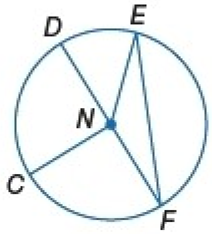
10-1 Circles and Circumference

For Exercises 1–4, refer to $\odot N$.



1. Name the circle.

SOLUTION:

The center of the circle is N. So, the circle is $\odot N$.

ANSWER:

$\odot N$

2. Identify each.

a. a chord
b. a diameter
c. a radius

SOLUTION:

a. A chord is a segment with endpoints on the circle. So, here $\overline{EF}$, $\overline{DF}$ are chords.
b. A diameter of a circle is a chord that passes through the center. Here, $\overline{DF}$ is a diameter.
c. A radius is a segment with endpoints at the center and on the circle. Here, $\overline{NC}$, $\overline{ND}$, $\overline{NE}$, or $\overline{NF}$ is radius.

ANSWER:

a. $\overline{EF}$, $\overline{DF}$
b. $\overline{DF}$
c. $\overline{NC}$, $\overline{ND}$, $\overline{NE}$, or $\overline{NF}$

3. If CN = 8 centimeters, find DN.

SOLUTION:

Here, $\overline{CN}$ and $\overline{DN}$ are radii of the same circle. So, they are equal in length. Therefore, DN = 8 cm.

ANSWER:

8 cm

4. If EN = 13 feet, what is the diameter of the circle?

SOLUTION:

Here, $\overline{EN}$ is a radius and the diameter is twice the radius.

$d = 2r$ Diameter Formula

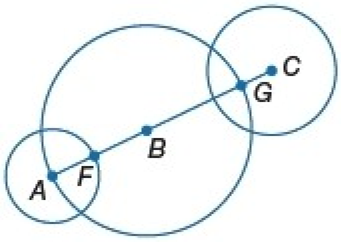
$d = 2(13)$ or 26 Substitute and simplify.

Therefore, the diameter is 26 ft.

ANSWER:

26 ft

The diameters of $\odot A$, $\odot B$, and $\odot C$ are 8 inches, 18 inches, and 11 inches, respectively. Find each measure.



5. *FG*

SOLUTION:

Since the diameter of $\odot B$ is 18 inches and $\odot A$ is 8 inches, AG = 18 and $AF = \frac{1}{2}(8)$ or 4.

$$\begin{aligned} AF + FG &= FG && \text{Segment Addition Postulate} \\ 4 + FG &= 18 && \text{Substitution} \\ FG &= 18 - 4 && \text{Subtract 4 from each side.} \\ FG &= 14 && \text{Simplify.} \end{aligned}$$

Therefore, FG = 14 in.

ANSWER:

14 in.

6. *FB*

SOLUTION:

Since the diameter of $\odot B$ is 18 inches and $\odot A$ is 4 inches, $AB = \frac{1}{2}(18)$ or 9 and $AF = \frac{1}{2}(8)$ or 4.

$$\begin{aligned} AF + FB &= AB && \text{Segment Addition Postulate} \\ 4 + FB &= 9 && \text{Substitution} \\ FB &= 9 - 4 && \text{Subtract 4 from each side.} \\ FB &= 5 && \text{Simplify.} \end{aligned}$$

Therefore, FB = 5 inches.

ANSWER:

5 in.

7. **RIDES** The circular ride described at the beginning of the lesson has a diameter of 44 feet. What are the radius and circumference of the ride? Round to the nearest hundredth, if necessary.

SOLUTION:

The radius is half the diameter. So, the radius of the circular ride is 22 feet.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ &= \pi(44) && \text{Substitution} \\ &= 44\pi && \text{Simplify.} \\ &\approx 138.23 && \text{Use a calculator.} \end{aligned}$$

Therefore, the circumference of the ride is about 138.23 feet.

ANSWER:

22 ft; 138.23 ft

8. **CCSS MODELING** The circumference of the circular swimming pool shown is about 56.5 feet. What are the diameter and radius of the pool? Round to the nearest hundredth.

SOLUTION:

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ 56.5 &= \pi d && \text{Substitution} \\ \frac{56.5}{\pi} &= d && \text{Divide each side by } \pi. \\ 17.98 &\approx d && \text{Use a calculator.} \end{aligned}$$

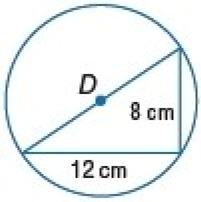
$$\begin{aligned} r &= \frac{1}{2}d && \text{Radius Formula} \\ &= \frac{1}{2}(17.98) && d \approx 17.98 \\ &\approx 8.99 && \text{Use a calculator.} \end{aligned}$$

The diameter of the pool is about 17.98 feet and the radius of the pool is about 8.99 feet.

ANSWER:

17.98 ft; 8.99 ft

9. **SHORT RESPONSE** The right triangle shown is inscribed in $\odot D$. Find the exact circumference of $\odot D$.



SOLUTION:

The diameter of the circle is the hypotenuse of the right triangle.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$12^2 + 8^2 = c^2$ Substitution

$208 = c^2$ Simplify.

$4\sqrt{13} = c$ Take the positive square root of each side.

The diameter of the circle is $4\sqrt{13}$ centimeters.

$C = \pi d$ Circumference Formula

$= \pi\left(4\sqrt{13}\right)$ $d = 4\sqrt{13}$

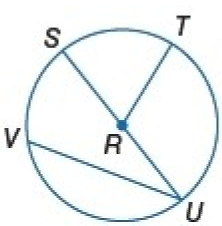
$= 4\pi\sqrt{13}$ Simplify.

The circumference of the circle is $4\pi\sqrt{13}$ centimeters.

ANSWER:

$4\pi\sqrt{13}$ cm

For Exercises 10–13, refer to $\odot R$.



10. Name the center of the circle.

SOLUTION:

R

ANSWER:

R

11. Identify a chord that is also a diameter.

SOLUTION:

Two chords are shown: $\overline{SU}$ and $\overline{VU}$. $\overline{SU}$ goes through the center, *R*, so $\overline{SU}$ is a diameter.

ANSWER:

$\overline{SU}$

12. Is $\overline{VU}$ a radius? Explain.

SOLUTION:

A radius is a segment with endpoints at the center and on the circle. But $\overline{VU}$ has both the end points on the circle, so it is a chord.

ANSWER:

No; it is a chord.

13. If *SU* = 16.2 centimeters, what is *RT*?

SOLUTION:

Here, $\overline{SU}$ is a diameter and $\overline{RT}$ is a radius.

