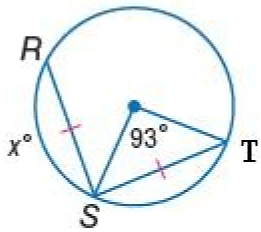
10-3 Arcs and Chords

ALGEBRA Find the value of *x*.

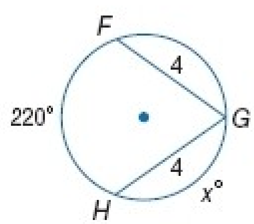
1. 

SOLUTION:

Arc *ST* is a minor arc, so *m*(arc *ST*) is equal to the measure of its related central angle or 93.
$\overline{RS}$ and $\overline{ST}$ are congruent chords, so the corresponding arcs *RS* and *ST* are congruent.
m(arc *RS*) = *m*(arc *ST*) and by substitution, *x* = 93.

ANSWER:

93

2. 

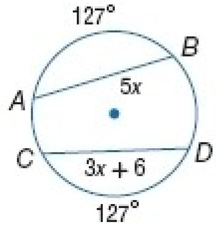
SOLUTION:

Since *HG* = 4 and *FG* = 4, $\overline{HG}$ and $\overline{FG}$ are congruent chords and the corresponding arcs *HG* and *FG* are congruent.
m(arc *HG*) = *m*(arc *FG*) = *x*
Arc *HG*, arc *GF*, and arc *FH* are adjacent arcs that form the circle, so the sum of their measures is 360.

$$\begin{aligned} x + x + 220 &= 360 && \text{Sum of arcs is 360.} \\ 2x + 220 &= 360 && \text{Simplify.} \\ 2x &= 140 && \text{Subtract 220 from each side.} \\ x &= 70 && \text{Divide each side by 2.} \end{aligned}$$

ANSWER:

70

3. 

SOLUTION:

In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. Since *m*(arc *AB*) = *m*(arc *CD*) = 127, arc *AB* $\cong$ arc *CD* and $\overline{AB} \cong \overline{CD}$.

$AB = CD$ Definition of congruent segments

$5x = 3x + 6$ Substitution

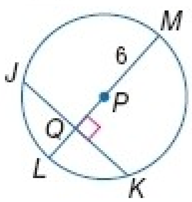
$2x = 6$ Subtract 3x from each side.

$x = 3$ Divide each side by 2.

ANSWER:

3

In $\odot P$, *JK* = 10 and $m\widehat{JLK} = 134$. Find each measure. Round to the nearest hundredth.



4. $m\widehat{JL}$

SOLUTION:

Radius $\overline{PL}$ is perpendicular to chord $\overline{JK}$. So, by Theorem 10.3, $\overline{PL}$ bisects arc *JKL*. Therefore, *m*(arc *JL*) = *m*(arc *LK*).

By substitution, *m*(arc *JL*) = $\frac{134}{2}$ or 67.

ANSWER:

67

5. *PQ*

SOLUTION:

Draw radius $\overline{PJ}$ and create right triangle *PJQ*. *PM* = 6 and since all radii of a circle are congruent, *PJ* = 6. Since the radius $\overline{PL}$ is perpendicular to $\overline{JK}$, $\overline{PL}$ bisects $\overline{JK}$ by Theorem 10.3. So, *JQ* = $\frac{1}{2}$(10) or 5.
Use the Pythagorean Theorem to find *PQ*.

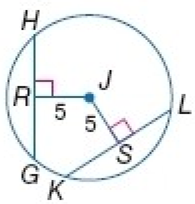
$$\begin{aligned} PQ^2 + JQ^2 &= PJ^2 && \text{Pythagorean Theorem} \\ PQ^2 + 5^2 &= 6^2 && JQ = 5, PJ = 6 \\ PQ^2 + 25 &= 36 && \text{Simplify.} \\ PQ^2 &= 11 && \text{Subtract 25 from each side.} \\ PQ &= \sqrt{11} \text{ or about } 3.32 && \text{Take the positive square root of each side.} \end{aligned}$$

So, *PQ* is about 3.32 units long.

ANSWER:

3.32

6. In $\odot J$, *GH* = 9, *KL* = 4*x* + 1. Find *x*.



SOLUTION:

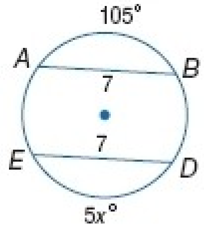
In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center. Since *JS* = *JR*, *KL* = *GH*.

$$\begin{aligned} 4x + 1 &= 9 && \text{Substitution} \\ 4x &= 8 && \text{Subtract 1 from each side.} \\ x &= 2 && \text{Divide each side by 4.} \end{aligned}$$

ANSWER:

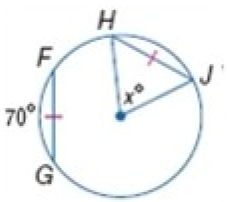
2

ALGEBRA Find the value of *x*.

7. 

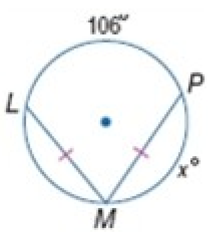
SOLUTION:
In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. Therefore, $m\widehat{AB} = m\widehat{ED}.$
$5x = 105$
$x = 21$

ANSWER:
21

8. 

SOLUTION:
In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. Therefore, $x = m\widehat{FG} = 70.$

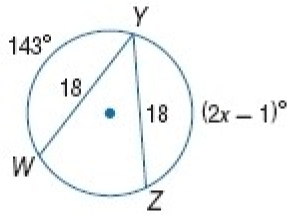
ANSWER:
70

9. 

SOLUTION:
In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. So, $m\widehat{LM} = m\widehat{PM} = x.$
The sum of the measures of the central angles of a circle with no interior points in common is 360. So,
$106 + x + x = 360.$
$2x = 254$
$x = 127$

ANSWER:
127

10-3 Arcs and Chords

10. 

SOLUTION:

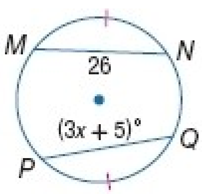
In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. Therefore, $m\widehat{YW} = m\widehat{YZ}.$

$2x - 1 = 143$

$x = 72$

ANSWER:

72

11. 

SOLUTION:

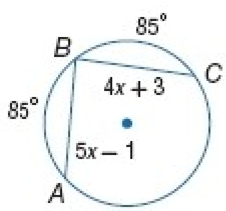
In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. Here, $m\widehat{MN} = m\widehat{PQ}.$ Therefore,

$3x + 5 = 26$

$x = 7$

ANSWER:

7

12. 

