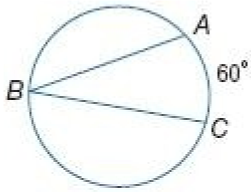
Find each measure.

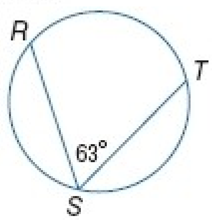
1. $m\angle B$



SOLUTION:
If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.
Therefore, $m\angle B = \frac{1}{2}\left(m\widehat{AC}\right) = 30.$

ANSWER:
30

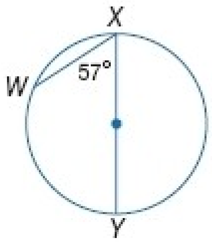
2. $m\widehat{RT}$



SOLUTION:
If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.
Therefore, $m\widehat{RT} = 2\left(m\angle S\right) = 126.$

ANSWER:
126

3. $m\widehat{WX}$



SOLUTION:

Here, $\widehat{XY}$ is a semi-circle. So, $m\widehat{XWY} = 180.$

$m\widehat{WX} + m\widehat{WY} = 180.$

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

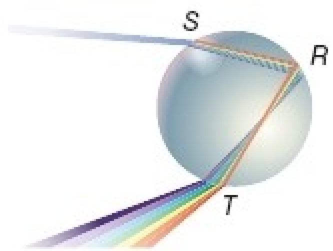
So, $m\widehat{WY} = 2(m\angle X) = 114.$

Therefore, $m\widehat{WX} = 180 - m\widehat{WY} = 66.$

ANSWER:

66

4. **SCIENCE** The diagram shows how light bends in a raindrop to make the colors of the rainbow. If $m\widehat{ST} = 144$, what is $m\angle R$?



SOLUTION:

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

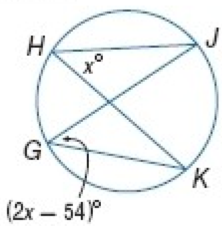
Therefore, $m\angle R = \frac{1}{2}\left(m\widehat{ST}\right) = 72.$

ANSWER:

72

ALGEBRA Find each measure.

5. $m\angle H$



SOLUTION:

If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent. So, $m\angle G = m\angle H$.

$2x - 54 = x$

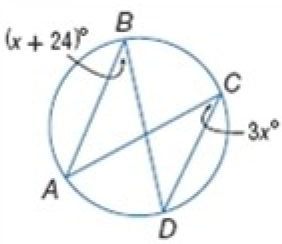
$x = 54$

Therefore, $m\angle H = 54$.

ANSWER:

54

6. $m\angle B$



SOLUTION:

If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent. So, $m\angle B = m\angle C$.

$3x = x + 24$

$2x = 24$

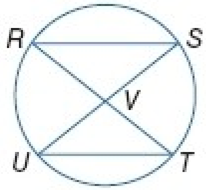
$x = 12$

Therefore, $m\angle B = 12 + 24 = 36$.

ANSWER:

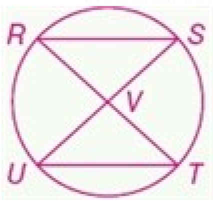
36

7. **PROOF Write a two-column proof.**
Given: $\overline{RT}$ bisects $\overline{SU}$.
Prove: $\Delta RVS \cong \Delta UVT$



SOLUTION:

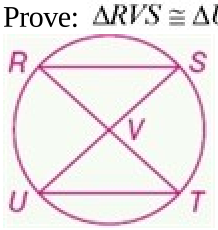
Given: $\overline{RT}$ bisects $\overline{SU}$.
Prove: $\Delta RVS \cong \Delta UVT$



Proof:
Statements (Reasons)
1. $\overline{RT}$ bisects $\overline{SU}$. (Given)
2. $\overline{SV} \cong \overline{VU}$ (Def. of segment bisector)
3. $\angle SRT$ intercepts $\widehat{ST}$. $\angle SUT$ intercepts $\widehat{ST}$. (Def. of intercepted arc)
4. $\angle SRT \cong \angle SUT$ (Inscribed $\angle$s of same arc are $\cong$.)
5. $\angle RVS \cong \angle UVT$ (Vertical $\angle$s are $\cong$.)
6. $\Delta RVS \cong \Delta UVT$ (AAS)

ANSWER:

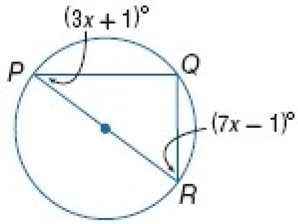
Given: $\overline{RT}$ bisects $\overline{SU}$.
Prove: $\Delta RVS \cong \Delta UVT$



Proof:
Statements (Reasons)
1. $\overline{RT}$ bisects $\overline{SU}$. (Given)
2. $\overline{SV} \cong \overline{VU}$ (Def. of segment bisector)
3. $\angle SRT$ intercepts $\widehat{ST}$. $\angle SUT$ intercepts $\widehat{ST}$. (Def. of intercepted arc)
4. $\angle SRT \cong \angle SUT$ (Inscribed $\angle$s of same arc are $\cong$.)
5. $\angle RVS \cong \angle UVT$ (Vertical $\angle$s are $\cong$.)
6. $\Delta RVS \cong \Delta UVT$ (AAS)

CCSS STRUCTURE Find each value.

8. $m\angle R$



SOLUTION:

An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle. So, $m\angle Q = 90.$

The sum of the measures of the angles of a triangle is 180. So,

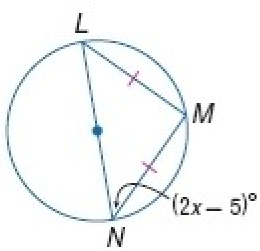
$$(3x+1)+(7x-1)+90=180.$$
$$10x=90$$
$$x=9$$

Therefore, $m\angle R = 7(9)-1=62.$

ANSWER:

62

9. x



SOLUTION:

An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle. So, $m\angle Q = 90.$ Since $LM = NM, m\angle L = m\angle N.$

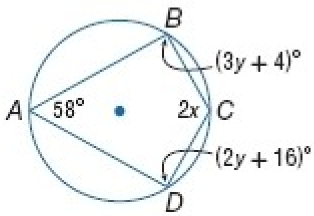
The sum of the measures of the angles of a triangle is 180. So,

$$(2x-5)+(2x-5)+90=180.$$
$$4x-10=90$$
$$x=25$$

ANSWER:

25

10. $m\angle C$ and $m\angle D$



SOLUTION:

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

$58 + 2x = 180$ and $(3y + 4) + (2y + 16) = 180$

Solve the two equations to find the values of x and y.