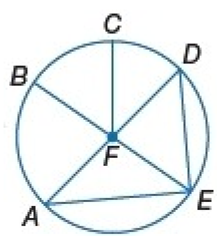
$$\begin{aligned} r &= \frac{1}{2}d && \text{Radius Formula} \\ &= \frac{1}{2}(16.2) && d = 16.2 \\ &= 8.1 && \text{Use a calculator.} \end{aligned}$$

Therefore, *RT* = 8.1 cm.

ANSWER:

8.1 cm

For Exercises 14–17, refer to $\odot F$.



14. Identify a chord that is not a diameter.

SOLUTION:

The chords $\overline{DE}$ and $\overline{AE}$ do not pass through the center. So, they are not diameters.

ANSWER:

$\overline{DE}$, $\overline{AE}$

15. If *CF* = 14 inches, what is the diameter of the circle?

SOLUTION:

Here, $\overline{CF}$ is a radius and the diameter is twice the radius. .

$$\begin{aligned} d &= 2r && \text{Diameter Formula} \\ &= 2(14) && r = 14 \\ &= 28 && \text{Use a calculator.} \end{aligned}$$

Therefore, the diameter of the circle is 28 inches.

ANSWER:

28 in.

16. Is $\overline{AF} \cong \overline{EF}$? Explain.

SOLUTION:

All radii of a circle are congruent. Since $\overline{AF}$ and $\overline{EF}$ are both radii of $\odot F$, $\overline{AF} \cong \overline{EF}$.

ANSWER:

Yes; they are both radii of $\odot F$.

17. If *DA* = 7.4 centimeters, what is *EF*?

SOLUTION:

Here, $\overline{DA}$ is a diameter and $\overline{EF}$ is a radius.

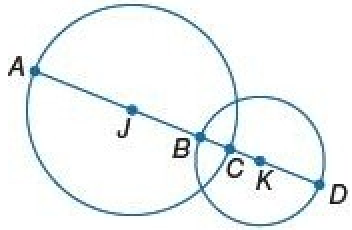
$$\begin{aligned} r &= \frac{1}{2}d && \text{Radius Formula} \\ &= \frac{1}{2}(7.4) && d = 7.4 \\ &= 3.7 && \text{Use a calculator.} \end{aligned}$$

Therefore, *EF* = 3.7 cm.

ANSWER:

3.7 cm

Circle *J* has a radius of 10 units, $\odot K$ has a radius of 8 units, and *BC* = 5.4 units. Find each measure.



18. *CK*

SOLUTION:

Since the radius of $\odot K$ is 8 units, *BK* = 8.

$$\begin{aligned} CK + BC &= BK && \text{Segment Addition Postulate} \\ CK + 5.4 &= 8 && \text{Substitution} \\ CK &= 2.6 && \text{Subtract 5.4 from each side.} \end{aligned}$$

Therefore, *CK* = 2.6 units.

ANSWER:

2.6

19. *AB*

SOLUTION:

Since $\overline{AC}$ is a diameter of circle *J* and the radius is 10, *AC* = 2(10) or 20 units.

$$\begin{aligned} AB + BC &= AC && \text{Segment Addition Postulate} \\ AB + 5.4 &= 20 && \text{Substitution} \\ AB &= 14.6 && \text{Subtract 5.4 from each side.} \end{aligned}$$

Therefore, *AB* = 14.6 units.

ANSWER:

14.6

20. *JK*

SOLUTION:

First find *CK*. Since circle *K* has a radius of 8 units, *BK* = 8.

$$\begin{aligned} CK + BC &= BK && \text{Segment Addition Postulate} \\ CK + 5.4 &= 8 && \text{Substitution} \\ CK &= 2.6 && \text{Subtract 5.4 from each side.} \end{aligned}$$

Since circle *J* has a radius of 10 units, *JC* = 10.

$$\begin{aligned} JK &= JC + CK && \text{Segment Addition Postulate} \\ JK &= 10 + 2.6 && \text{Substitution} \\ JK &= 12.6 && \text{Simplify.} \end{aligned}$$

Therefore, *JK* = 12.6 units.

ANSWER:

12.6

21. *AD*

SOLUTION:

We can find *AD* using *AD* = *AC* + *CK* + *KD*. The radius of circle *K* is 8 units and the radius of circle *J* is 10 units, so *KD* = 8 and *AC* = 2(10) or 20. Before finding *AD*, we need to find *CK*.

$CK + BC = BK$ Segment Addition Postulate

$CK + 5.4 = 8$ Substitution

$CK = 2.6$ Subtract 5.4 from each side.

$AD = AC + CK + KD$ Segment Addition Postulate

$AD = 20 + 2.6 + 8$ Substitution

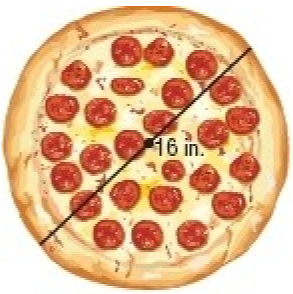
$AD = 30.6$ Simplify.

Therefore, *AD* = 30.6 units.

ANSWER:

30.6

22. **PIZZA** Find the radius and circumference of the pizza shown. Round to the nearest hundredth, if necessary.



SOLUTION:

The diameter of the pizza is 16 inches.

$r = \frac{1}{2}d$ Radius Formula

$= \frac{1}{2}(16)$ Substitution

$= 8$ Simplify.

So, the radius of the pizza is 8 inches.

The circumference *C* of a circle of diameter *d* is given by $C = \pi d$.

$C = \pi d$ Circumference Formula

$= \pi(16)$ Substitution

≈ 50.27 Use a calculator.

Therefore, the circumference of the pizza is about 50.27 inches.

ANSWER:

8 in.; 50.27 in.

23. **BICYCLES** A bicycle has tires with a diameter of 26 inches. Find the radius and circumference of a tire. Round to the nearest hundredth, if necessary.

SOLUTION:

The diameter of a tire is 26 inches. The radius is half the diameter.

$$\begin{aligned} r &= \frac{1}{2}d && \text{Radius Formula} \\ &= \frac{1}{2}(26) && \text{Substitution} \\ &= 13 && \text{Simplify.} \end{aligned}$$

So, the radius of the tires is 13 inches.

The circumference C of a circle with diameter d is given by $C = \pi d$.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ &= \pi(26) && \text{Substitution} \\ &\approx 81.68 && \text{Use a calculator.} \end{aligned}$$

Therefore, the circumference of the bicycle tire is about 81.68 inches.

ANSWER:

13 in.; 81.68 in.

Find the diameter and radius of a circle by with the given circumference. Round to the nearest hundredth.

24. C = 18 in.

SOLUTION:

The circumference C of a circle with diameter d is given by $C = \pi d$.