SOLUTION:

In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. Here, $m\widehat{AB} = m\widehat{BC}.$ Therefore,

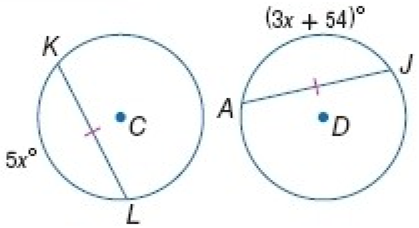
$5x - 1 = 4x + 3$

$x = 4$

ANSWER:

4

13. $\odot C \cong \odot D$



SOLUTION:

In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. Here, $\overline{KL} \cong \overline{AJ}$. Therefore,

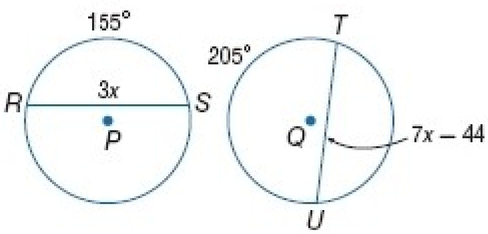
$5x = 3x + 54$

$x = 4$

ANSWER:

27

14. $\odot P \cong \odot Q$



SOLUTION:

In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. Here, measure of minor arc $\widehat{TU} = 360 - 205 = 155$.

So, $m\widehat{TU} = m\widehat{RS}$. Therefore,

$3x = 7x - 44$

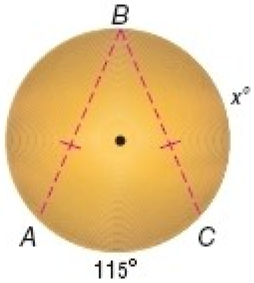
$4x = 44$

$x = 11$

ANSWER:

11

15. **CCSS MODELING** Angie is in a jewelry making class at her local arts center. She wants to make a pair of triangular earrings from a metal circle. She knows that $\widehat{AC}$ is $115°$. If she wants to cut two equal parts off so that $\widehat{AB} = \widehat{BC}$, what is *x*?



SOLUTION:

In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. So, $m\widehat{AB} = m\widehat{BC} = x.$

The sum of the measures of the central angles of a circle with no interior points in common is 360.

So, $115 + x + x = 360.$

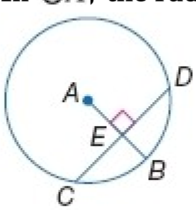
$2x = 245$

$x = 122.5$

ANSWER:

$122.5°$

In $\odot A$, the radius is 14 and *CD* = 22. Find each measure. Round to the nearest hundredth, if necessary.



16. *CE*

SOLUTION:

Radius $\overline{AB}$ is perpendicular to chord $\overline{CD}$. So, by Theorem 10.3, $\overline{AB}$ bisects $\overline{CD}$. Therefore, *CE* = *ED*.

By substitution, *CE* = $\frac{1}{2}$(22) or 11 units.

ANSWER:

11

10-3 Arcs and Chords

17. *EB*

SOLUTION:

First find *AE*. Draw radius $\overline{AC}$ and create right triangle *ACE*. The radius of the circle is 14, so *AC* = 14. Since the radius $\overline{AB}$ is perpendicular to $\overline{CD}$, $\overline{AB}$ bisects $\overline{CD}$ by Theorem 10.3. So, *CE* = $\frac{1}{2}$(22) or 11.

Use the Pythagorean Theorem to find *AE*.

$CE^2 + AE^2 = AC^2$	Pythagorean Theorem
$11^2 + AE^2 = 14^2$	$CE = 11, AC = 14$
$121 + AE^2 = 196$	Simplify.
$AE^2 = 75$	Subtract 121 from each side.
$AE = \sqrt{75}$ or about 8.66	Take the positive square root of each side.

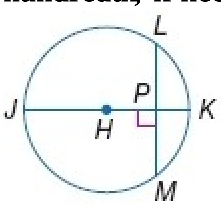
By the Segment Addition Postulate, *EB* = *AB* - *AE*.

Therefore, *EB* is 14 - 8.66 or about 5.34 units long.

ANSWER:

5.34

In $\odot H$, the diameter is 18, *LM* = 12, and $m\widehat{LM} = 84$. Find each measure. Round to the nearest hundredth, if necessary.



18. $m\widehat{LK}$

SOLUTION:

Diameter $\overline{JK}$ is perpendicular to chord $\overline{LM}$. So, by Theorem 10.3, $\overline{JK}$ bisects arc *LKM*. Therefore, *m*(arc *LK*) = *m*(arc *KM*).

By substitution, *m*(arc *LK*) = $\frac{1}{2}$(84) or 42.

ANSWER:

42

10-3 Arcs and Chords

19. *HP*

SOLUTION:

Draw radius $\overline{HL}$ and create right triangle *HLP*. Diameter *JK* = 18 and the radius of a circle is half of the diameter, so *HL* = 9. Since the diameter $\overline{JK}$ is perpendicular to $\overline{LM}$, $\overline{JK}$ bisects $\overline{LM}$ by Theorem 10.3. So, *LP* = $\frac{1}{2}$(12) or 6. Use the Pythagorean Theorem to find *HP*.

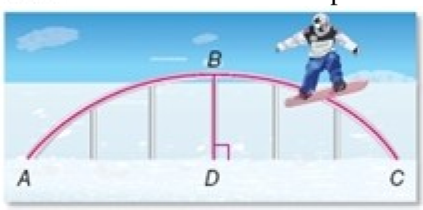
$$
\begin{aligned}
HP^2 + LP^2 &= HL^2 && \text{Pythagorean Theorem} \\
HP^2 + 6^2 &= 9^2 && LP = 6, HL = 9 \\
HP^2 + 36 &= 81 && \text{Simplify.} \\
HP^2 &= 45 && \text{Subtract 36 from each side.} \\
HP &= \sqrt{45} \text{ or about } 6.71 && \text{Take the positive square root of each side.}
\end{aligned}
$$

Therefore, *HP* is about 6.71 units long.

ANSWER:

6.71

20. **SNOWBOARDING** The snowboarding rail shown is an arc of a circle in which $\overline{BD}$ is part of the diameter. If $\widehat{ABC}$ is about 32% of a complete circle, what is $m\widehat{AB}$?



SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360.

$$
\begin{aligned}
m(\text{arc } ABC) &= 32\% \text{ of } 360 \\
&= 0.32(360) && 32\% = 0.32. \\
&= 115.2 && \text{Simplify.}
\end{aligned}
$$

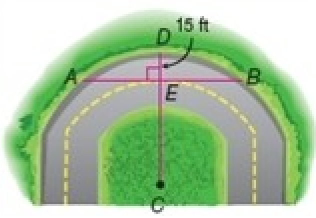
The diameter containing $\overline{BD}$ is perpendicular to chord $\overline{AC}$. So, by Theorem 10.3, $\overline{BD}$ bisects arc *ABC*. Therefore, *m*(arc *AB*) = *m*(arc *BC*). By substitution, *m*(arc *AB*) = $\frac{1}{2}$(115.2) or 57.6.

ANSWER:

57.6

10-3 Arcs and Chords

21. **ROADS** The curved road at the right is part of $\odot C$, which has a radius of 88 feet. What is *AB*? Round to the nearest tenth.