$2x = 122 \qquad 5y = 160$

$x = 61 \qquad y = 32$

Use the values of the variables to find $m\angle C$ and $m\angle D$.

$m\angle C = 2(61) = 122$

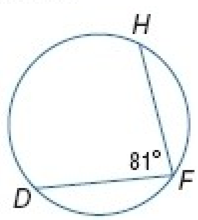
$m\angle D = 2(32) + 16 = 80$

ANSWER:

122; 80

ALGEBRA Find each measure.

11. $m\widehat{DH}$



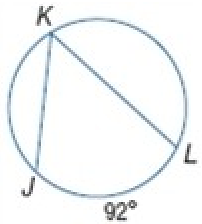
SOLUTION:

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, $m\widehat{DH} = 2(m\angle F) = 162.$

ANSWER:

162

12. $m\angle K$



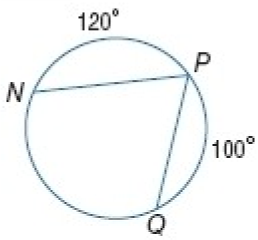
SOLUTION:

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, $m\angle K = \frac{1}{2}\left(m\widehat{JL}\right) = 46.$

ANSWER:

46

13. $m\angle P$



SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360. So,

$120 + 100 + m\widehat{QN} = 360.$

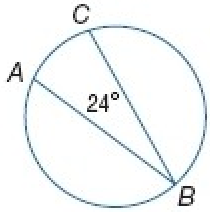
$m\widehat{QN} = 140$

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, $m\angle P = \frac{1}{2}\left(m\widehat{QN}\right) = 70.$

ANSWER:

70

14. $m\widehat{AC}$



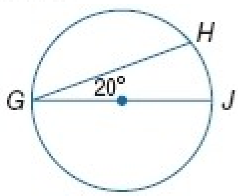
SOLUTION:

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, $m\widehat{AC} = 2(m\angle B) = 48.$

ANSWER:

48

15. $m\widehat{GH}$



SOLUTION:

Here, $m\widehat{GH} = m\widehat{GHJ} - m\widehat{HJ}.$

The arc $\widehat{GHJ}$ is a semicircle. So, $m\widehat{GHJ} = 180.$

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

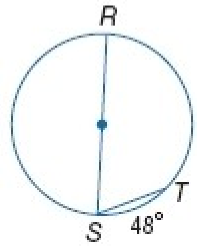
So, $m\widehat{HJ} = 2(m\angle G) = 40.$

Therefore, $m\widehat{GH} = 140.$

ANSWER:

140

16. $m\angle S$



SOLUTION:

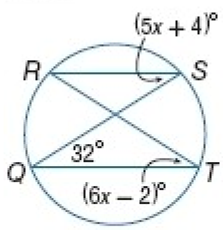
Here, $m\widehat{RT} = m\widehat{RTS} - m\widehat{TS}.$

The arc $\widehat{RTS}$ is a semicircle. So, $m\widehat{RTS} = 180.$ Then, $m\widehat{RT} = 180 - 48 = 132.$
If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

Therefore, $m\angle S = \frac{1}{2}\left(m\widehat{RT}\right) = 66.$

ANSWER:
66

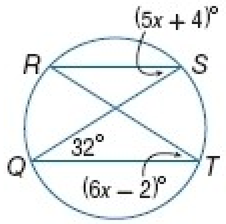
17. $m\angle R$



SOLUTION:
If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent. So, $m\angle R = m\angle Q = 32.$

ANSWER:
32

18. $m\angle S$



SOLUTION:

If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent. So, $m\angle T = m\angle S$.

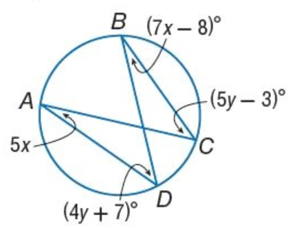
$$6x - 2 = 5x + 4$$
$$x = 6$$

Therefore, $m\angle S = 5(6) + 4 = 34$.

ANSWER:

34

19. $m\angle A$



SOLUTION:

If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent. Since $\angle A$ and $\angle B$ both intercept arc CD , $m\angle A = m\angle B$.

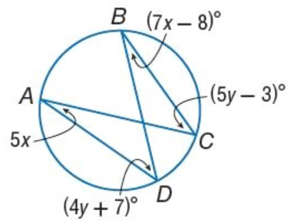
$$5x = 7x - 8$$
$$4 = x$$

Therefore, $m\angle A =$ 5(4) or 20.

ANSWER:

20

20. $m\angle C$



SOLUTION:

If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent. Since $\angle C$ and $\angle D$ both intercept arc *AB*, $m\angle C = m\angle D$.

$$5y - 3 = 4y + 7$$
$$y = 10$$

Therefore, $m\angle C = 5(10) - 3$ or 47.

ANSWER:

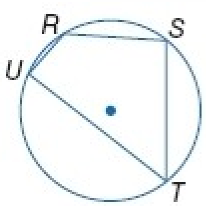
47

PROOF Write the specified type of proof.

21. paragraph proof

Given: $m\angle T = \frac{1}{2}m\angle S$

Prove: $m\widehat{TUR} = 2\left(m\widehat{URS}\right)$



SOLUTION:

Proof: Given $m\angle T = \frac{1}{2}m\angle S$ means that $m\angle S = 2m\angle T$. Since $m\angle S = \frac{1}{2}m\widehat{TUR}$ and $m\angle T = \frac{1}{2}m\widehat{URS}$, the equation becomes $\frac{1}{2}m\widehat{TUR} = 2\left(\frac{1}{2}m\widehat{URS}\right)$. Multiplying each side of the equation by 2 results in $m\widehat{TUR} = 2m\widehat{URS}$.

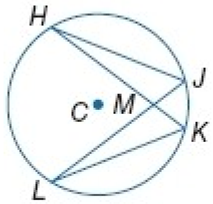
ANSWER:

Proof: Given $m\angle T = \frac{1}{2}m\angle S$ means that $m\angle S = 2m\angle T$. Since $m\angle S = \frac{1}{2}m\widehat{TUR}$ and $m\angle T = \frac{1}{2}m\widehat{URS}$, the equation becomes $\frac{1}{2}m\widehat{TUR} = 2\left(\frac{1}{2}m\widehat{URS}\right)$. Multiplying each side of the equation by 2 results in $m\widehat{TUR} = 2m\widehat{URS}$.

