}$ or about 8.5 in.

42. **ALGEBRA** Which of the following shows $25x^2 - 5x$ factored completely?

F $5x(x)$
G $5x(5x - 1)$
H $x(x - 5)$
J $x(5x - 1)$

SOLUTION:

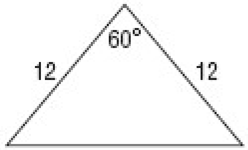
$25x^2 - 5x = 5x(5x - 1)$

So, the correct choice is G.

ANSWER:

G

43. **SAT/ACT** What is the perimeter of the triangle shown below?



A 12 units
B 24 units
C 34.4 units
D 36 units
E 104 units

SOLUTION:

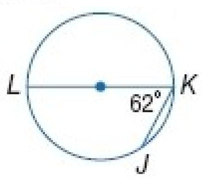
The given triangle is an isosceles triangle, since the two sides are congruent.
Since the triangle is isosceles, the base angles are congruent.
We know that the sum of all interior angles of a triangle is 180.
So, the measure of base angles is 60 each. So, the triangle is an equilateral triangle. Since the triangle is equilateral, all the sides are congruent.
Perimeter = 12 + 12 + 12 = 36
So, the correct choice is D.

ANSWER:

D

Find each measure.

44. $m\widehat{JK}$



SOLUTION:

Here, $\widehat{LK}$ is a semi-circle. So, $m\widehat{LJK} = 180.$

$m\widehat{LJ} + m\widehat{JK} = 180.$

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

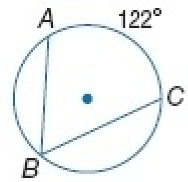
So, $m\widehat{LJ} = 2(m\angle K) = 124.$

Therefore, $m\widehat{JK} = 180 - 124 = 56.$

ANSWER:

56

45. $m\angle B$



SOLUTION:

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

So, $m\widehat{AC} = 2(m\angle B)$.

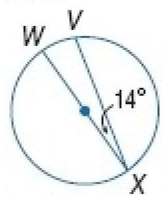
Substitute.

$$122 = 2(m\angle B)$$
$$m\angle B = 61$$

ANSWER:

61

46. $m\widehat{VX}$



SOLUTION:

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, the arc is twice the measure of the angle.

$$\begin{aligned} m(\text{arc}WV) &= 2m\angle WVX && \text{Arc equals twice the inscribed angle.} \\ &= 2(14) && \text{Substitution} \\ &= 28 && \text{Simplify.} \end{aligned}$$

Since $\overline{WX}$ is a diameter, arc *WVX* is a semicircle and has a measure of 180. Use the Arc Addition Postulate to find the measure of arc *VX*.

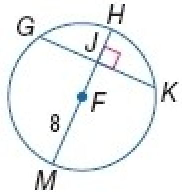
$$\begin{aligned} m(\text{arc}WV) + m(\text{arc}VX) &= m(\text{arc}WVX) && \text{Arc Addition Postulate} \\ 28 + m(\text{arc}VX) &= 180 && \text{Substitution} \\ m(\text{arc}VX) &= 152 && \text{Subtract 28 from each side.} \end{aligned}$$

Therefore, the measure of arc *VX* is 152.

ANSWER:

152

In $\odot F$, GK = 14 and $m\widehat{GHK} = 142$. Find each measure. Round to the nearest hundredth.



47. $m\widehat{GH}$

SOLUTION:

If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, $\overline{FH}$ bisects $\widehat{GHK}$. Therefore, $m\widehat{GH} = \frac{1}{2}\left(m\widehat{GHK}\right) = 71$.

ANSWER:

71

48. *JK*

SOLUTION:

If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, $\overline{FJ}$ bisects $\overline{GK}$. Then *JK* = 7 units.

ANSWER:

7

49. $m\widehat{KM}$

SOLUTION:

Here, segment HM is a diameter of the circle. We know that $m\widehat{HM} = 180$.