Here, C = 18 in. Use the formula to find the diameter. Then find the radius.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ 18 &= \pi d && \text{Substitution} \\ \frac{18}{\pi} &= d && \text{Divide each side by } \pi. \\ 5.73 &\approx d && \text{Use a calculator.} \end{aligned}$$

$$\begin{aligned} r &= \frac{1}{2}d && \text{Radius Formula} \\ &= \frac{1}{2}(5.73) && \text{Substitution} \\ &\approx 2.86 && \text{Use a calculator.} \end{aligned}$$

Therefore, the diameter is about 5.73 inches and the radius is about 2.86 inches.

ANSWER:

5.73 in.; 2.86 in.

10-1 Circles and Circumference

25. C = 124 ft

SOLUTION:

The circumference C of a circle with diameter d is given by $C = \pi d$.
Here, C = 124 ft. Use the formula to find the diameter. Then find the radius.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ 124 &= \pi d && \text{Substitution} \\ \frac{124}{\pi} &= d && \text{Divide each side by } \pi. \\ 39.47 &\approx d && \text{Use a calculator.} \end{aligned}$$

$$\begin{aligned} r &= \frac{1}{2}d && \text{Radius Formula} \\ &= \frac{1}{2}(39.47) && \text{Substitution} \\ &\approx 19.74 && \text{Use a calculator.} \end{aligned}$$

Therefore, the diameter is about 39.47 feet and the radius is about 19.74 feet.

ANSWER:

39.47 ft; 19.74 ft

26. C = 375.3 cm

SOLUTION:

The circumference C of a circle with diameter d is given by $C = \pi d$.
Here, C = 375.3 cm. Use the formula to find the diameter. Then find the radius.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ 375.3 &= \pi d && \text{Substitution} \\ \frac{375.3}{\pi} &= d && \text{Divide each side by } \pi. \\ 119.46 &\approx d && \text{Use a calculator.} \end{aligned}$$

$$\begin{aligned} r &= \frac{1}{2}d && \text{Radius Formula} \\ &= \frac{1}{2}(119.46) && \text{Substitution} \\ &\approx 59.73 && \text{Use a calculator.} \end{aligned}$$

Therefore, the diameter is about 119.46 centimeters and the radius is about 59.73 centimeters.

ANSWER:

119.46 cm; 59.73 cm

27. C = 2608.25 m

SOLUTION:

The circumference C of a circle with diameter d is given by $C = \pi d$.
Here, C = 2608.25 meters. Use the formula to find the diameter. Then find the radius.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ 2608.25 &= \pi d && \text{Substitution} \\ \frac{2608.25}{\pi} &= d && \text{Divide each side by } \pi. \\ 830.23 &\approx d && \text{Use a calculator.} \end{aligned}$$

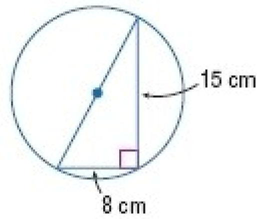
$$\begin{aligned} r &= \frac{1}{2}d && \text{Radius Formula} \\ &= \frac{1}{2}(830.23) && \text{Substitution} \\ &\approx 415.12 && \text{Use a calculator.} \end{aligned}$$

Therefore, the diameter is about 830.23 meters and the radius is about 415.12 meters.

ANSWER:

830.23 m; 415.12 m

CCSS SENSE-MAKING Find the exact circumference of each circle by using the given inscribed or circumscribed polygon.

28. 

SOLUTION:

The hypotenuse of the right triangle is a diameter of the circle. Use the Pythagorean Theorem to find the diameter.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 8^2 + 15^2$ Substitution

$c^2 = 289$ Simplify.

$c = 17$ Take the positive square root of each side.

The diameter of the circle is 17 cm. The circumference *C* of a circle is given by $C = \pi d$.

$C = \pi d$ Circumference Formula

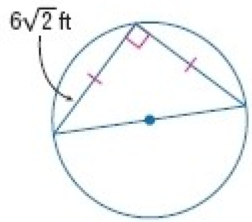
$= \pi(17)$ Substitution

$= 17\pi$ Simplify.

Therefore, the circumference of the circle is 17π centimeters.

ANSWER:

17π cm

29. 

SOLUTION:

The hypotenuse of the right triangle is a diameter of the circle. Use the Pythagorean Theorem to find the diameter.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = (6\sqrt{2})^2 + (6\sqrt{2})^2$ Substitution

$c^2 = 144$ Simplify.

$c = 12$ Take the positive square root of each side.

The diameter of the circle is 12 feet. The circumference *C* of a circle is given by $C = \pi d$.

$C = \pi d$ Circumference Formula

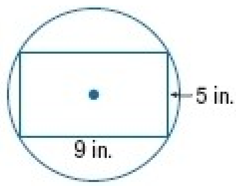
$= \pi(12)$ Substitution

$= 12\pi$ Simplify.

Therefore, the circumference of the circle is 12π feet.

ANSWER:

12π ft

30. 

SOLUTION:

Each diagonal of the inscribed rectangle will pass through the origin, so it is a diameter of the circle. Use the Pythagorean Theorem to find the diameter of the circle.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 9^2 + 5^2$ Substitution

$c^2 = 106$ Simplify.

$c = \sqrt{106}$ Take the positive square root of each side.

The diameter of the circle is $\sqrt{106}$ inches. The circumference *C* of a circle is given by $C = \pi d$.

$C = \pi d$ Circumference Formula

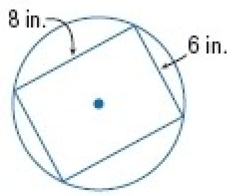
$= \pi\left(\sqrt{106}\right)$ Substitution

$= \sqrt{106}\,\pi$ Simplify.

Therefore, the circumference of the circle is $\sqrt{106}\,\pi$ inches.

ANSWER:

$\sqrt{106}\,\pi$ in.

31. 

SOLUTION:

Each diagonal of the inscribed rectangle will pass through the origin, so it is a diameter of the circle.
Use the Pythagorean Theorem to find the diameter of the circle.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 8^2 + 6^2$ Substitution

$c^2 = 100$ Simplify.

$c = 10$ Take the positive square root of each side.

The diameter of the circle is 10 inches. The circumference *C* of a circle is given by $C = \pi d$.

$C = \pi d$ Circumference Formula

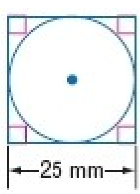
$= \pi(10)$ Substitution

$= 10\pi$ Simplify.

Therefore, the circumference of the circle is 10π inches.

ANSWER:

10π in.

32. 

SOLUTION:

A diameter perpendicular to a side of the square and the length of each side will measure the same. So, *d* = 25 millimeters.
The circumference *C* of a circle is given by $C = \pi d$.