22. two-column proof

Given: $\odot C$

Prove: $\Delta KML \sim \Delta JMH$



SOLUTION:

Statements (Reasons):

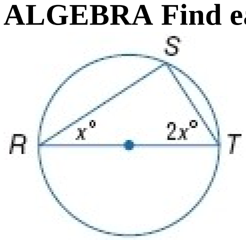
1. $\odot C$ (Given)
2. $\angle H \cong \angle L$ (Inscribed $\angle$s intercepting same arc are $\cong$.)
3. $\angle KML \cong \angle JMH$ (Vertical $\angle$s are $\cong$.)
4. $\Delta KML \sim \Delta JMH$ (AA Similarity)

ANSWER:

Statements (Reasons):

1. $\odot C$ (Given)
2. $\angle H \cong \angle L$ (Inscribed $\angle$s intercepting same arc are $\cong$.)
3. $\angle KML \cong \angle JMH$ (Vertical $\angle$s are $\cong$.)
4. $\Delta KML \sim \Delta JMH$ (AA Similarity)

ALGEBRA Find each value.



23. x

SOLUTION:

An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle. So, $m\angle S = 90.$

The sum of the measures of the angles of a triangle is 180. So,

$$\begin{aligned} x + 2x + 90 &= 180. \\ 3x &= 90 \\ x &= 30 \end{aligned}$$

ANSWER:

30

24. $m\angle T$

SOLUTION:

An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle. So, $m\angle S = 90.$

The sum of the measures of the angles of a triangle is 180. So,
$$\begin{aligned} x + 2x + 90 &= 180. \\ 3x &= 90 \\ x &= 30 \end{aligned}$$

Therefore, $m\angle T = 2(30) = 60.$

ANSWER:

60

25. x

SOLUTION:

An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle. So, $m\angle D = 90.$

The sum of the measures of the angles of a triangle is 180. So,
$$\begin{aligned} (5x - 12) + (3x) + 90 &= 180. \\ 8x &= 102 \\ x &= 12.75 \end{aligned}$$

ANSWER:

12.75

26. $m\angle C$

SOLUTION:

An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle. So, $m\angle D = 90.$

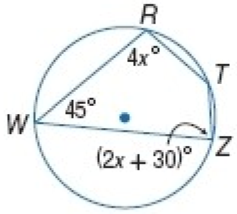
The sum of the measures of the angles of a triangle is 180. So,
$$\begin{aligned} (5x - 12) + (3x) + 90 &= 180. \\ 8x &= 102 \\ x &= 12.75 \end{aligned}$$

Therefore, $m\angle C = 5(12.75) - 12 = 51.75.$

ANSWER:

51.75

CCSS STRUCTURE Find each measure.



27. $m\angle T$

SOLUTION:

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

$45 + m\angle T = 180.$

Therefore, $m\angle T = 135.$

ANSWER:

135

28. $m\angle Z$

SOLUTION:

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

$4x + 2x + 30 = 180.$

$6x = 150$

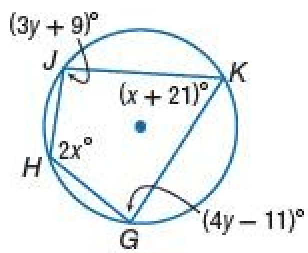
$x = 25$

Therefore, $m\angle Z = 2(25) + 30 = 80.$

ANSWER:

80

CCSS STRUCTURE Find each measure.



29. $m\angle H$

SOLUTION:

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

$2x + x + 21 = 180.$

$3x = 159$

$x = 53$

Therefore, $m\angle H = 2(53) = 106.$

ANSWER:

106

30. $m\angle G$

SOLUTION:

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

$3y+9+4y-11=180.$

$7x=182$

$x=26$

Therefore, $m\angle G=4(26)-11=93.$

ANSWER:

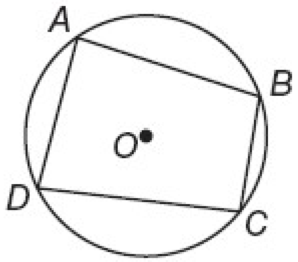
93

10-4 Inscribed Angles

31. **PROOF** Write a paragraph proof for Theorem 10.9.

SOLUTION:

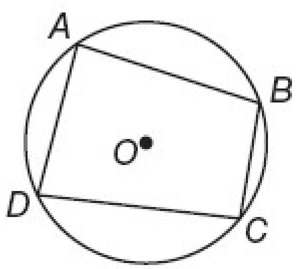
Given: Quadrilateral $ABCD$ is inscribed in $\odot O$.
Prove: $\angle A$ and $\angle C$ are supplementary.
$\angle B$ and $\angle D$ are supplementary.



Proof: By arc addition and the definitions of arc measure and the sum of central angles, $m\widehat{DCB} + m\widehat{DAB} = 360$. Since by Theorem 10.6 $m\angle C = \frac{1}{2}m\widehat{DAB}$ and $m\angle A = \frac{1}{2}m\widehat{DCB}, m\angle C + m\angle A = \frac{1}{2}\left(m\widehat{DCB} + m\widehat{DAB}\right)$, but $m\widehat{DCB} + m\widehat{DAB} = 360$, so $m\angle C + m\angle A = \frac{1}{2}(360)$ or 180. This makes $\angle C$ and $\angle A$ supplementary. Because the sum of the measures of the interior angles of a quadrilateral is 360, $m\angle A + m\angle C + m\angle B + m\angle D = 360$. But $m\angle A + m\angle C = 180$, so $m\angle B + m\angle D = 180$, making them supplementary also.

ANSWER:

Given: Quadrilateral $ABCD$ is inscribed in $\odot O$.
Prove: $\angle A$ and $\angle C$ are supplementary.
$\angle B$ and $\angle D$ are supplementary.