SOLUTION:

The radius of the circle is 88 ft. So, *CD* = *CB* = 88. Also, *CE* = *CD* – *ED* = 88 – 15 = 73.
Use the Pythagorean Theorem to find *EB*, the length of a leg of the right triangle *CEB*.

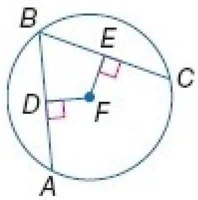
$EB = \sqrt{88^2 - 73^2} = \sqrt{2415} \approx 49.14$ ft.

If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, $\overline{CE}$ bisects $\overline{AB}$. Therefore, $AB = 2(EB) = 98.3$ ft.

ANSWER:

98.3 ft

22. **ALGEBRA** In $\odot F$, $\overline{AB} \cong \overline{BC}$, *DF* = 3*x* – 7, and *FE* = *x* + 9. What is *x*?



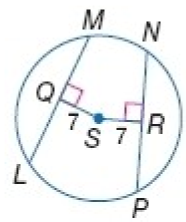
SOLUTION:

In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center. Since $\overline{AB} \cong \overline{BC}$, *DF* = *EF*.

3*x* – 7 = *x* + 9
2*x* = 16
x = 8

ANSWER:

8



23. **ALGEBRA** In $\odot S$, $LM = 16$ and $PN = 4x$. What is x?

SOLUTION:

In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center. Since $SQ = SR$, $LM = PN$.

$4x = 16$

$x = 4$

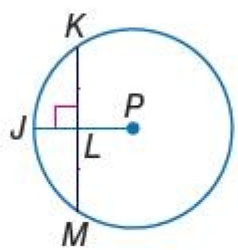
ANSWER:

4

PROOF Write a two-column proof.

24. **Given:** $\odot P, \overline{KM} \perp \overline{JP}$

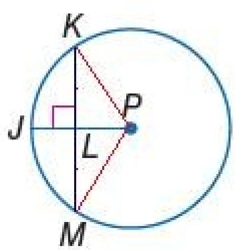
Prove: $\overline{JP}$ bisects $\overline{KM}$ and $\widehat{KM}$



SOLUTION:

Given: $\odot P, \overline{KM} \perp \overline{JP}$

Prove: $\overline{JP}$ bisects $\overline{KM}$ and $\widehat{KM}$



Proof:

Statements (Reasons)

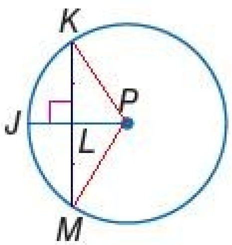
1. $\overline{KM} \perp \overline{JP}$ (Given)
2. Draw radii $\overline{PK}$ and $\overline{PM}$. (2 points determine a line.)
3. $\overline{PK} \cong \overline{PM}$ (All radii of a $\odot$ are $\cong$.)
4. $\overline{PL} \cong \overline{PL}$ (Reflex. Prop. of $\cong$)
5. $\angle PLM$ and $\angle PLK$ are right $\angle$s. (Def. of $\perp$)
6. $\angle PLM \cong \angle PLK$ (All right $\angle$s are $\cong$.)

7. $\Delta PLM \cong \Delta PLK$ (SAS)
8. $\overline{ML} \cong \overline{KL}$ (CPCTC)
9. $\overline{PJ}$ bisects $\overline{KM}$.(Def. of bisect)
10. $\angle MPJ \cong \angle KPJ$ (CPCTC)
11. $\widehat{MJ} \cong \widehat{KJ}$ (In the same circle, two arcs are congruent if their corresponding central angles are congruent.)
12. $\overline{JP}$ bisects $\widehat{KM}$. (Def. of bisect)

ANSWER:

Given: $\odot P, \overline{KM} \perp \overline{JP}$

Prove: $\overline{JP}$ bisects $\overline{KM}$ and $\widehat{KM}$



Proof:
<u>Statements (Reasons)</u>

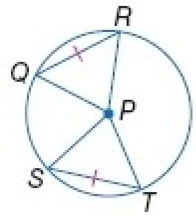
1. $\overline{KM} \perp \overline{JP}$ (Given)
2. Draw radii $\overline{PK}$ and $\overline{PM}$. (2 points determine a line.)
3. $\overline{PK} \cong \overline{PM}$ (All radii of a $\odot$ are $\cong$.)
4. $\overline{PL} \cong \overline{PL}$ (Reflex. Prop. of $\cong$)
5. $\angle PLM$ and $\angle PLK$ are right $\angle$s. (Def. of $\perp$)
6. $\angle PLM \cong \angle PLK$ (All right $\angle$s are $\cong$.)
7. $\Delta PLM \cong \Delta PLK$ (SAS)
8. $\overline{ML} \cong \overline{KL}$ (CPCTC)
9. $\overline{PJ}$ bisects $\overline{KM}$.(Def. of bisect)
10. $\angle MPJ \cong \angle KPJ$ (CPCTC)
11. $\widehat{MJ} \cong \widehat{KJ}$ (In the same circle, two arcs are congruent if their corresponding central angles are congruent.)
12. $\overline{JP}$ bisects $\widehat{KM}$. (Def. of bisect)

PROOF Write the specified type of proof.

25. paragraph proof of Theorem 10.2, part 2

Given: $\odot P, \overline{QR} \cong \overline{ST}$

Prove: $\widehat{QR} \cong \widehat{ST}$



SOLUTION:

Proof:

Because all radii are congruent, $\overline{QP} \cong \overline{PR} \cong \overline{SP} \cong \overline{PT}$. You are given that $\overline{QR} \cong \overline{ST}$, so $\Delta PQR \cong \Delta PST$ by SSS. Thus, $\angle QPR \cong \angle SPT$ by CPCTC. Since the central angles have the same measure, their intercepted arcs have the same measure and are therefore congruent. Thus, $\widehat{QR} \cong \widehat{ST}$.

ANSWER:

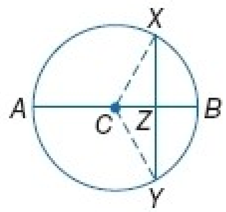
Proof:

Because all radii are congruent, $\overline{QP} \cong \overline{PR} \cong \overline{SP} \cong \overline{PT}$. You are given that $\overline{QR} \cong \overline{ST}$, so $\Delta PQR \cong \Delta PST$ by SSS. Thus, $\angle QPR \cong \angle SPT$ by CPCTC. Since the central angles have the same measure, their intercepted arcs have the same measure and are therefore congruent. Thus, $\widehat{QR} \cong \widehat{ST}$.