$C = \pi d$ Circumference Formula

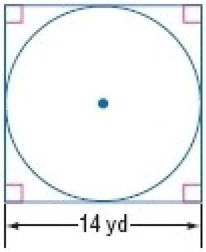
$= \pi(25)$ Substitution

$= 25\pi$ Simplify.

Therefore, the circumference of the circle is 25π millimeters.

ANSWER:

25π mm

33. 

SOLUTION:

A diameter perpendicular to a side of the square and the length of each side will measure the same. So, d = 14 yards.

The circumference C of a circle is given by $C = \pi d$.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ &= \pi(14) && \text{Substitution} \\ &= 14\pi && \text{Simplify.} \end{aligned}$$

Therefore, the circumference of the circle is 14π yards.

ANSWER:

14π yd

34. **DISC GOLF** Disc golf is similar to regular golf, except that a flying disc is used instead of a ball and clubs. For professional competitions, the maximum weight of a disc in grams is 8.3 times its diameter in centimeters. What is the maximum allowable weight for a disc with circumference 66.92 centimeters? Round to the nearest tenth.

SOLUTION:

The circumference C of a circle with diameter d is given by $C = \pi d$.

Here, C = 66.92 cm. Use the formula to find the diameter.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ 66.92 &= \pi d && \text{Substitution} \\ \frac{66.92}{\pi} &= d && \text{Divide each side by } \pi. \\ 21.3 &\approx d && \text{Use a calculator.} \end{aligned}$$

The diameter of the disc is about 21.3 centimeters and the maximum weight allowed is 8.3 times the diameter in centimeters.

Therefore, the maximum weight is 8.3 × 21.3 or about 176.8 grams.

ANSWER:

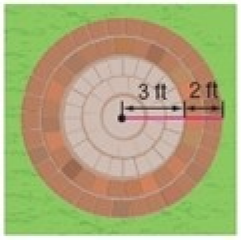
176.8 g

10-1 Circles and Circumference

35. **PATIOS** Mr. Martinez is going to build the patio shown.

a. What is the patio's approximate circumference?

b. If Mr. Martinez changes the plans so that the inner circle has a circumference of approximately 25 feet, what should the radius of the circle be to the nearest foot?



SOLUTION:

a. The radius of the patio is 3 + 2 = 5 ft. The circumference C of a circle with radius r is given by $C = 2\pi r$. Use the formula to find the circumference.

$$\begin{aligned} C &= 2\pi r && \text{Circumference Formula} \\ &= 2\pi(5) && \text{Substitution} \\ &\approx 31.42 && \text{Use a calculator.} \end{aligned}$$

Therefore, the circumference is about 31.42 feet.

b. Here, C_{inner} = 25 ft. Use the formula to find the radius of the inner circle.

$$\begin{aligned} C &= 2\pi r && \text{Circumference Formula} \\ 25 &= 2\pi r && \text{Substitution} \\ \frac{25}{2\pi} &= r && \text{Divide each side by } 2\pi. \\ 4.0 &\approx r && \text{Use a calculator.} \end{aligned}$$

So, the radius of the inner circle is about 4 feet.

ANSWER:

a. 31.42 ft

b. 4 ft

The radius, diameter, or circumference of a circle is given. Find each missing measure to the nearest hundredth.

36. $d = 8\frac{1}{2}$ in., $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$

SOLUTION:

The radius is half the diameter and the circumference C of a circle with diameter d is given by $C = \pi d$.

$$\begin{aligned} r &= \frac{1}{2}d && \text{Radius Formula} \\ &= \frac{1}{2}(8.5) && d = 8\tfrac{1}{2} \text{ or } 8.5 \\ &= 4.25 && \text{Use a calculator.} \end{aligned}$$

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ &= \pi(8.5) && \text{Substitution} \\ &\approx 26.70 && \text{Use a calculator.} \end{aligned}$$

Therefore, the radius is 4.25 inches and the circumference is about 26.70 inches.

ANSWER:

4.25 in.; 26.70 in.

37. $r = 11\frac{2}{5}$ ft, $d =$ __?__, $C =$ __?__

SOLUTION:

The diameter is twice the radius and the circumference C of a circle with diameter d is given by $C = \pi d$.

$$\begin{aligned} d &= 2r && \text{Diameter Formula} \\ &= 2(11.4) && r = 11\tfrac{2}{5} \text{ or } 11.4 \\ &= 22.80 && \text{Use a calculator.} \end{aligned}$$

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ &= \pi(22.80) && \text{Substitution} \\ &\approx 71.63 && \text{Use a calculator.} \end{aligned}$$

So, the diameter is 22.80 feet and the circumference is about 71.63 feet.

ANSWER:

22.80 ft; 71.63 ft

38. $C = 35x$ cm, $d =$ __?__, $r =$ __?__

SOLUTION:

The circumference C of a circle with diameter d is given by $C = \pi d$.
Here, $C = 35x$ cm. Use the formula to find the diameter.Then find the radius.

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ 35x &= \pi d && \text{Substitution} \\ \frac{35x}{\pi} &= d && \text{Divide each side by } \pi. \\ 11.14x &\approx d && \text{Use a calculator.} \end{aligned}$$

$$\begin{aligned} r &= \frac{1}{2}d && \text{Radius Formula} \\ &\approx \frac{1}{2}(11.14x) && d \approx 11.14x \\ &\approx 5.57x && \text{Use a calculator.} \end{aligned}$$

Therefore, the diameter is about 11.14x centimeters and the radius is about 5.57x centimeters.

ANSWER:

11.14x cm; 5.57x cm

39. $r = \frac{x}{8}$, $d =$ ___, $C =$ __?__

SOLUTION:

The diameter is twice the radius and the circumference C of a circle with diameter d is given by $C = \pi d$.

$$\begin{aligned} d &= 2r && \text{Diameter Formula} \\ &= 2\left(\frac{x}{8}\right) && r = \frac{x}{8} \\ &= \frac{x}{4} \text{ or } 0.25x && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} C &= \pi d && \text{Circumference Formula} \\ &= \pi(0.25x) && d = 0.25x \\ &\approx 0.79x && \text{Use a calculator.} \end{aligned}$$

So, the diameter is 0.25x units and the circumference is about 0.79x units.

ANSWER:

0.25x; 0.79x

Determine whether the circles in the figures below appear to be *congruent, concentric,* or *neither*.

40. Refer to the photo on page 703.

SOLUTION:

The circles have the same center and they are in the same plane. So, they are concentric circles.

ANSWER:

concentric

41. Refer to the photo on page 703.

SOLUTION:

The circles neither have the same radius nor are they concentric.

ANSWER:

neither

42. Refer to the photo on page 703.

SOLUTION:

The circles appear to have the same radius. So, they appear to be congruent.