Proof: By arc addition and the definitions of arc measure and the sum of central angles, $m\widehat{DCB} + m\widehat{DAB} = 360$. Since by Theorem 10.6 $m\angle C = \frac{1}{2}m\widehat{DAB}$ and $m\angle A = \frac{1}{2}m\widehat{DCB}, m\angle C + m\angle A = \frac{1}{2}\left(m\widehat{DCB} + m\widehat{DAB}\right)$, but $m\widehat{DCB} + m\widehat{DAB} = 360$, so $m\angle C + m\angle A = \frac{1}{2}(360)$ or 180. This makes $\angle C$ and $\angle A$ supplementary. Because the sum of the measures of the interior angles of a quadrilateral is 360, $m\angle A + m\angle C + m\angle B + m\angle D = 360$. But $m\angle A + m\angle C = 180$, so $m\angle B + m\angle D = 180$, making them supplementary also.

SIGNS: A stop sign in the shape of a regular octagon is inscribed in a circle. Find each measure.

32. $m\widehat{NPQ}$

SOLUTION:

Since all the sides of the stop sign are congruent chords of the circle, the corresponding arcs are congruent. There are eight adjacent arcs that make up the circle, so their sum is 360. Thus, the measure of each arc joining consecutive vertices is $\frac{1}{8}$ of 360 or 45.

$$\begin{aligned} m(\text{arc}NPQ) &= m(\text{arc}NO) + m(\text{arc}OP) + m(\text{arc}PQ) && \text{Arc Addition Postulate} \\ &= 45 + 45 + 45 && \text{Substitution} \\ &= 135 && \text{Simplify.} \end{aligned}$$

Therefore, measure of arc *NPQ* is 135.

ANSWER:

135

33. $m\angle RLQ$

SOLUTION:

Since all the sides of the stop sign are congruent chords of the circle, all the corresponding arcs are congruent. There are eight adjacent arcs that make up the circle, so their sum is 360. Thus, the measure of each arc joining consecutive vertices is $\frac{1}{8}$ of 360 or 45. Since $\angle RLQ$ is inscribed in the circle, its measure equals one half the measure of its intercepted arc *QR*.

$$\begin{aligned} m\angle RLQ &= \frac{1}{2} m(\text{arc}QR) \\ &= \frac{1}{2}(45) \\ &= 22.5 \end{aligned}$$

Therefore, the measure of $\angle RLQ$ is 22.5.

ANSWER:

22.5

34. $m\angle LRQ$

SOLUTION:

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

So, $m\angle LRQ = \frac{1}{2}\left(m\widehat{LNQ}\right)$.

The sum of the measures of the central angles of a circle with no interior points in common is 360. Here, 360 is divided into 8 equal arcs and each arc measures $\frac{360}{8} = 45$.

Then, $m\widehat{LNQ} = m\widehat{LM} + m\widehat{MN} + m\widehat{NO} + m\widehat{OP} + m\widehat{PQ} = 5(45) = 225$.

Therefore, $m\angle LRQ = \frac{1}{2}(225) = 112.5$.

ANSWER:

112.5

35. $m\angle LSR$

SOLUTION:

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

So, $m\angle LSR = \frac{1}{2}\left(m\widehat{LSR}\right)$.

The sum of the measures of the central angles of a circle with no interior points in common is 360. Here, 360 is divided into 8 equal arcs and each arc measures $\frac{360}{8} = 45$.

Then, $m\widehat{LSR} = 6\left(m\widehat{LM}\right) = 6(45) = 270$.

Therefore, $m\angle LSR = \frac{1}{2}(270) = 135$.

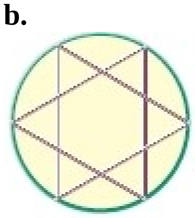
ANSWER:

135

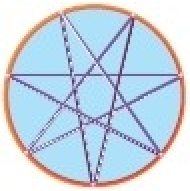
36. **ART** Four different string art star patterns are shown. If all of the inscribed angles of each star shown are congruent, find the measure of each inscribed angle.

a.

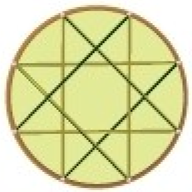
b.



c.



d.



SOLUTION:

a. The sum of the measures of the central angles of a circle with no interior points in common is 360. Here, 360 is divided into 5 equal arcs and each arc measures $\frac{360}{5} = 72.$

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, the measure of each inscribed angle is 36.

b. Here, 360 is divided into 6 equal arcs and each arc measures $\frac{360}{6} = 60.$ Each inscribed angle is formed by intercepting alternate vertices of the star. So, each intercepted arc measures 120. If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, the measure of each inscribed angle is 60.

c. Here, 360 is divided into 7 equal arcs and each arc measures $\frac{360}{7} \approx 51.43.$ If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, the measure of each inscribed angle is about 25.7.

d. The sum of the measures of the central angles of a circle with no interior points in common is 360. Here, 360 is divided into 8 equal arcs and each arc measures $\frac{360}{8} = 45.$ Each inscribed angle is formed by intercepting alternate vertices of the star. So, each intercepted arc measures 90. If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, the measure of each inscribed angle is 45.

ANSWER:

a. 36

b. 60

c. $\frac{180}{7}$ or about 25.7

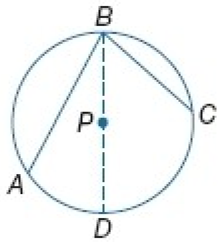
d. 45

PROOF Write a two-column proof for each case of Theorem 10.6.

37. Case 2

Given: P lies inside $\angle ABC$. $\overline{BD}$ is a diameter.

Prove: $m\angle ABC = \frac{1}{2}m\widehat{AC}$



SOLUTION:

Proof:

Statements (Reasons)

1. $m\angle ABC = m\angle ABD + m\angle DBC$ ($\angle$ Addition Postulate)
2. $m\angle ABD = \frac{1}{2}m(\text{arc } AD)$

   $m\angle DBC = \frac{1}{2}m(\text{arc } DC)$ (The measure of an inscribed $\angle$ whose side is a diameter is half the measure of the intercepted arc (Case 1)).
3. $m\angle ABC = \frac{1}{2}m(\text{arc } AD) + \frac{1}{2}m(\text{arc } DC)$ (Substitution)
4. $m\angle ABC = \frac{1}{2}[m(\text{arc } AD) + m(\text{arc } DC)]$ (Factor)
5. $m(\text{arc } AD) + m(\text{arc } DC) = m(\text{arc } AC)$ (Arc Addition Postulate)
6. $m\angle ABC = \frac{1}{2}m(\text{arc } AC)$ (Substitution)

ANSWER:

Proof:

Statements (Reasons)

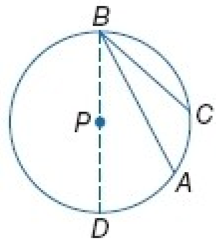
1. $m\angle ABC = m\angle ABD + m\angle DBC$ ($\angle$ Addition Postulate)
2. $m\angle ABD = \frac{1}{2}m(\text{arc } AD)$

   $m\angle DBC = \frac{1}{2}m(\text{arc } DC)$ (The measure of an inscribed $\angle$ whose side is a diameter is half the measure of the intercepted arc (Case 1)).
3. $m\angle ABC = \frac{1}{2}m(\text{arc } AD) + \frac{1}{2}m(\text{arc } DC)$ (Substitution)
4. $m\angle ABC = \frac{1}{2}[m(\text{arc } AD) + m(\text{arc } DC)]$ (Factor)
5. $m(\text{arc } AD) + m(\text{arc } DC) = m(\text{arc } AC)$ (Arc Addition Postulate)
6. $m\angle ABC = \frac{1}{2}m(\text{arc } AC)$ (Substitution)

38. Case 3

Given: P lies outside $\angle ABC$. $\overline{BD}$ is a diameter.

Prove: $m\angle ABC = \frac{1}{2} m\widehat{AC}$



SOLUTION:

Proof:

Statements (Reasons)

1. $m\angle ABC = m\angle DBC - m\angle DBA$ ($\angle$ Addition Postulate, Subtraction Property of Equality)
2. $m\angle DBC = \frac{1}{2}m(\text{arc } DC)$
   $m\angle DBA = \frac{1}{2}m(\text{arc } DA)$ (The measure of an inscribed $\angle$ whose side is a diameter is half the measure of the intercepted arc (Case 1)).
3. $m\angle ABC = \frac{1}{2}m(\text{arc } DC) - \frac{1}{2}m(\text{arc } DA)$ (Substitution)
4. $m\angle ABC = \frac{1}{2}[m(\text{arc } DC) - m(\text{arc } DA)]$ (Factor)
5. $m(\text{arc } DA) + m(\text{arc } AC) = m(\text{arc } DC)$ (Arc Addition Postulate)
6. $m(\text{arc } AC) = m(\text{arc } DC) - m(\text{arc } DA)$ (Subtraction Property of Equality)
7. $m\angle ABC = \frac{1}{2}m(\text{arc } AC)$ (Substitution)

ANSWER:

Proof:

Statements (Reasons)

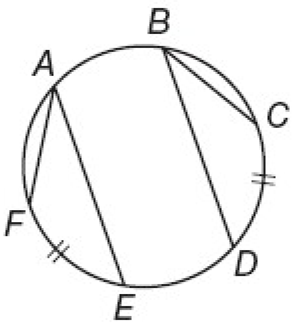
1. $m\angle ABC = m\angle DBC - m\angle DBA$ ($\angle$ Addition Postulate, Subtraction Property of Equality)
2. $m\angle DBC = \frac{1}{2}m(\text{arc } DC)$
   $m\angle DBA = \frac{1}{2}m(\text{arc } DA)$ (The measure of an inscribed $\angle$ whose side is a diameter is half the measure of the intercepted arc (Case 1)).
3. $m\angle ABC = \frac{1}{2}m(\text{arc } DC) - \frac{1}{2}m(\text{arc } DA)$ (Substitution)
4. $m\angle ABC = \frac{1}{2}[m(\text{arc } DC) - m(\text{arc } DA)]$ (Factor)
5. $m(\text{arc } DA) + m(\text{arc } AC) = m(\text{arc } DC)$ (Arc Addition Postulate)
6. $m(\text{arc } AC) = m(\text{arc } DC) - m(\text{arc } DA)$ (Subtraction Property of Equality)
7. $m\angle ABC = \frac{1}{2}m(\text{arc } AC)$ (Substitution)

PROOF Write the specified proof for each theorem.

39. Theorem 10.7, two-column proof

SOLUTION:

Given: $\angle FAE$ and $\angle CBD$ are inscribed; $\overset{\frown}{EF} \cong \overset{\frown}{DC}$
Prove: $\angle FAE \cong \angle CBD$
Proof:

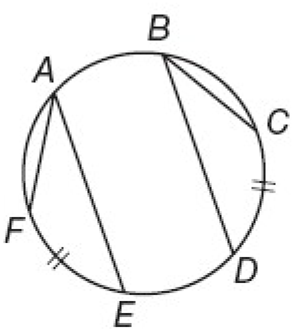


Statements (Reasons)

1. $\angle FAE$ and $\angle CBD$ are inscribed; $\overset{\frown}{EF} \cong \overset{\frown}{DC}$ (Given)
2. $m\angle FAE = \frac{1}{2}m\overset{\frown}{EF}$; $m\angle CBD = \frac{1}{2}m\overset{\frown}{DC}$ (Measure of an inscribed $\angle$ = half measure of intercepted arc.)
3. $m\overset{\frown}{EF} = m\overset{\frown}{DC}$ (Def. of $\cong$ arcs)
4. $\frac{1}{2}m\overset{\frown}{EF} = \frac{1}{2}m\overset{\frown}{DC}$ (Mult. Prop. of Equality)
5. $m\angle FAE = m\angle CBD$ (Substitution)
6. $\angle FAE \cong \angle CBD$ (Def. of $\cong$ $\angle s$)

ANSWER:

Given: $\angle FAE$ and $\angle CBD$ are inscribed; $\overset{\frown}{EF} \cong \overset{\frown}{DC}$
Prove: $\angle FAE \cong \angle CBD$
Proof:

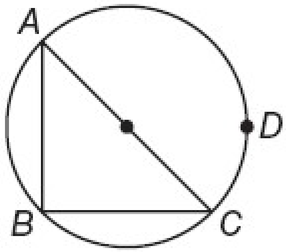


Statements (Reasons)

1. $\angle FAE$ and $\angle CBD$ are inscribed; $\overset{\frown}{EF} \cong \overset{\frown}{DC}$ (Given)
2. $m\angle FAE = \frac{1}{2}m\overset{\frown}{EF}$; $m\angle CBD = \frac{1}{2}m\overset{\frown}{DC}$ (Measure of an inscribed $\angle$ = half measure of intercepted arc.)
3. $m\overset{\frown}{EF} = m\overset{\frown}{DC}$ (Def. of $\cong$ arcs)
4. $\frac{1}{2}m\overset{\frown}{EF} = \frac{1}{2}m\overset{\frown}{DC}$ (Mult. Prop. of Equality)
5. $m\angle FAE = m\angle CBD$ (Substitution)
6. $\angle FAE \cong \angle CBD$ (Def. of $\cong$ $\angle s$)