26. two-column proof of Theorem 10.3

Given: $\odot C, \overline{AB} \perp \overline{XY}$

Prove: $\overline{XZ} \cong \overline{YZ}, \widehat{XB} \cong \widehat{YB}$



SOLUTION:

Proof:

Statements (Reasons)

1. $\odot C, \overline{AB} \perp \overline{XY}$ (Given)
2. $\overline{CX} \cong \overline{CY}$ (All radii of a $\odot$ are $\cong$.)
3. $\overline{CZ} \cong \overline{CZ}$ (Reflexive Prop.)
4. $\angle XZC$ and $\angle YZC$ are rt. $\angle$s (Definition of $\perp$ lines)
5. $\Delta XZC \cong \Delta YZC$ (HL)
6. $\overline{XZ} \cong \overline{YZ}$, $\angle XCZ \cong \angle YCZ$ (CPCTC)
7. $\widehat{XB} \cong \widehat{YB}$ (If central $\angle$s are $\cong$, intercepted arcs are $\cong$.)

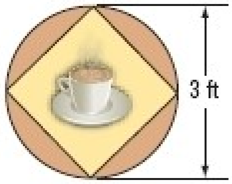
ANSWER:

Proof:

Statements (Reasons)

1. $\odot C, \overline{AB} \perp \overline{XY}$ (Given)
2. $\overline{CX} \cong \overline{CY}$ (All radii of a $\odot$ are $\cong$.)
3. $\overline{CZ} \cong \overline{CZ}$ (Reflexive Prop.)
4. $\angle XZC$ and $\angle YZC$ are rt. $\angle$s (Definition of $\perp$ lines)
5. $\Delta XZC \cong \Delta YZC$ (HL)
6. $\overline{XZ} \cong \overline{YZ}$, $\angle XCZ \cong \angle YCZ$ (CPCTC)
7. $\widehat{XB} \cong \widehat{YB}$ (If central $\angle$s are $\cong$, intercepted arcs are $\cong$.)

27. **DESIGN** Roberto is designing a logo for a friend's coffee shop according to the design at the right, where each chord is equal in length. What is the measure of each arc and the length of each chord?



SOLUTION:

The four chords are equal in length. So, the logo is a square inscribed in a circle. Each diagonal of the square is a diameter of the square and it is 3 ft long. The length of each side of a square of diagonal *d* units long is given by $\frac{d}{\sqrt{2}}$. Therefore, the length of each chord is $\frac{3}{\sqrt{2}} \approx 2.12$ ft.

In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. Here, all the four chords are equal in length and hence the corresponding arcs are equal in measure. Therefore, each arc $90°$.

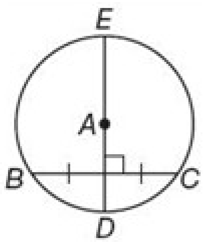
ANSWER:

Each arc is $90°$ and each chord is 2.12 ft.

28. **PROOF** Write a two-column proof of Theorem 10.4.

SOLUTION:

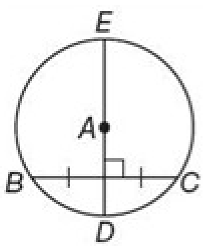
Given: $\odot A$, $\overline{ED}$ is the $\perp$ bisector of $\overline{BC}$.
Prove: $\overline{ED}$ is a diameter of $\odot A$.
Proof:



Statements (Reasons)
1. $\overline{ED}$ is the $\perp$ bisector of $\overline{BC}$ (Given)
2. A is equidistant from B and C. (All radii of a $\odot$ are $\cong$.)
3. A lies on the $\perp$ bisector of $\overline{BC}$. (Conv. of the $\perp$ Bisector Thm.)
4. $\overline{ED}$ is a diameter of $\odot A$. (Def. of diameter)

ANSWER:

Given: $\odot A$, $\overline{ED}$ is the $\perp$ bisector of $\overline{BC}$.
Prove: $\overline{ED}$ is a diameter of $\odot A$.



Proof:
Statements (Reasons)
1. $\overline{ED}$ is the $\perp$ bisector of $\overline{BC}$ (Given)
2. A is equidistant from B and C. (All radii of a $\odot$ are $\cong$.)
3. A lies on the $\perp$ bisector of $\overline{BC}$. (Conv. of the $\perp$ Bisector Thm.)
4. $\overline{ED}$ is a diameter of $\odot A$. (Def. of diameter)

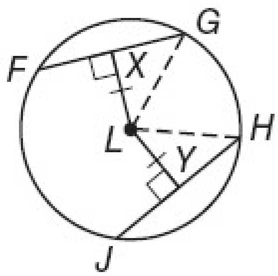
CCSS ARGUMENTS Write a two-column proof of the indicated part of Theorem 10.5.

29. In a circle, if two chords are equidistant from the center, then they are congruent.

SOLUTION:

Given: $\odot L, \overline{LX} \perp \overline{FG}, \overline{LY} \perp \overline{JH}, \overline{LX} \cong \overline{LY}$
Prove: $\overline{FG} \cong \overline{JH}$
Proof:

10-3 Arcs and Chords



Statements (Reasons)

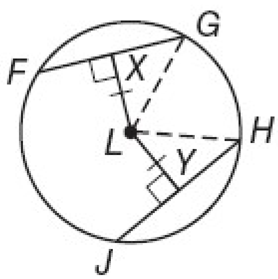
1. $\overline{LG} \cong \overline{LH}$ (All radii of a $\odot$ are $\cong$.)
2. $\overline{LX} \perp \overline{FG}, \overline{LY} \perp \overline{JH}, \overline{LX} \cong \overline{LY}$ (Given)
3. $\angle LXG$ and $\angle LYH$ are right $\angle$s. (Definition of $\perp$ lines)
4. $\Delta XGL \cong \Delta YHL$ (HL)
5. $\overline{XG} \cong \overline{YH}$ (CPCTC)
6. $XG = YH$ (Definition of $\cong$ segments)
7. $2(XG) = 2(YH)$ (Multiplication Property of Equality)
8. $\overline{LX}$ bisects $\overline{FG}$; $\overline{LY}$ bisects $\overline{JH}$. ($\overline{LX}$ and $\overline{LY}$ are contained in radii. A radius $\perp$ to a chord bisects the chord.)
9. $FG = 2(XG)$, $JH = 2(YH)$ (Definition of segment bisector)
10. $FG = JH$ (Substitution)
11. $\overline{FG} \cong \overline{JH}$ (Definition of $\cong$ segments)

ANSWER:

Given: $\odot L, \overline{LX} \perp \overline{FG}, \overline{LY} \perp \overline{JH}, \overline{LX} \cong \overline{LY}$

Prove: $\overline{FG} \cong \overline{JH}$

Proof:



Statements (Reasons)

1. $\overline{LG} \cong \overline{LH}$ (All radii of a $\odot$ are $\cong$.)
2. $\overline{LX} \perp \overline{FG}, \overline{LY} \perp \overline{JH}, \overline{LX} \cong \overline{LY}$ (Given)
3. $\angle LXG$ and $\angle LYH$ are right $\angle$s. (Definition of $\perp$ lines)
4. $\Delta XGL \cong \Delta YHL$ (HL)
5. $\overline{XG} \cong \overline{YH}$ (CPCTC)
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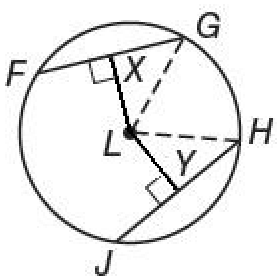
9. $FG = 2(XG)$, $JH = 2(YH)$ (Definition of segment bisector)
10. $FG = JH$ (Substitution)
11. $\overline{FG} \cong \overline{JH}$ (Definition of $\cong$ segments)