ANSWER:

congruent

43. **HISTORY** The *Indian Shell Ring* on Hilton Head Island approximates a circle. If each unit on the coordinate grid represents 25 feet, how far would someone have to walk to go completely around the ring? Round to the nearest tenth.

SOLUTION:

The radius of the circle is 3 units, that is 75 feet. So, the diameter is 150 ft. The circumference *C* of a circle with diameter *d* is given by $C = \pi d$.

So, the circumference is $C = \pi(150) \approx 471.2$ ft. Therefore, someone have to walk about 471.2 feet to go completely around the ring.

ANSWER:

471.2 ft

44. **CCSS MODELING** A brick path is being installed around a circular pond. The pond has a circumference of 68 feet. The outer edge of the path is going to be 4 feet from the pond all the way around. What is the approximate circumference of the path? Round to the nearest hundredth.

SOLUTION:

The circumference C of a circle with radius r is given by $C = 2\pi r$. Here, C_{pond} = 68 ft. Use the formula to find the radius of the pond.

$$\begin{aligned} C_{\text{pond}} &= 2\pi r && \text{Circumference Formula} \\ 68 &= 2\pi r && \text{Substitution} \\ \frac{68}{2\pi} &= r && \text{Divide each side by } 2\pi. \\ 10.82 &\approx r && \text{Use a calculator.} \end{aligned}$$

So, the radius of the pond is about 10.82 feet and the radius to the outer edge of the path will be 10.82 + 4 = 14.82 ft. Use the radius to find the circumference of the brick path.

$$\begin{aligned} C_{\text{path}} &= 2\pi r && \text{Circumference Formula} \\ &\approx 2\pi(14.82) && r \approx 14.82 \\ &\approx 93.13 && \text{Use a calculator.} \end{aligned}$$

Therefore, the circumference of the path is about 93.13 feet.

ANSWER:

93.13 ft

45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore changing dimensions in circles.

a. GEOMETRIC Use a compass to draw three circles in which the scale factor from each circle to the next larger circle is 1 : 2.

b. TABULAR Calculate the radius (to the nearest tenth) and circumference (to the nearest hundredth) of each circle. Record your results in a table.

c. VERBAL Explain why these three circles are geometrically similar.

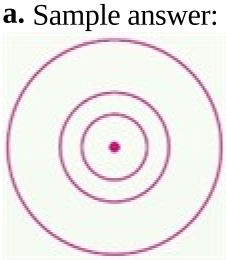
d. VERBAL Make a conjecture about the ratio between the circumferences of two circles when the ratio between their radii is 2.

e. ANALYTICAL The scale factor from $\odot A$ to $\odot B$ is $\frac{b}{a}$. Write an equation relating the circumference (C_A) of $\odot A$ to the circumference (C_B) of $\odot B$.

f. NUMERICAL If the scale factor from $\odot A$ to $\odot B$ is $\frac{1}{3}$, and the circumference of $\odot A$ is 12 inches, what is the circumference of $\odot B$?

SOLUTION:

a. Sample answer:



b. Sample answer.

Circle Radius (cm)	Circumference (cm)
0.5	3.14
1	6.28
2	12.57

c. They all have the same shape–circular.
d. The ratio of their circumferences is also 2.
e. The circumference is directly proportional to radii. So, if the radii are in the ratio $b : a$, so will the circumference.

Therefore, $(C_B) = \frac{b}{a}(C_A)$.

f. Use the formula from part **e.** Here, the circumference of $\odot A$ is 12 inches and $\frac{b}{a} = \frac{1}{3}$.

$(C_B) = \frac{b}{a}(C_A)$ Circumference Proportion

$C_B = \frac{1}{3}(12)$ Substitution

$C_B = 4$ Simplify.

Therefore, the circumference of $\odot B$ is 4 inches.

ANSWER:

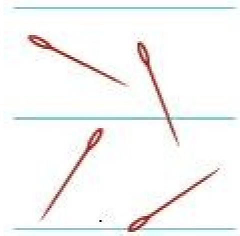
a. Sample answer:

b. Sample answer.

Circle Radius (cm)	Circumference (cm)
0.5	3.14
1	6.28
2	12.57

c. They all have the same shape—circular.
d. The ratio of their circumferences is also 2.
e. $(C_B) = \frac{b}{a}(C_A)$
f. 4 in.

46. BUFFON'S NEEDLE Measure the length ℓ of a needle (or toothpick) in centimeters. Next, draw a set of horizontal lines that are ℓ centimeters apart on a sheet of plain white paper.



a. Drop the needle onto the paper. When the needle lands, record whether it touches one of the lines as a hit. Record the number of hits after 25, 50, and 100 drops.

b. Calculate the ratio of two times the total number of drops to the number of hits after 25, 50, and 100 drops.

c. How are the values you found in part **b** related to π?

SOLUTION:

a. See students' work.
b. See students' work.
c. Sample answer: The values are approaching 3.14, which is approximately equal to π.

ANSWER:

a. See students' work.
b. See students' work.
c. Sample answer: The values are approaching 3.14, which is approximately equal to π.

47. **MAPS** The concentric circles on the map below show the areas that are 5, 10, 15, 20, 25, and 30 miles from downtown Phoenix.

a. How much greater is the circumference of the outermost circle than the circumference of the center circle?
b. As the radii of the circles increases by 5 miles, by how much does the circumference increase?

SOLUTION:

a. The radius of the outermost circle is 30 miles and that of the center circle is 5 miles. Find the circumference of each circle.

$$\begin{aligned} C_{\text{outer}} &= 2\pi r && \text{Circumference Formula} \\ &= 2\pi(30) && r_{\text{outer}} = 30 \\ &= 60\pi && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} C_{\text{center}} &= 2\pi r && \text{Circumference Formula} \\ &= 2\pi(5) && r_{\text{center}} = 5 \\ &= 10\pi && \text{Simplify.} \end{aligned}$$

The difference of the circumferences of the outermost circle and the center circle is $60\pi - 10\pi = 50\pi$ or about 157.1 miles.

b. Let the radius of one circle be x miles and the radius of the next larger circle be $x + 5$ miles. Find the circumference of each circle.

$$\begin{aligned} C &= 2\pi r && \text{Circumference Formula} \\ &= 2\pi(x) && r = x \\ &= 2x\pi && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} C &= 2\pi r && \text{Circumference Formula} \\ &= 2\pi(x+5) && r = x+5 \\ &= 2x\pi + 10\pi && \text{Simplify.} \end{aligned}$$

The difference of the two circumferences is $(2x\pi + 10\pi) - (2x\pi) = 10\pi$ or about 31.4 miles.
Therefore, as the radius of each circle increases by 5 miles, the circumference increases by about 31.4 miles.