40. Theorem 10.8, paragraph proof

SOLUTION:

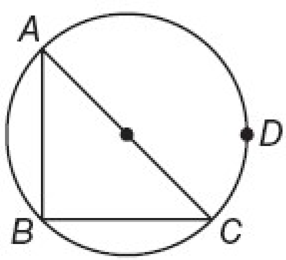


Part I: **Given:** $\widehat{ADC}$ is a semicircle.
Prove: $\angle ABC$ is a right angle.
Proof: Since $\widehat{ADC}$ is a semicircle, then $m\widehat{ADC} = 180$. Since $\angle ABC$ is an inscribed angle, then $m\angle ABC = \frac{1}{2}m\widehat{ADC}$ or 90. So, by definition, $\angle ABC$ is a right angle.

Part II: **Given**: $\angle ABC$ is a right angle.
Prove: $\widehat{ADC}$ is a semicircle.

Proof: Since $\angle ABC$ is an inscribed angle, then $m\angle ABC = \frac{1}{2}m\widehat{ADC}$ and by the Multiplication Property of Equality, $m\widehat{ADC} = 2m\angle ABC$. Because $\angle ABC$ is a right angle, $m\angle ABC = 90$. Then $m\widehat{ADC}$ = 2(90) or 180. So by definition, $\widehat{ADC}$ is a semicircle.

ANSWER:



Part I: Given: $\widehat{ADC}$ is a semicircle.
Prove: $\angle ABC$ is a right angle.
Proof: Since $\widehat{ADC}$ is a semicircle, then $m\widehat{ADC} = 180$. Since $\angle ABC$ is an inscribed angle, then $m\angle ABC = \frac{1}{2}m\widehat{ADC}$ or 90. So, by definition, $\angle ABC$ is a right angle.

Part II: Given: $\angle ABC$ is a right angle.
Prove: $\widehat{ADC}$ is a semicircle.

Proof: Since $\angle ABC$ is an inscribed angle, then $m\angle ABC = \frac{1}{2}m\widehat{ADC}$ and by the Multiplication Property of Equality,

$m\widehat{ADC} = 2m\angle ABC$. Because $\angle ABC$ is a right angle, $m\angle ABC = 90$. Then $m\widehat{ADC} = 2(90)$ or 180. So by definition, $\widehat{ADC}$ is a semicircle.

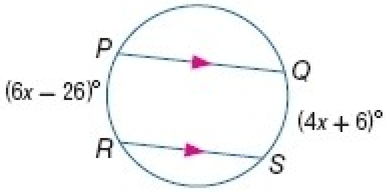
41. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between the arcs of a circle that are cut by two parallel chords.

a. GEOMETRIC Use a compass to draw a circle with parallel chords $\overline{AB}$ and $\overline{CD}$. Connect points A and D by drawing segment $\overline{AD}$.

b. NUMERICAL Use a protractor to find $m\angle A$ and $m\angle D$. Then determine $m\widehat{AC}$ and $m\widehat{BD}$. What is true about these arcs? Explain.

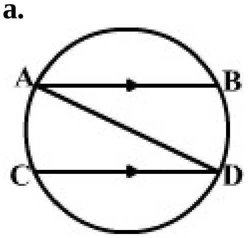
c. VERBAL Draw another circle and repeat parts **a** and **b.** Make a conjecture about arcs of a circle that are cut by two parallel chords.

d. ANALYTICAL Use your conjecture to find $m\widehat{PR}$ and $m\widehat{QS}$ in the figure at the right. Verify by using inscribed angles to find the measures of the arcs.



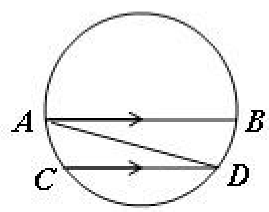
SOLUTION:

a.



b. Sample answer: $m\angle A = 30$, $m\angle D = 30$; $m\widehat{AC} = 60$, $m\widehat{BD} = 60$; The arcs are congruent because they have equal measures.

c. Sample answer:



$m\angle A = 15$, $m\angle D = 15$, $m\widehat{BD} = 30$, and $m\widehat{AC} = 30$. The arcs are congruent.
In a circle, two parallel chords cut congruent arcs.

d. In a circle, two parallel chords cut congruent arcs. So, $m\widehat{PR} = m\widehat{QS}$.

$6x - 26 = 4x + 6.$

$2x = 32$

$x = 16$

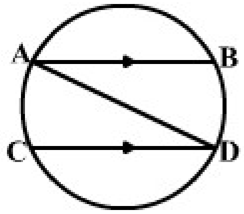
$m\widehat{QS} = 4(16) + 6$ or 70

$m\widehat{PR} = 6(16) - 26$ or 70

Therefore, the measure of arcs *QS* and *PR* are each 70.

ANSWER:

a.



b. Sample answer: $m\angle A = 30$, $m\angle D = 30$; $m\widehat{AC} = 60$, $m\widehat{BD} = 60$; The arcs are congruent because they have equal measures.

c. Sample answer: In a circle, two parallel chords cut congruent arcs. See students' work.

d. 70; 70

CCSS ARGUMENTS Determine whether the quadrilateral can always, sometimes, or never be inscribed in a circle. Explain your reasoning.

42. square

SOLUTION:

Squares have right angles at each vertex, therefore each pair of opposite angles will be supplementary and inscribed in a circle. Therefore, the statement is *always* true.

ANSWER:

Always; squares have right angles at each vertex, therefore each pair of opposite angles will be supplementary and inscribed in a circle.

43. rectangle

SOLUTION:

Rectangles have right angles at each vertex, therefore each pair of opposite angles will be supplementary and inscribed in a circle. Therefore, the statement is *always* true.

ANSWER:

Always; rectangles have right angles at each vertex, therefore each pair of opposite angles will be supplementary and inscribed in a circle.

44. parallelogram

SOLUTION:

The opposite angles of a parallelogram are always congruent. They will only be supplementary when they are right angles. So, a parallelogram can be inscribed in a circle as long as it is a rectangle. Therefore, the statement is *sometimes* true.