30. In a circle, if two chords are congruent, then they are equidistant from the center.

SOLUTION:

Given: $\odot L, \overline{FG} \cong \overline{JH}$ $\overline{LG}$ and $\overline{LH}$ are radii.
$\overline{LX} \perp \overline{FG}, \overline{LY} \perp \overline{JH}$
Prove: $\overline{LX} \cong \overline{LY}$
Proof:

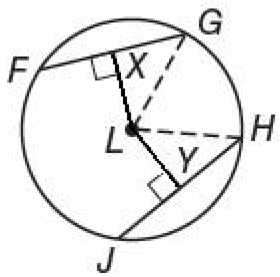


Statements (Reasons)

1. $\odot L, \overline{FG} \cong \overline{JH}$ and $\overline{LG}$ and $\overline{LH}$ are radii.
$\overline{LX} \perp \overline{FG}, \overline{LY} \perp \overline{JH}$ (Given)
2. $\overline{LX}$ bisects $\overline{FG}$; $\overline{LY}$ bisects $\overline{JH}$. ($\overline{LX}$ and $\overline{LY}$ are contained in radii. A radius $\perp$ to a chord bisects the chord.)
3. $XG = \frac{1}{2}FG$, $YH = \frac{1}{2}JH$ (Definition of bisector)
4. $FG = JH$ (Definition of $\cong$ segments)
5. $\frac{1}{2}FG = \frac{1}{2}JH$ (Multiplication Property of Equality)
6. $XG = YH$ (Substitution)
7. $\overline{XG} \cong \overline{YH}$ (Definition of $\cong$ segments)
8. $\overline{LG} \cong \overline{LH}$ (All radii of a circle are $\cong$.)
9. $\angle GXL$ and $\angle HYL$ are right $\angle$s (Def. of $\perp$ lines)
10. $\Delta XLG \cong \Delta YLH$ (HL)
11. $\overline{LX} \cong \overline{LY}$ (CPCTC)

ANSWER:

Given: $\odot L, \overline{FG} \cong \overline{JH}$ $\overline{LG}$ and $\overline{LH}$ are radii.
$\overline{LX} \perp \overline{FG}, \overline{LY} \perp \overline{JH}$
Prove: $\overline{LX} \cong \overline{LY}$
Proof:

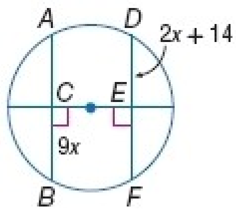


Statements (Reasons)

1. $\odot L, \overline{FG} \cong \overline{JH}$ and $\overline{LG}$ and $\overline{LH}$ are radii.
$\overline{LX} \perp \overline{FG}, \overline{LY} \perp \overline{JH}$ (Given)
2. $\overline{LX}$ bisects $\overline{FG}$; $\overline{LY}$ bisects $\overline{JH}$. ($\overline{LX}$ and $\overline{LY}$ are contained in radii. A radius $\perp$ to a chord bisects the chord.)
3. $XG = \frac{1}{2}FG$, $YH = \frac{1}{2}JH$ (Definition of bisector)
4. $FG = JH$ (Definition of $\cong$ segments)
5. $\frac{1}{2}FG = \frac{1}{2}JH$ (Multiplication Property of Equality)
6. $XG = YH$ (Substitution)
7. $\overline{XG} \cong \overline{YH}$ (Definition of $\cong$ segments)
8. $\overline{LG} \cong \overline{LH}$ (All radii of a circle are $\cong$.)
9. $\angle GXL$ and $\angle HYL$ are right $\angle$s (Def. of $\perp$ lines)
10. $\Delta XLG \cong \Delta YLH$ (HL)
11. $\overline{LX} \cong \overline{LY}$ (CPCTC)

Find the value of *x*.

31. $\overline{AB} \cong \overline{DF}$



SOLUTION:

If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, $AC = BC = \frac{1}{2}(AB)$ and $DE = FE = \frac{1}{2}(DF)$.

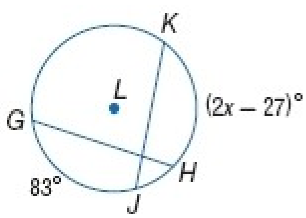
We have $\overline{AB} \cong \overline{DF}$. Then, $\frac{1}{2}(AB) = \frac{1}{2}(DF)$ or $BC = DE$.

$9x = 2x + 14$
$7x = 14$
$x = 2$

ANSWER:

2

32. $\overline{GH} \cong \overline{KJ}$



SOLUTION:

$\overline{GH}$ and $\overline{KJ}$ are congruent chords, so the corresponding arcs *GH* and *KJ* are congruent.

$$\begin{aligned} m(\text{arc}\,GH) &= m(\text{arc}\,KJ) && \text{Definition of congruent arcs} \\ m(\text{arc}\,GH) &= m(\text{arc}\,GJ) + m(\text{arc}\,JH) && \text{Arc Addition Postulate} \\ m(\text{arc}\,KJ) &= m(\text{arc}\,JH) + m(\text{arc}\,HK) && \text{Arc Addition Postulate} \\ m(\text{arc}\,GJ) + m(\text{arc}\,JH) &= m(\text{arc}\,JH) + m(\text{arc}\,HK) && \text{Substitution} \\ m(\text{arc}\,GJ) &= m(\text{arc}\,HK) && \text{Subtraction Property of Equality.} \end{aligned}$$

Use the labeled values of the arcs on the figure to find x.

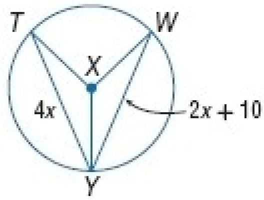
$$\begin{aligned} 83 &= 2x - 27 && \text{Substitution} \\ 110 &= 2x && \text{Add 27 to each side.} \\ 55 &= x && \text{Divide each side by 2.} \end{aligned}$$

Therefore, the value of x is 55.

ANSWER:

55

33. $\widehat{WTY} \cong \widehat{TWY}$



SOLUTION:

To find x we need to show that chords $\overline{TY}$ and $\overline{WY}$ are congruent.

$arc\,WTY \cong arc\,TWY$	Given
$m(arc\,WTY) = m(arc\,TWY)$	Definition of congruent arcs
$m(arc\,WTY) = m(arc\,WT) + m(arc\,TY)$	Arc Addition Postulate
$m(\text{arc}\,TWY) = m(\text{arc}\,WT) + m(\text{arc}\,WY)$	Arc Addition Postulate
$m(arc\,WT) + m(arc\,TY) = m(arc\,WT) + m(\text{arc}\,WY)$	Substitution
$m(arc\,TY) = m(\text{arc}\,WY)$	Subtraction Property of Equality
$arc\,TY \cong arc\,WY$	Defintion of congruent arcs
$\overline{TY} \cong \overline{WY}$	Two chords are $\cong$ if their corr. arcs are $\cong$.
$TY = WY$	Defintion of congruent segments

Use the values of the segments shown on the figure to find x.