ANSWER:

a. 157.1 mi
b. by about 31.4 mi

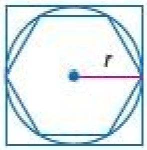
48. **WRITING IN MATH** How can we describe the relationships that exist between circles and lines?

SOLUTION:

Sample answer: A line and a circle may intersect in one point, two points, or may not intersect at all. A line that intersects a circle in one point can be described as a tangent. A line that intersects a circle in exactly two points can be described as a secant. A line segment with endpoints on a circle can be described as a chord. If the chord passes through the center of the circle, it can be described as a diameter. A line segment with endpoints at the center and on the circle can be described as a radius.

ANSWER:

Sample answer: A line that intersects a circle in one point can be described as a tangent. A line that intersects a circle in exactly two points can be described as a secant. A line segment with endpoints on a circle can be described as a chord. If the chord passes through the center of the circle, it can be described as a diameter. A line segment with endpoints at the center and on the circle can be described as a radius.

49. 

REASONING In the figure, a circle with radius r is inscribed in a regular polygon and circumscribed about another.

a. What are the perimeters of the circumscribed and inscribed polygons in terms of r? Explain.

b. Is the circumference C of the circle greater or less than the perimeter of the circumscribed polygon? the inscribed polygon? Write a compound inequality comparing C to these perimeters.

c. Rewrite the inequality from part **b** in terms of the diameter d of the circle and interpret its meaning.

d. As the number of sides of both the circumscribed and inscribed polygons increase, what will happen to the upper and lower limits of the inequality from part **c**, and what does this imply?

SOLUTION:

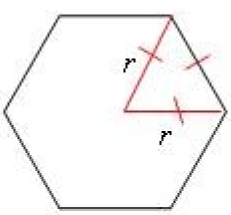
a. The circumscribed polygon is a square with sides of length $2r$.

$P = 4s$ Perimeter formula for square

$= 4(2r)$ Substitution

$= 8r$ Multiply.

The inscribed polygon is a hexagon with sides of length r. (Each triangle formed using a central angle will be equilateral.)



$P = 6s$ Perimeter formula for regular hexagon

$= 6r$ Substitution

So, the perimeters are $8r$ and $6r$.

b.

$C = 2\pi r$ Circumference Formula

$\approx 6.28\pi$ Multiply.

The circumference is less than the perimeter of the circumscribed polygon and greater than the perimeter of the inscribed polygon.

$8r < C < 6r$

c. Since the diameter is twice the radius, $8r = 4(2r)$ or $4d$ and $6r = 3(2r)$ or $3d$.

The inequality is then $4d < C < 3d$. Therefore, the circumference of the circle is between 3 and 4 times its diameter.

d. These limits will approach a value of πd, implying that $C = \pi d$.

ANSWER:

a. $8r$ and $6r$; Twice the radius of the circle, $2r$ is the side length of the square, so the perimeter of the square is 4 $(2r)$ or $8r$. The

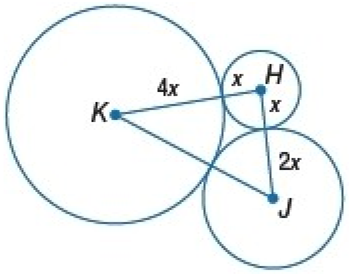
regular hexagon is made up of six equilateral triangles with side length r, so the perimeter of the hexagon is $6(r)$ or $6r$.

b. less; greater; $6r < C < 8r$

c. $3d < C < 4d$; The circumference of the circle is between 3 and 4 times its diameter.

d. These limits will approach a value of πd, implying that $C = \pi d$.

50. **CHALLENGE** The sum of the circumferences of circles *H*, *J*, and *K* shown at the right is 56π units. Find *KJ*.



SOLUTION:

The radius of $\odot K$ is 4*x* units, the radius of $\odot H$ is *x* units, and the radius of $\odot J$ is 2*x* units. Find the circumference of each circle.

$$\begin{aligned} C_{\odot K} &= 2\pi r && \text{Circumference Formula} \\ &= 2\pi(4x) && \text{Substitution} \\ &= 8\pi x && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} C_{\odot H} &= 2\pi r && \text{Circumference Formula} \\ &= 2\pi(x) && \text{Substitution} \\ &= 2\pi x && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} C_{\odot J} &= 2\pi r && \text{Circumference Formula} \\ &= 2\pi(2x) && \text{Substitution} \\ &= 4\pi x && \text{Simplify.} \end{aligned}$$

The sum of the circumferences is 56π units. Substitute the three circumferences and solve for *x*.

$$\begin{aligned} 8\pi x + 4\pi x + 2\pi x &= 56\pi && \text{Sum Equation} \\ 14\pi x &= 56\pi && \text{Add like terms.} \\ \frac{14\pi x}{14\pi} &= \frac{56\pi}{14\pi} && \text{Divide each side by } 14\pi. \\ x &= 4 && \text{Simplify.} \end{aligned}$$

So, the radius of is 4 units, the radius of is 2(4) or 8 units, and the radius of is 4(4) or 16 units.
The measure of *KJ* is equal to the sum of the radii of and .
Therefore, *KJ* = 16 + 8 or 24 units.

ANSWER:

24 units

51. **REASONING** Is the distance from the center of a circle to a point in the interior of a circle *sometimes, always,* or *never* less than the radius of the circle? Explain.

SOLUTION:

A radius is a segment drawn between the center of the circle and a point on the circle. A segment drawn from the center to a point inside the circle will always have a length less than the radius of the circle.

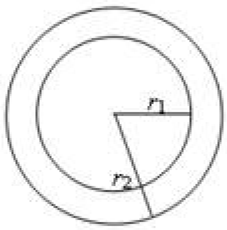
ANSWER:

Always; a radius is a segment drawn between the center of the circle and a point on the circle. A segment drawn from the center to a point inside the circle will always have a length less than the radius of the circle.

52. **PROOF** Use the locus definition of a circle and dilations to prove that all circles are similar.

SOLUTION:

A circle is a locus of points in a plane equidistant from a given point. For any two circles $\odot A$ and $\odot B$, there exists a translation that
maps center A onto center B, moving $\odot A$ so that it is concentric with $\odot B$. There also exists a dilation with scale factor k such
that each point that makes up $\odot A$ is moved to be the same distance from center A as the points that make up $\odot B$ are from center B.



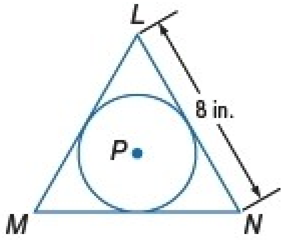
For some value k, $r_2 = kr_1$.

Therefore, $\odot A$ is mapped onto $\odot B$. Since there exists a rigid motion followed by a scaling that maps $\odot A$ onto $\odot B$, the circles are
similar. Thus, all circles are similar.