ANSWER:

Sometimes; a parallelogram can be inscribed in a circle as long as it is a rectangle.

45. rhombus

SOLUTION:

The opposite angles of a rhombus are always congruent. They will only be supplementary when they are right angles. So, a rhombus can be inscribed in a circle as long as it is a square. Since the opposite angles of rhombi that are not squares are not supplementary, they can not be inscribed in a circle. Therefore, the statement is *sometimes* true.

ANSWER:

Sometimes; a rhombus can be inscribed in a circle as long as it is a square. Since the opposite angles of rhombi that are not squares are not supplementary, they can not be inscribed in a circle.

46. kite

SOLUTION:

Exactly one pair of opposite angles of a kite are congruent. To be supplementary, they must each be a right angle. If one pair of opposite angles for a quadrilateral is supplementary, the other pair must also be supplementary. So, as long as the angles that compose the pair of congruent opposite angles are right angles, a kite can be inscribed in a circle. Therefore, the statement is *sometimes* true.

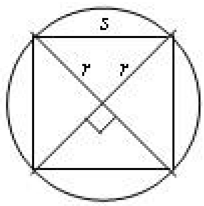
ANSWER:

Sometimes; as long as the angles that compose the pair of congruent opposite angles are right angles.

47. **CHALLENGE** A square is inscribed in a circle. What is the ratio of the area of the circle to the area of the square?

SOLUTION:

A square with side s is inscribed in a circle with radius *r*.



Using the Pythagorean Theorem,

$s^2 = r^2 + r^2$

$s^2 = 2r^2$

$s = r\sqrt{2}$

The area of a square of side *s* is $A = s^2$ and the area of a circle of radius *r* is $A = \pi r^2$.

$$\frac{A_{\text{circle}}}{A_{\text{square}}} = \frac{\pi r^2}{s^2}$$

$$= \frac{\pi r^2}{(r\sqrt{2})^2}$$

$$= \frac{\pi}{2}$$

Therefore, the ratio of the area of the circle to the area of the inscribed square is $\frac{\pi}{2}$

ANSWER:

$\frac{\pi}{2}$

48. **WRITING IN MATH** A $45°-45°-90°$ right triangle is inscribed in a circle. If the radius of the circle is given, explain how to find the lengths of the right triangle's legs.

SOLUTION:

Sample answer: A $45°-45°-90°$ triangle will have two inscribed angles of 45 and one of 90. The hypotenuse is across from the 90, or right angle. According to theorem 10.8, an inscribed angle of a triangle intercepts a diameter if the angle is a right angle. Therefore, the hypotenuse is a diameter and has a length of $2r$. Use trigonometry to find the length of the equal legs.

$$\sin 45 = \frac{\text{leg}}{2r} \quad \sin\theta = \frac{\text{opp}}{\text{hyp}}; \theta = 45; \text{hyp} = 2r$$

$$\frac{\sqrt{2}}{2} = \frac{\text{leg}}{2r} \quad \sin 45 = \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2}(2r) = \text{leg} \quad \text{Multiply each side by } 2r.$$

$$\sqrt{2}r = \text{leg} \quad \text{Simplify.}$$

Therefore, the length of each leg of the 45-45-90 triangle can be found by multiplying the radius of the circle in which it is inscribed by $\sqrt{2}$.

ANSWER:

Sample answer: According to theorem 10.8, an inscribed angle of a triangle intercepts a diameter if the angle is a right angle. Therefore, the hypotenuse is a diameter and has a length of $2r$. Using trigonometry, each leg = sin $45° \cdot 2r$ or $\sqrt{2}r$.

49. **OPEN ENDED** Find and sketch a real-world logo with an inscribed polygon.

SOLUTION:

See students' work.

ANSWER:

See students' work.

50. **WRITING IN MATH** Compare and contrast inscribed angles and central angles of a circle. If they intercept the same arc, how are they related?

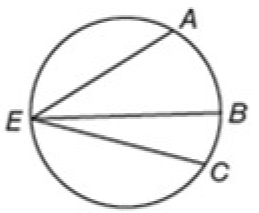
SOLUTION:

An inscribed angle has its vertex on the circle. A central angle has its vertex at the center of the circle. If a central angle intercepts arc AB, then the measure of the central angle is equal to m(arc AB). If an inscribed angle also intercepts arc AB, then the measure of the inscribed angle is equal to $\frac{1}{2}m$(arc AB). So, if an inscribed angle and a central angle intercept the same arc, then the measure of the inscribed angle is one-half the measure of the central angle.

ANSWER:

An inscribed angle has its vertex on the circle. A central angle has its vertex at the center of the circle. If an inscribed angle and a central angle intercept the same arc, then the measure of the inscribed angle is one-half the measure of the central angle.

51. In the circle below, $m\widehat{AC} = 160$ and $m\angle BEC = 38$. What is $m\angle AEB$?



A 42
B 61
C 80
D 84

SOLUTION:
If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.

Therefore, $m\angle AEC = \frac{1}{2}\left(m\widehat{AC}\right) = 80.$

But, $m\angle AEC = m\angle BEC + m\angle AEB.$
Therefore, $m\angle AEB = 80 - 38 = 42.$
The correct choice is A.

ANSWER:
A

52. **ALGEBRA** Simplify
$4(3x - 2)(2x + 4) + 3x^2 + 5x - 6$.
F $9x^2 + 3x - 14$
G $9x^2 + 13x - 14$
H $27x^2 + 37x - 38$
J $27x^2 + 27x - 26$

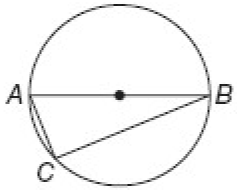
SOLUTION:
Use the Distributive Property to simplify the first term.

$$4(3x-2)(2x+4)+3x^2+5x-6$$
$$=4(6x^2+12x-4x-8)+3x^2+5x-6$$
$$=24x^2+32x-32+3x^2+5x-6$$
$$=27x^2+37x-38$$

Therefore, the correct choice is H.

ANSWER:
H

53. In the circle below, $\overline{AB}$ is a diameter, AC = 8 inches, and BC = 15 inches. Find the diameter, the radius, and the circumference of the circle.



SOLUTION:

Use the Pythagorean Theorem to find the hypotenuse of the triangle which is also the diameter of the circle.