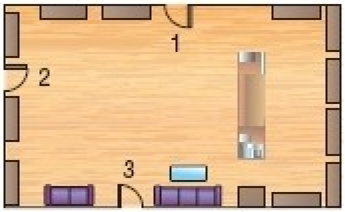
$4x = 2x + 10$	Substitution
$2x = 10$	Subtract $2x$ from each side.
$x = 5$	Divide each side by 2.

Therefore, the value of x is 5.

ANSWER:

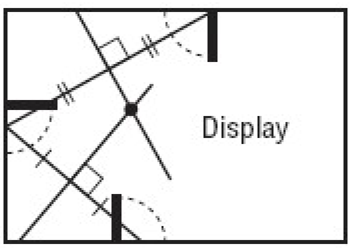
5

34. **ADVERTISING** A bookstore clerk wants to set up a display of new books. If there are three entrances into the store as shown in the figure at the right, where should the display be to get maximum exposure?

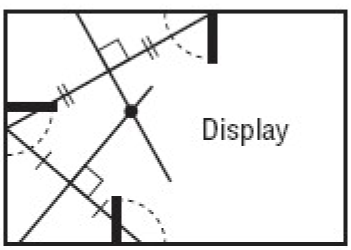


SOLUTION:

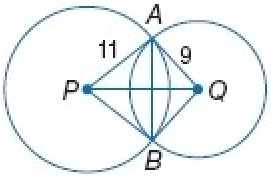
The display should be placed at the incenter of the triangle having the three entrances as vertices. Draw two segments connecting the centers of the three entrances. Next, construct the perpendicular bisector of each segment and mark the point of intersection. The display should be placed at this point.



ANSWER:



35. **CHALLENGE** The common chord $\overline{AB}$ between $\odot P$ and $\odot Q$ is perpendicular to the segment connecting the centers of the circles. If $AB = 10$, what is the length of $\overline{PQ}$? Explain your reasoning.



SOLUTION:

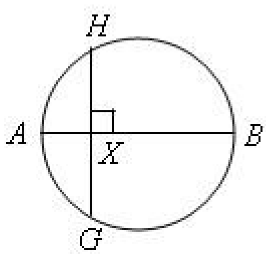
Here, P and Q are equidistant from the endpoints of $\overline{AB}$ so they both lie on the perpendicular bisector of $\overline{AB}$, so $\overline{PQ}$ is the perpendicular bisector of $\overline{AB}$. Let S be the point of intersection of $\overline{AB}$ and $\overline{PQ}$. Hence, $PS = QS = 5$. Since $\overline{PS}$ is perpendicular to chord $\overline{AB}$, $\angle PSA$ is a right angle. So, ΔPSA is a right triangle. By the Pythagorean Theorem, $PS = \sqrt{(PA)^2 - (AS)^2}$. By substitution, $PS = \sqrt{11^2 - 5^2}$ or $\sqrt{96}$. Similarly, ΔASQ is a right triangle with $SQ = \sqrt{(AQ)^2 - (AS)^2} = \sqrt{9^2 - 5^2}$ or $\sqrt{56}$. Since $PQ = PS + SQ$, $PQ = \sqrt{96} + \sqrt{56}$ or about 17.3.

ANSWER:

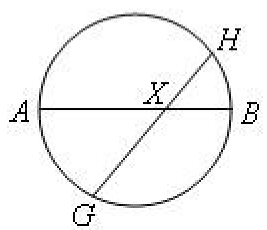
About 17.3; P and Q are equidistant from the endpoints of $\overline{AB}$ so they both lie on the perpendicular bisector of $\overline{AB}$, so $\overline{PQ}$ is the perpendicular bisector of $\overline{AB}$. Let S be the point of intersection of $\overline{AB}$ and $\overline{PQ}$. Hence, $PS = QS = 5$. Since $\overline{PS}$ is perpendicular to chord $\overline{AB}$, $\angle PSA$ is a right angle. So, ΔPSA is a right triangle. By the Pythagorean Theorem, $PS = \sqrt{(PA)^2 - (AS)^2}$. By substitution, $PS = \sqrt{11^2 - 5^2}$ or $\sqrt{96}$. Similarly, ΔASQ is a right triangle with $SQ = \sqrt{(AQ)^2 - (AS)^2} = \sqrt{9^2 - 5^2}$ or $\sqrt{56}$. Since $PQ = PS + SQ$, $PQ = \sqrt{96} + \sqrt{56}$ or about 17.3.

36. **REASONING** In a circle, $\overline{AB}$ is a diameter and $\overline{HG}$ is a chord that intersects $\overline{AB}$ at point X. Is it *sometimes*, *always*, or *never* true that $HX = GX$? Explain.

SOLUTION:



$\overline{HX} \perp \overline{AB}$, $HX = GX$



$\overline{HX}$ is not perpendicular to $\overline{AB}$, $HX \neq GX$

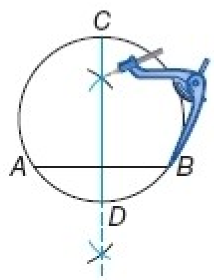
If the diameter is perpendicular to the chord, then it bisects the chord. Therefore, the statement is *sometimes* true.

ANSWER:

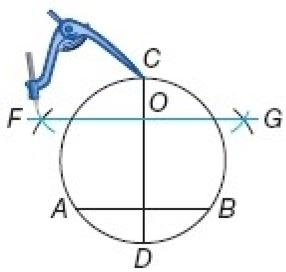
Sometimes; if the diameter is perpendicular to the chord, then it bisects the chord.

37. **CHALLENGE** Use a compass to draw a circle with chord $\overline{AB}$. Refer to this construction for the following problem.

Step 1 Construct $\overline{CD}$, the perpendicular bisector of $\overline{AB}$.



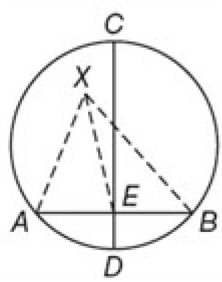
Step 2 Construct $\overline{FG}$, the perpendicular bisector of $\overline{CD}$. Label the point of intersection *O*.



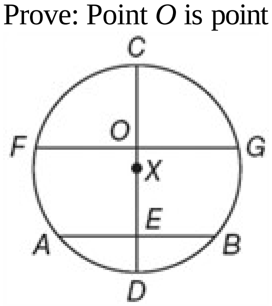
a. Use an indirect proof to show that $\overline{CD}$ passes through the center of the circle by assuming that the center of the circle is not on $\overline{CD}$.
b. Prove that *O* is the center of the circle.

SOLUTION:

a. Given: $\overline{CD}$ is the perpendicular bisector of chord $\overline{AB}$ in $\odot X$.
Prove: $\overline{CD}$ contains point *X*.