$m\widehat{HM} = m\widehat{HK} + m\widehat{KM}$

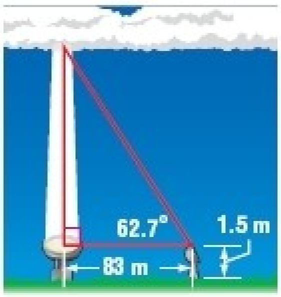
$180 = 71 + m\widehat{KM}$

$m\widehat{KM} = 109$

ANSWER:

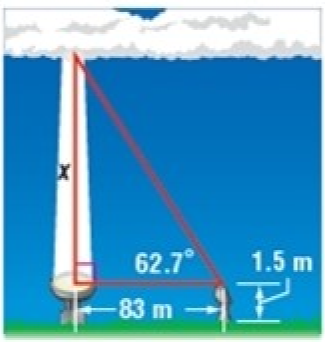
109

50. **METEOROLOGY** The altitude of the base of a cloud formation is called the ceiling. To find the ceiling one night, a meteorologist directed a spotlight vertically at the clouds. Using a theodolite, an optical instrument with a rotatable telescope, placed 83 meters from the spotlight and 1.5 meters above the ground, he found the angle of elevation to be $62.7°$. How high was the ceiling?



SOLUTION:

First, use a trigonometric ratio to find the value of x in the diagram.



$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 62.7° = \frac{x}{83}$$

$$83 \tan 62.7° = x$$

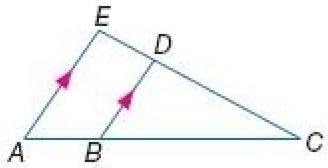
$$x \approx 160.8$$

The height of ceiling is about 160.8 + 1.5 or 162.3 meters.

ANSWER:

about 162.3 m

Determine whether the triangles are similar. If so, write a similarity statement. Explain your reasoning.

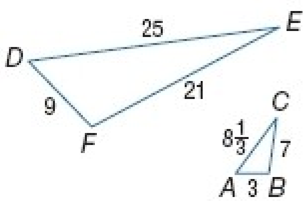
51. 

SOLUTION:

Yes; From the markings, $\overline{AE} \parallel \overline{BD}$. So, $\angle EAB$ and $\angle DBC$ are congruent corresponding angles. By the reflexive property, $\angle C$ is congruent to $\angle C$. Therefore, $\Delta AEC \sim \Delta BDC$ by AA Similarity.

ANSWER:

Yes; $\Delta AEC \sim \Delta BDC$ by AA Similarity.

52. 

SOLUTION:

Yes; Since the ratio of the corresponding sides, $\frac{3}{9}$, $\frac{7}{21}$, and $\frac{8\frac{1}{3}}{25}$ all equal $\frac{1}{3}$, the corresponding sides are proportional. Therefore, $\Delta DEF \sim \Delta ACB$ by SSS Similarity.

ANSWER:

Yes; $\Delta DEF \sim \Delta ACB$ by SSS Similarity.

Solve each equation.

53. $15 = \frac{1}{2}\left[(360 - x) - 2x\right]$

SOLUTION:

$$15 = \frac{1}{2}\left[(360 - x) - 2x\right]$$
$$15 = \frac{1}{2}\left[360 - x - 2x\right]$$
$$15 = \frac{1}{2}\left[360 - 3x\right]$$
$$30 = 360 - 3x$$
$$3x = 330$$
$$x = 110$$

ANSWER:

110

54. $x+12=\frac{1}{2}\left[(180-120)\right]$

SOLUTION:

$$x+12=\frac{1}{2}\left[(180-120)\right]$$
$$x+12=\frac{1}{2}[60]$$
$$x+12=30$$
$$x=18$$

ANSWER:

18

55. $x=\frac{1}{2}\left[(180-64)\right]$

SOLUTION:

$$x=\frac{1}{2}\left[(180-64)\right]$$
$$x=\frac{1}{2}[116]$$
$$x=58$$

ANSWER:

58