$AB = \sqrt{8^2 + 15^2} = \sqrt{289} \approx 17$ in.

The radius is half the diameter. So, the radius of the circle is about 8.5 in.

The circumference C of a circle of a circle of diameter d is given by $C = \pi d$.

Therefore, the circumference of the circle is $C = \pi(17) \approx 53.4$ in.

ANSWER:

d = 17 in., r = 8.5 in ., $C = 17\pi$ or about 53.4 in.

54. **SAT/ACT** The sum of three consecutive integers is –48. What is the least of the three integers?

A –15
B –16
C –17
D –18
E –19

SOLUTION:

Let the three consecutive integers be x , x +1, and x + 2. Then,

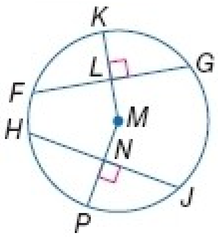
$$\begin{aligned} x+x+1+x+2 &= -48 && \text{Sum of numbers is } -48 \\ 3x+3 &= -48 && \text{Simplify.} \\ 3x &= -51 && \text{Subtract 3 from each side.} \\ x &= -17 && \text{Divide each side by 3.} \end{aligned}$$

So, the three integers are –17, –16, and –15 and the least of these is –17.
Therefore, the correct choice is C.

ANSWER:

C

In $\odot M$, $FL = 24$, $HJ = 48$, and $m\widehat{HP} = 65$. Find each measure.



55. *FG*

SOLUTION:

If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, $\overline{ML}$ bisects $\overline{FG}$. Therefore, $FG = 2(FL) = 48$ units.

ANSWER:
48

56. $m\widehat{PJ}$

SOLUTION:

If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, $\overline{MN}$ bisects $\widehat{HPJ}$. Therefore, $m\widehat{PJ} = m\widehat{HP} = 65$.

ANSWER:
65

57. *NJ*

SOLUTION:

If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, $\overline{MN}$ bisects $\overline{HJ}$. Therefore, $NJ = \frac{1}{2}(HJ) = 24$ units.

ANSWER:
24

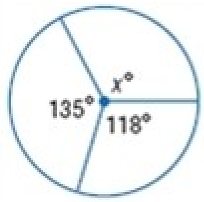
58. $m\widehat{HJ}$

SOLUTION:

If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, $\overline{MN}$ bisects $\widehat{HPJ}$. Therefore, $m\widehat{HJ} = 2\left(m\widehat{HP}\right) = 130$.

ANSWER:
130

Find *x*.

59. 

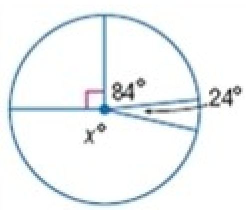
SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360. So,

$$135 + 118 + x = 360.$$
$$235 + x = 360$$
$$x = 107$$

ANSWER:

107

60. 

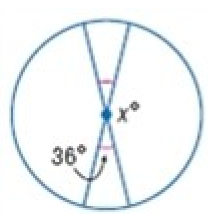
SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360. So,

$$24 + 84 + 90 + x = 360.$$
$$198 + x = 360$$
$$x = 162$$

ANSWER:

162

61. 

SOLUTION:

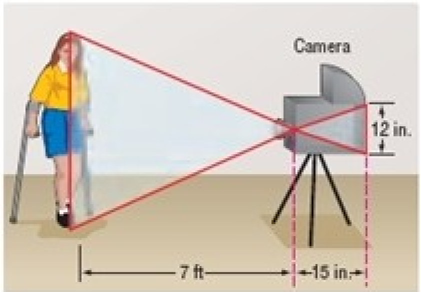
The sum of the measures of the central angles of a circle with no interior points in common is 360. So,

$$x + x + 36 + 36 = 360.$$
$$2x + 72 = 360$$
$$2x = 288$$
$$x = 144$$

ANSWER:

144

62. **PHOTOGRAPHY** In one of the first cameras invented, light entered an opening in the front. An image was reflected in the back of the camera, upside down, forming similar triangles. Suppose the image of the person on the back of the camera is 12 inches, the distance from the opening to the person is 7 feet, and the camera itself is 15 inches long. How tall is the person being photographed?



SOLUTION:

The height of the person being photographed is the length of the base of the larger triangle. The two triangles are similar. So, their corresponding sides will be proportional. One foot is equivalent to 12 in. Then, the height of the larger triangle is 7(12) = 84 in. Let *x* be the length of the base of the larger triangle.

$$\frac{x}{84} = \frac{12}{15}$$
$$15x = 1008$$
$$x = 67.2 \text{ in. or } 5.6 \text{ ft.}$$

Therefore, the person being photographed is 5.6 ft tall.

ANSWER:

5.6 ft

ALGEBRA Suppose *B* is the midpoint of $\overline{AC}$. Use the given information to find the missing measure.

63. $AB = 4x - 5$, $BC = 11 + 2x$, $AC = ?$

SOLUTION:

Since *B* is the midpoint of $\overline{AC}, AB = BC.$

$$4x - 5 = 11 + 2x$$
$$2x = 16$$
$$x = 8$$
$$\begin{aligned} AC &= 2(4(8) - 5) \\ &= 2(27) \\ &= 54 \end{aligned}$$

ANSWER:

54

64. $AB = 6y - 14$, $BC = 10 - 2y$, $AC = ?$

SOLUTION:

Since B is the midpoint of $\overline{AC}, AB = BC$.

$$
\begin{aligned}
6y - 14 &= 10 - 2y \\
8y &= 24 \\
y &= 3 \\
AC &= 2\left(6(3) - 14\right) \\
&= 2(4) \\
&= 8
\end{aligned}
$$

ANSWER:

8

65. $BC = 6 - 4m$, $AC = 8$, $m = ?$

SOLUTION:

Since B is the midpoint of $\overline{AC}, AC = 2(BC)$.

$$
\begin{aligned}
8 &= 2(6 - 4m) \\
-4 &= -8m \\
\frac{1}{2} &= m
\end{aligned}
$$

ANSWER:

$\frac{1}{2}$

66. $AB = 10s + 2$, $AC = 40$, $s = ?$

SOLUTION:

Since B is the midpoint of $\overline{AC}, AC = 2(AC)$.

$$
\begin{aligned}
40 &= 2(10s + 2) \\
36 &= 20s \\
1.8 &= s
\end{aligned}
$$

ANSWER:

1.8