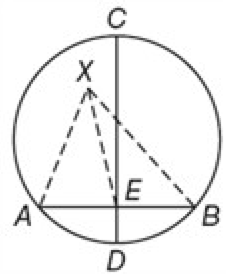
Proof: Suppose *X* is not on $\overline{CD}$.Draw $\overline{XE}$ and radii $\overline{XA}$ and $\overline{XB}$. Since $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$, *E* is the midpoint of $\overline{AB}$ and $\overline{AE} \cong \overline{EB}$. Also, $\overline{XA} \cong \overline{XB}$, since all radii of a $\odot$ are $\cong$. $\overline{XE} \cong \overline{XE}$ by the Reflexive Property. So, $\Delta AXE \cong \Delta BXE$ by SSS. By CPCTC, $\angle XEA \cong \angle XEB$. Since they also form a linear pair, $\angle XEA$ and $\angle XEB$ are right angles.So, $\overline{XE} \perp \overline{AB}$. By definition, $\overline{XE}$ is the perpendicular bisector of $\overline{AB}$. But $\overline{CD}$ is also the perpendicular bisector of $\overline{AB}$. This contradicts the uniqueness of a perpendicular bisector of a segment. Thus, the assumption is false, and center *X* must be on $\overline{CD}$.
b. Given: In $\odot X$, *X* is on $\overline{CD}$ and $\overline{FG}$ bisects $\overline{CD}$ at *O*.
Prove: Point *O* is point *X*.



Proof:
Since point X is on $\overline{CD}$ and C and D are on $\odot X$, $\overline{CD}$ is a diameter of $\odot X$. Since $\overline{FG}$ bisects $\overline{CD}$ at O, O is the midpoint of $\overline{CD}$. Since the midpoint of a diameter is the center of a circle, O, is the center of the circle. Therefore, point O is point X.

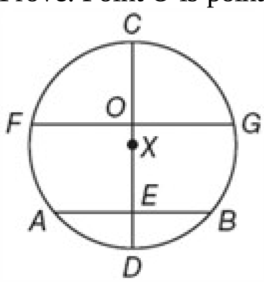
ANSWER:

a. Given: $\overline{CD}$ is the perpendicular bisector of chord $\overline{AB}$ in $\odot X$.
Prove: $\overline{CD}$ contains point X.



Proof: Suppose X is not on $\overline{CD}$. Draw $\overline{XE}$ and radii $\overline{XA}$ and $\overline{XB}$. Since $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$, E is the midpoint of $\overline{AB}$ and $\overline{AE} \cong \overline{EB}$. Also, $\overline{XA} \cong \overline{XB}$, since all radii of a $\odot$ are $\cong$. $\overline{XE} \cong \overline{XE}$ by the Reflexive Property. So, $\Delta AXE \cong \Delta BXE$ by SSS. By CPCTC, $\angle XEA \cong \angle XEB$. Since they also form a linear pair, $\angle XEA$ and $\angle XEB$ are right angles. So, $\overline{XE} \perp \overline{AB}$. By definition, $\overline{XE}$ is the perpendicular bisector of $\overline{AB}$. But $\overline{CD}$ is also the perpendicular bisector of $\overline{AB}$. This contradicts the uniqueness of a perpendicular bisector of a segment. Thus, the assumption is false, and center X must be on $\overline{CD}$.

b. Given: In $\odot X$, X is on $\overline{CD}$ and $\overline{FG}$ bisects $\overline{CD}$ at O.
Prove: Point O is point X.

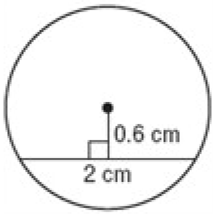


Proof:
Since point X is on $\overline{CD}$ and C and D are on $\odot X$, $\overline{CD}$ is a diameter of $\odot X$. Since $\overline{FG}$ bisects $\overline{CD}$ at O, O is the midpoint of $\overline{CD}$. Since the midpoint of a diameter is the center of a circle, O, is the center of the circle. Therefore, point O is point X.

38. **OPEN ENDED** Construct a circle and draw a chord. Measure the chord and the distance that the chord is from the center. Find the length of the radius.

SOLUTION:

Sample answer:



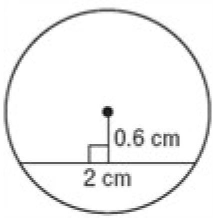
Draw the radius from the center to one end of the chord to create a right triangle. Since the distance is a perpendicular from the center point, it will bisect the chord. The two legs of the triangle will measure 0.6 cm and 1 cm. Use the Pythagorean Theorem to find the radius.

$r = \sqrt{1^2 + 0.6^2}$ or about 1.2.

Therefore, the radius $\approx$ 1.2 cm.

ANSWER:

Sample answer:

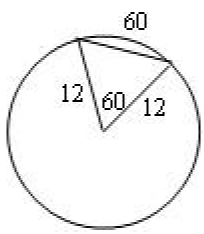


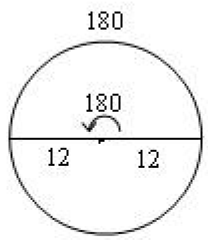
radius $\approx$ 1.2 cm

39. **WRITING IN MATH** If the measure of an arc in a circle is tripled, will the chord of the new arc be three times as long as the chord of the original arc? Explain your reasoning.

SOLUTION:

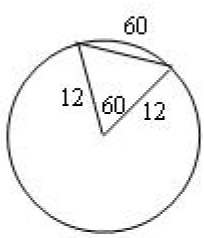
No; sample answer: In a circle with a radius of 12, an arc with a measure of 60 determines a chord of length 12. (The triangle related to a central angle of 60 is equilateral.) If the measure of the arc is tripled to 180, then the chord determined by the arc is a diameter and has a length of 2(12) or 24, which is not three times as long as the original chord.

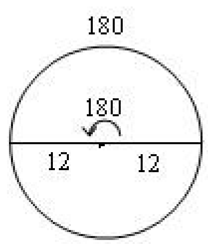




ANSWER:

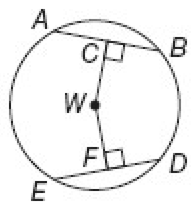
No; sample answer: In a circle with a radius of 12, an arc with a measure of 60 determines a chord of length 12. (The triangle related to a central angle of 60 is equilateral.) If the measure of the arc is tripled to 180, then the chord determined by the arc is a diameter and has a length of 2(12) or 24, which is not three times as long as the original chord.





40. If $CW = WF$ and $ED = 30$, what is DF?

A 60
B 45
C 30
D 15



SOLUTION:

If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So,

$DF = \frac{1}{2}(ED) = 15.$

Therefore, the correct choice is D.

ANSWER:

D

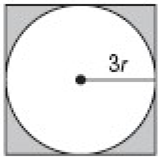
41. **ALGEBRA** Write the ratio of the area of the circle to the area of the square in simplest form.