ANSWER:

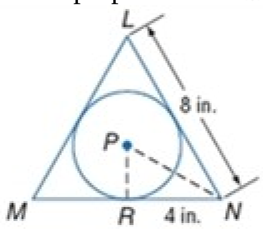
A circle is a locus of points in a plane equidistant from a given point. For any two circles $\odot A$ and $\odot B$, there exists a translation that maps center A onto center B, moving $\odot A$ so that it is concentric with $\odot B$. There also exists a dilation with scale factor k such that each point that makes up $\odot A$ is moved to be the same distance from center A as the points that make up $\odot B$ are from center B. Therefore, $\odot A$ is mapped onto $\odot B$. Since there exists a rigid motion followed by a scaling that maps $\odot A$ onto $\odot B$, the circles are similar. Thus, all circles are similar.

53. **CHALLENGE** In the figure, $\odot P$ is inscribed in equilateral triangle *LMN*. What is the circumference of $\odot P$?



SOLUTION:

Draw a perpendicular $\overline{PR}$ to the side $\overline{MN}$. Also join $\overline{PN}$.



Since ΔLMN is equilateral triangle, *MN* = 8 inches and *RN* = 4 inches. Also, $\overline{PN}$ bisects $\angle N$, so $m\angle PNR = 30°$. Use the definition of tangent of an angle to find *PR*, which is the radius of $\odot P$.

$$\tan 30° = \frac{PR}{4}$$

$$PR = 4 \cdot \frac{1}{\sqrt{3}} \text{ or } \frac{4\sqrt{3}}{3}$$

Therefore, the radius of $\odot P$ is $\frac{4\sqrt{3}}{3}$.

The circumference *C* of a circle with radius *r* is given by $C = 2\pi r$.

$$\begin{aligned} C &= 2\pi r && \text{Circumference Formula} \\ &= 2\pi\left(\frac{4\sqrt{3}}{3}\right) && \text{Substitution} \\ &= \frac{8\sqrt{3}\pi}{3} && \text{Simplify.} \end{aligned}$$

The circumference of $\odot P$ is $\frac{8\sqrt{3}\pi}{3}$ inches.

ANSWER:

$\frac{8\pi}{\sqrt{3}}$ or $\frac{8\pi\sqrt{3}}{3}$ in.

54. **WRITING IN MATH** Research and write about the history of pi and its importance to the study of geometry.

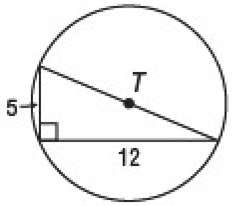
SOLUTION:

See students' work.

ANSWER:

See students' work.

55. **GRIDDED RESPONSE** What is the circumference of $\odot T$? Round to the nearest tenth.



SOLUTION:

Use the Pythagorean Theorem to find the diameter of the circle.

$d = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$

The circumference *C* of a circle with diameter *d* is given by $C = \pi d$.
Use the formula to find the circumference.

$$C = \pi(13)$$
$$= 13\pi$$
$$\approx 40.8 \text{ units}$$

ANSWER:

40.8

56. What is the radius of a table with a circumference of 10 feet?

A 1.6 ft
B 2.5 ft
C 3.2 ft
D 5 ft

SOLUTION:

The circumference *C* of a circle with radius *r* is given by $C = 2\pi r$.
Here, *C* = 10 ft. Use the formula to find the radius.

$C = 2\pi r$ Circumference Formula

$10 = 2\pi r$ Substitution

$\frac{10}{2\pi} = r$ Divide each side by 2π.

$1.6 \approx r$ Use a calculator.

The radius of the circle is about 1.6 feet, so the correct choice is A.

ANSWER:

A

57. **ALGEBRA** Bill is planning a circular vegetable garden with a fence around the border. If he can use up to 50 feet of fence, what radius can he use for the garden?
F 10
G 9
H 8
J 7

SOLUTION:

The circumference *C* of a circle with radius *r* is given by $C = 2\pi r$.
Because 50 feet of fence can be used to border the garden, $C_{max} = 50$. Use the circumference formula to find the maximum radius.

$$C_{max} = 2\pi r \quad \text{Circumference Formula}$$
$$50 = 2\pi r \quad \text{Substitution}$$
$$\frac{50}{2\pi} = r \quad \text{Divide each side by } 2\pi.$$
$$7.96 \approx r \quad \text{Use a calculator.}$$

Therefore, the radius of the garden must be less than or equal to about 7.96 feet. Therefore, the only correct choice is J.

ANSWER:
J

58. **SAT/ACT** What is the radius of a circle with an area of $\frac{\pi}{4}$ square units?

A 0.4 units
B 0.5 units
C 2 units
D 4 units
E 16 units

SOLUTION:

The area of a circle of radius *r* is given by the formula $A = \pi r^2$. Here $A = \frac{\pi}{4}$ square units. Use the formula to find the radius.

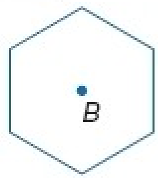
$$\pi r^2 = A \quad \text{Area of Circle Formula}$$
$$\pi r^2 = \frac{\pi}{4} \quad \text{Substitution}$$
$$r^2 = \frac{1}{4} \quad \text{Divide each side by } \pi.$$
$$r = \frac{1}{2} \text{ or } 0.5 \quad \text{Take the positive square root of each side.}$$

Therefore, the correct choice is B.

ANSWER:
B

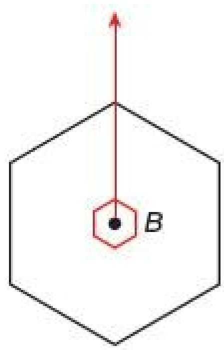
Copy each figure and point *B*. Then use a ruler to draw the image of the figure under a dilation with center *B* and the scale factor *r* indicated.

59. $r = \frac{1}{5}$

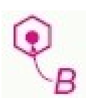


SOLUTION:

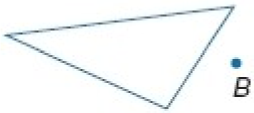
Draw a ray from point *B* through a vertex of the hexagon. Use the ruler to measure the distance from *B* to the vertex. Mark a point on the ray that is $\frac{1}{5}$ of that distance from *B*. Repeat the process for rays through the other five vertices. Connect the six points on the rays to construct the dilated image of the hexagon.



ANSWER:

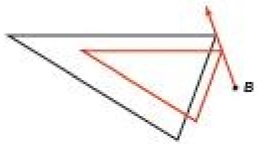


60. $r = \frac{2}{5}$

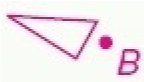


SOLUTION:

Draw a ray from point *B* through a vertex of the triangle. Use the ruler to measure the distance from *B* to the vertex. Mark a point on the ray that is $\frac{2}{3}$ of that distance from *B*. Repeat the process for rays through the other two vertices. Connect the three points on the rays to construct the dilated image of the triangle.