F $\frac{\pi}{4}$

G $\frac{\pi}{2}$

H $\frac{3\pi}{4}$

J π



SOLUTION:

The radius of the circle is 3*r* and the length of each side of the square is 2(3*r*) or 6*r*.

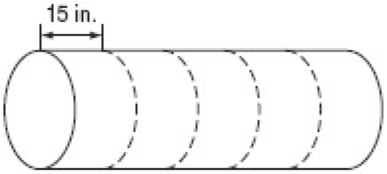
$$\begin{aligned} \frac{A_{\text{circle}}}{A_{\text{square}}} &= \frac{\pi r^2}{s^2} && \text{Area formulas} \\ &= \frac{\pi(3r)^2}{(6r)^2} && \text{Substitution} \\ &= \frac{9\pi r^2}{36r^2} && \text{Simplify.} \\ &= \frac{\pi}{4} && \text{Simplify.} \end{aligned}$$

Therefore, the correct choice is F.

ANSWER:

F

42. **SHORT RESPONSE** The pipe shown is divided into five equal sections. How long is the pipe in feet (ft) and inches (in.)?



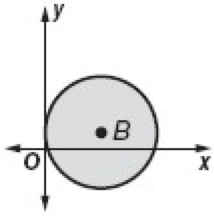
SOLUTION:

The length of each section is 15 inches. So, the total length of the pipe is 5(15) = 75 in. We have, 12 inches = 1 foot. Therefore, 75 in. = 6 ft 3in. or 6.25 ft.

ANSWER:

6 ft 3 in.

43. **SAT/ACT** Point B is the center of a circle tangent to the y-axis and the coordinates of Point B are (3, 1). What is the area of the circle?



A π units^2
B 3π units^2
C 4π units^2
D 6π units^2
E 9π units^2

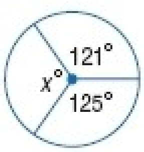
SOLUTION:
Since the coordinates of B are (3, 1) and the y-axis is a tangent to the circle, the radius of the circle is 3 units.

$$\begin{aligned} A &= \pi r^2 && \text{Formula for area of circle} \\ &= \pi(3)^2 && r = 3 \\ &= 9\pi && \text{Simplify.} \end{aligned}$$

The area of the circle is 9π units^2.
Therefore, the correct choice is E.

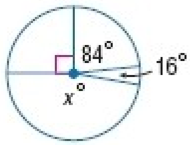
ANSWER:
E

Find *x*.

44. 

SOLUTION:
The sum of the measures of the central angles of a circle with no interior points in common is 360. So,
$121 + 125 + x = 360.$
$246 + x = 360$
$x = 114$

ANSWER:
114

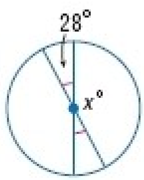
45. 

SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360. So,
$90 + 84 + 16 + x = 360.$
$190 + x = 360$
$x = 170$

ANSWER:

170

46. 

SOLUTION:

The sum of the measures of the central angles of a circle with no interior points in common is 360. So,
$28 + 28 + x + x = 360.$
$56 + 2x = 360$
$x = 152$

ANSWER:

152

47. **CRAFTS** Ruby created a pattern to sew flowers onto a quilt by first drawing a regular pentagon that was 3.5 inches long on each side. Then she added a semicircle onto each side of the pentagon to create the appearance of five petals. How many inches of gold trim does she need to edge 10 flowers? Round to the nearest inch.

SOLUTION:

The diameter of each semi circle is 3.5 inches. The circumference C of a circle of a circle of diameter d is given by $C = \pi d.$

Since the pattern contains semi circles, the circumference of each semi circle is $\frac{1}{2}\pi d.$

The total number of semi circles in 10 flowers is 10(5) = 50. So, the total circumference is $50\left(\frac{1}{2}\pi d\right) = 25\pi d.$

Substitute for d.
$25\pi(3.5) \approx 275$
Therefore, she will need about 275 inches of gold trim for the purpose.

ANSWER:

275 in.

Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *obtuse*, or *right*. Justify your answer.

48. 8, 15, 17

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$8+15>17\checkmark$

$15+17>8\checkmark$

$8+17>15\checkmark$

Therefore, the set of numbers can be measures of a triangle.
Classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$17^2\stackrel{?}{=}15^2+8^2$

$289\stackrel{?}{=}225+64$

$289=289\checkmark$

Therefore, by the converse of Pythagorean Theorem, a triangle with the given measures will be a right triangle.

ANSWER:

yes; right

$17^2\stackrel{?}{=}8^2+15^2$

$289=64+225$

49. 20, 21, 31

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$20+21>31\checkmark$

$21+31>20\checkmark$

$20+31>21\checkmark$

Therefore, the set of numbers can be measures of a triangle.
Now, classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$31^2\stackrel{?}{=}20^2+21^2$

$961\stackrel{?}{=}400+441$

$961>841$

Therefore, by Pythagorean Inequality Theorem, a triangle with the given measures will be an obtuse triangle.

ANSWER:

yes; obtuse

$31^2\stackrel{?}{=}20^2+21^2$

$961>400+441$

50. 10, 16, 18

SOLUTION:

By the triangle inequality theorem, the sum of the lengths of any two sides should be greater than the length of the third side.

$10 + 16 > 18$ ✓

$16 + 18 > 10$ ✓

$10 + 18 > 16$ ✓

Therefore, the set of numbers can be measures of a triangle.
Now, classify the triangle by comparing the square of the longest side to the sum of the squares of the other two sides.

$18^2 \stackrel{?}{=} 16^2 + 10^2$

$324 \stackrel{?}{=} 256 + 100$

$324 < 356$

Therefore, by Pythagorean Inequality Theorem, a triangle with the given measures will be an acute triangle.

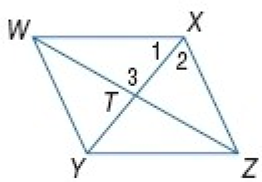
ANSWER:

yes; acute

$18^2 \stackrel{?}{=} 10^2 + 16^2$

$324 < 100 + 256$

ALGEBRA Quadrilateral *WXZY* is a rhombus. Find each value or measure.



51. If $m\angle 3 = y^2 - 31$, find y.

SOLUTION:

The diagonals of a rhombus are perpendicular to each other. So, $m\angle 3 = 90$.

$y^2 - 31 = 90$ Substitution

$y^2 = 121$ Add 31 to each side.

$y = \pm 11$ Take the square root of each side.

ANSWER:

± 11

52. If $m\angle XZY = 56$, find $m\angle YWZ$.

SOLUTION:

The opposite angles of a rhombus are congruent. So, $m\angle YWX = 56$. In a rhombus, each diagonal bisects a pair of opposite angles.

$$\begin{aligned} m\angle YWZ &= \frac{1}{2}(m\angle YWX) && \text{Definition of angle bisector} \\ &= \frac{1}{2}(56) && \text{Substitution.} \\ &= 28 && \text{Simplify.} \end{aligned}$$

ANSWER:

28