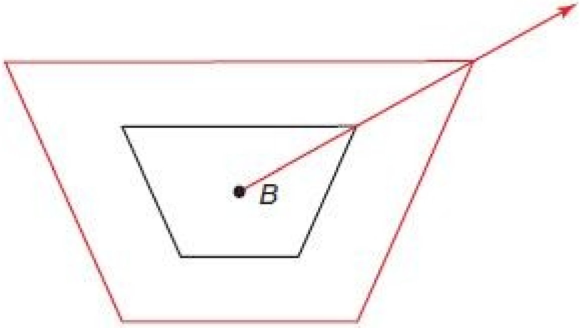
ANSWER:



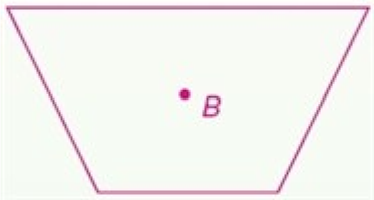
61. $r = 2$

SOLUTION:

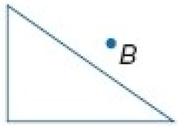
Draw a ray from point *B* through a vertex of the trapezoid. Use the ruler to measure the distance from *B* to the vertex. Mark a point on the ray that is 2 times that distance from *B*. Repeat the process for rays through the other three vertices. Connect the four points on the rays to construct the dilated image of the trapezoid.



ANSWER:

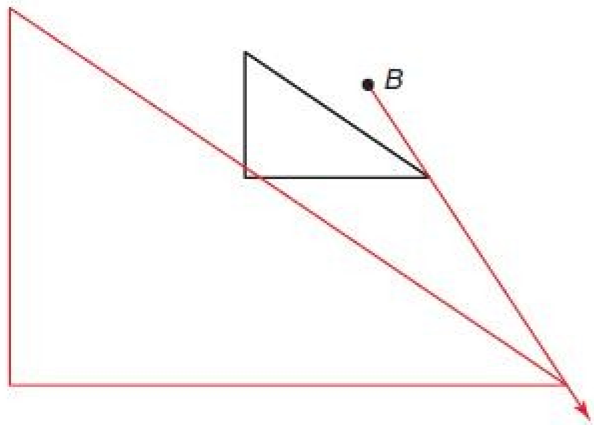


62. $r = 3$

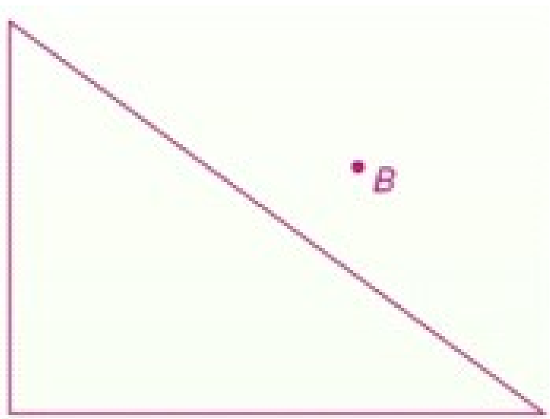


SOLUTION:

Draw a ray from point *B* through a vertex of the triangle. Use the ruler to measure the distance from *B* to the vertex. Mark a point on the ray that is 3 times that distance from *B*. Repeat the process for rays through the other two vertices. Connect the three points on the rays to construct the dilated image of the triangle.



ANSWER:



State whether each figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

63. Refer to the image on page 705.

SOLUTION:

The figure cannot be mapped onto itself by any rotation. So, it does not have a rotational symmetry.

ANSWER:

No

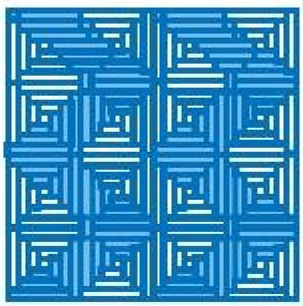
64. Refer to the image on page 705.

SOLUTION:

The figure can be mapped onto itself by a rotation of 90º. So, it has a rotational symmetry. So, the magnitude of symmetry is 90º and order of symmetry is 4.

ANSWER:

Yes; 4, 90

65. 

SOLUTION:

The given figure cannot be mapped onto itself by any rotation. So, it does not have a rotational symmetry.

ANSWER:

No

66. Refer to the image on page 705.

SOLUTION:

The given figure cannot be mapped onto itself by any rotation. So, it does not have a rotational symmetry.

ANSWER:

No

Determine the truth value of the following statement for each set of conditions. Explain your reasoning.
If you are over 18 years old, then you vote in all elections.

67. You are 19 years old and you vote.

SOLUTION:

True; sample answer: Since the hypothesis is true and the conclusion is true, then the statement is true for the conditions.

ANSWER:

True; sample answer: Since the hypothesis is true and the conclusion is true, then the statement is true for the conditions.

68. You are 21 years old and do not vote.

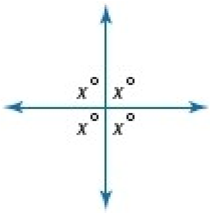
SOLUTION:

False; sample answer: Since the hypothesis is true and the conclusion is false, then the statement is false for the conditions.

ANSWER:

False; sample answer: Since the hypothesis is true and the conclusion is false, then the statement is false for the conditions.

Find *x*.

69. 

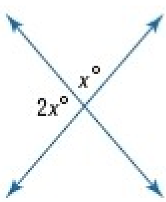
SOLUTION:

The sum of the measures of angles around a point is 360º.

$$x° + x° + x° + x° = 360°$$
$$4x = 360$$
$$x = 90$$

ANSWER:

90

70. 

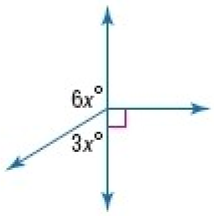
SOLUTION:

The angles with the measures *x*º and 2*x*º form a straight line. So, the sum is 180.

$$x° + 2x° = 180°$$
$$3x = 180$$
$$x = 60$$

ANSWER:

60

71. 

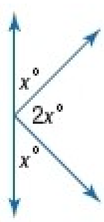
SOLUTION:

The angles with the measures $6x$º and $3x$º form a straight line. So, the sum is 180.

$$6x° + 3x° = 180°$$
$$9x = 180$$
$$x = 20$$

ANSWER:

20

72. 

SOLUTION:

The sum of the three angles is 180.

$$x° + 2x° + x° = 180°$$
$$4x = 180$$
$$x = 45$$

ANSWER:

45